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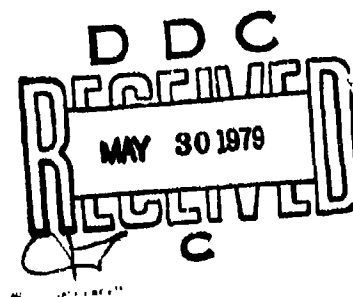
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INVESTIGATION OF STRESS-STRAIN
HISTORY MODELING AT STRESS RISERS

PHASE II

LOCKHEED-GEORGIA COMPANY
MARIETTA, GEORGIA 30063



December 1978

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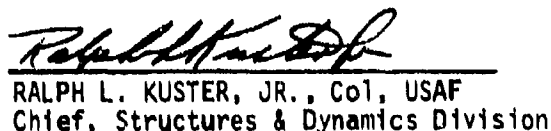
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20. ABSTRACT (Continued)

→ A four-part experimental program was conducted to generate constitutive data necessary for the formulation of a hysteresis analysis model. The experimental program included simple coupon specimens, a unique simplified stress concentration specimen, center circularly notched super-scale specimens, and notched fatigue specimens. Complex, load time test sequences and flight-by-flight spectrum loadings representative of transport aircraft loads were used in the program.

- ↳ Significant creep and stress relaxation was measured during the experimental program. These data were used in the development of creep/stress relaxation module for the hysteresis analysis. The automated hysteresis analysis developed during this program includes a material hardening/softening module, a creep/stress relaxation module, locus and branch curve definition for the stable material response, and a damage accumulation module. Correlation studies have been conducted using this analysis as well as a linear damage analysis method to compare predicted versus actual specimen life. ←

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FOREWORD

The research and development reported herein was conducted under Air Force Contract F33615-75-C-3078 by the Lockheed-Georgia Company, a Division of Lockheed Corporation. The work was sponsored by the Air Force Flight Dynamics Laboratory, Air Force Systems Command, Wright-Patterson Air Force Base, Ohio. This report covers the work conducted during the period of February, 1977, through October, 1978. This research was initiated under the direction of Dr. Robert L. Neulieb of the Fatigue, Fracture, and Reliability Group, Structural Integrity Branch, (FBE), Structural Mechanics Division as the Project Engineer. The program was completed under the direction of Mr. Robert M. Engle of the AFFDL/FBE as the Project Engineer. This report covers Phase II of the program conducted during the period of 15 February 1977 to 30 September 1978. The work was performed under Project 1367, Task 03, Work Unit 11.

This report was submitted for publication in December, 1978. Mr. James R. Carroll was the Lockheed-Georgia Company Program Manager. Finite element analysis was conducted by Dr. R. L. Brugh and Dr. L. S. Long. Mr. R. W. Wilkinson and Mr. L. W. Liu were responsible for the hysteresis analysis development, and Mr. J. F. Holliday conducted the computer programming. The experimental program was conducted by Mr. G. J. Gilbert and Mr. F. L. Amend.

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SECTION I

INTRODUCTION

Structural cracking is a major factor in assessing the useful life of aircraft structures. The potential crack initiation sites must be accurately identified and assessed, usually at or near stress concentrations such as fastener holes. Numerous analysis methods are available to predict crack initiation; however, few of these techniques consider the complete stress-strain history and the effects of plasticity on this history. Recent studies, including those in Phase I of this program and reported in Reference 1, have shown that plasticity induced by overloads produces a life-lengthening effect in the structures, but this plastic zone also undergoes time-dependant changes under sustained loading conditions. These time-dependent creep and stress relaxation changes can be a predominant factor in predicting time to initial cracking, crack growth, and in establishing test requirements and procedures.

The research reported here is an extension of the Phase I program that includes an analytical and experimental effort to develop an empirical analysis for stress-strain history accountability. An elastic-plastic finite element code simulation was used to model the stress-strain history and creep at the stress concentration. A four-part experimental program was conducted to collect material constitutive data necessary for the formulation of a hysteresis stress-strain analysis. This analysis method includes: an algorithm for the basic material response, a cyclic hardening or softening module, creep and stress relaxation, and a damage accumulation model. It is a very useful tool for the analyst in defining the stress-strain history and for predicting time to initial cracking.

SECTION II

PROGRAM OBJECTIVES AND SUMMARY

This program had as its main objective the development of an empirical analysis to model the stress-strain history at stress risers. It includes a study of the effects of load and mechanically induced plastic yielding at the stress riser; variations in the plastically induced stress-strain field resulting from sustained loading, constant amplitude cycles, and temperatures; and the effects of loading history on the stress-strain curve. A two phase analytical and experimental program was conducted to develop data necessary to formulate a hysteresis type stress-strain analysis method.

In Phase I, it was demonstrated that complex loading histories which include sufficiently high loads to induce plasticity and sustained loadings do, in fact, significantly affect the stress-strain history and the time to initial cracking of the structure. Experiments were conducted to measure creep at the stress riser. Based on these experimental data, it was hypothesized that the stress-strain history modeling is a complex function of both stress (relaxation) and creep (strain). Phase II included finite-element code simulation of the observed phase I results as well as an experimental program to develop constitutive data necessary to the formulation of the hysteresis analysis.

The results of the Phase I program are published in AFFDL-TR-76-150, Reference 1. This report documents the results of Phase II. For completeness, however, a brief summary of Phase I is included in this section.

2.1 PHASE I - SUMMARY

An extensive experimental program was conducted during this phase to evaluate the effects of overloads, sustained load hold periods, constant amplitude cycle block size, and temperature on the time to initial cracking of centrally notched, open hole test specimens. These data were then used in the formulation of a hysteresis type of analysis which includes modules for both the transient and stable stress-strain state.

Typical results from the complex load, time, temperature testing which was conducted are illustrated in Tables 1 and 2. Thirty-two different sequences were included. These

can be grouped into similar sequences as illustrated by A, B, C, and D in Table 1. Sequence A is the baseline constant amplitude test. Sequences B, C, and D include tensile overloads, compression underloads, 24-hour and 1-hour hold periods, and 15 thousand and one thousand cycle block size between overloads. The table illustrates the effect of these variables on time to crack initiation. The baseline constant amplitude cycling demonstrated, for example, 48,000 cycles to crack initiation. Tensile overloads as applied in Sequence B greatly extend the specimen life. However, a compression load immediately following the tensile overload (Sequence C) reduces the impact of the overload. An even more significant effect is demonstrated in the Sequence D results. Here time-dependent effects become important. For example, a tensile overload in B resulted in a life greater than two-million cycles; the underload when added reduced this to 715,000 cycles, but, when the compression load was held the life was reduced to 141,000 cycles. Similar data for increased underloads are included in Table 2.

The tension overloads were sufficient to induce a plastic zone around the stress riser. A super-scale test specimen (with a 2.0-inch diameter center hole) and a unique strain transducer were used to measure strain changes (creep) in this plastic zone during these hold periods. These measured changes, although not large, are sufficient to affect the specimen life. The effect of the measured strain changes can be illustrated by the data shown in Figure 1. These data are from tests which do not include hold periods (they are of the Sequence C grouping in Table 1) but the cyclic limits and times to failure tend to illustrate the time-dependent changes in the plastic zone. For example, cycling between the limits A-A is similar to a sequence with no underload. Cycling between B-B is typical of Sequence B and limits C-C tend to show the effect of the hold time or Sequence D. Note that as the cyclic limits change from A to C there is both a change in stress and strain. From this observation of measured test data, it was hypothesized that there is a complex, time dependent relationship between stress and strain which tend to significantly affect time to crack initiation.

This hypothesis is illustrated in Figure 2. In this figure both a time dependent creep, $\Delta\epsilon$, and a time dependent stress relaxation, $\Delta\sigma$, are shown.

These results and the hypothesis proposed (Figure 2) led to the development of the program scope for Phase II. Additional analyses and tests were conducted in this phase to model the observed Phase I results as well as to develop constitutive data for the hysteresis analysis model.

2.2 PHASE II - SUMMARY

Phase II in itself was a two-part program. The first part of the program involved a finite-element code simulation of the elastic-plastic stress-strain history from the Phase I experimental data. Part B was both an analytical and experimental effort, again using finite-element simulations in the analysis. The experimental program included development of constitutive data for the creep and stress relaxation modeling, super-scale specimen testing, and spectrum fatigue tests of notched coupons.

Several available elastic-plastic finite element analysis programs were evaluated to determine the one most applicable for the stress-strain model. The MARC General Purpose Program was selected primarily because of its flexibility in user subroutines and its available creep module. This finite-element program was used to model simple plates, the super-scale specimen used for the complex sequence tests, and the three bar, simplified stress concentration (SSC) specimen. The analysis, using this finite element code, demonstrated that the Phase I elastic-plastic stress-strain history and the creep could be modeled.

Following the analysis demonstration, a four part experimental program was initiated. Simple unnotched coupons were tested to obtain creep and stress relaxation data. The simplified stress-concentration (SSC) specimen was used in complex loading sequences to evaluate creep and the associate plastic stress and stress relaxation. Additional super-scale specimens were then tested to expand the data base developed in Phase I. Finally, notched coupon specimens were tested using flight-by-flight spectrum fatigue loadings. The finite-element code was used to model the SSC specimen and super-scale specimens to compare the predicted and experimental results.

The experimental data have been used in formulating a hysteresis type of an analysis for predicting the stress-strain history at stress concentrations.

SECTION III

FINITE ELEMENT ANALYSIS

A finite-element analysis technique was used for modeling the creep/relaxation behavior for this contract. The first task of this phase was to select an acceptable existing finite-element computer program. A computer program developed by Lockheed Missiles and Space Company entitled "Non-Linear Elastic Plastic Structural Analysis Program (NEPSAP)" was first examined. It was determined that this program had several deficiencies which prohibited proper modeling of the creep/relaxation phenomena. The second program examined was the MARC General-Purpose Program. The results obtained from MARC are discussed in the following paragraphs.

3.1 MARC - GENERAL PURPOSE PROGRAM

The MARC general purpose finite-element program is designed to cover a wide spectrum of linear and/or nonlinear problems encountered in structural analysis. MARC has a large library of elements which can be used to model virtually any geometry that may be encountered. To provide additional flexibility, user subroutines are available in this program so that modifications may be made by the user. The user subroutines utilized in this study allowed user-specified input for both a particular creep law and work-hardening behavior.

3.1.1 Creep Formulation

An evaluation of various creep laws was made to determine the best mathematical representation for creep of the material in this study. A summary of the various creep formulations which were considered and the limitations of each is shown in Table 3. The Nutting, Hoff, Scott-Blair formulation was considered the most applicable for the program. A user subroutine, WKSLP, was written to represent this formulation and incorporated into the run stream of MARC. Constants for this subroutine were obtained from experimental tests. Figure 3 shows a comparison between the results obtained from this subroutine and those obtained from coupon tests for an axially loaded square plate.

3.1.2 MARC Program Modifications

A simple unnotched flat plate (Figure 4) which is fixed at one edge and loaded axially at the opposite edge was used initially to gain an understanding of the nonlinear capability in MARC. The standard version of MARC allows only a bilinear representation of the stress-strain curve. Figure 5 illustrates a typical stress-strain curve obtained from loading the simple plate for 1 1/2 cycles. The open symbols are computed stress-strain data points. A comparison between the finite element analysis results and a typical coupon tested during Phase I is illustrated in Figure 6. Correlation between the two is within acceptable limits for the initial tension loading; however, it is rather poor on unloading. The area where the correlation is the poorest possibly corresponds to values of notch stress and notch strain in elements immediately adjacent to a hole when the specimen is loaded in compression during a hold period. In order to better represent this phenomena, in-house IRAD funds were used to develop a capability in MARC to allow the user to define multi-linear representation of the tension and compression yield surfaces. In addition, the capability of defining a different yield point in tension and compression was added. This involved modifying twelve subroutines in MARC. Figure 7 is a typical example of the stress-strain curve obtained from MARC after this feature was added.

3.2 PLATE/LAYERED MODEL ANALYSIS

A layered model was developed to evaluate through the thickness variances in stress and strain. Figure 8 illustrates the simple model utilized for this evaluation. Edge A of this model is restrained in the Y-direction and Edge B is restrained in the X-direction. Also, the model is restrained along the bottom so that no vertical displacements (Z-axis) are possible. This model as restrained represents a four-layered plate with a rectangular hole in the center. The plate was loaded in the X-direction as shown in Figure 8.

The evaluation of a finite-element analysis obtained for this model indicated that through-the-thickness variances in stress and strain cannot be detected. At the region of highest stress concentration both the stress and strain varied through the thickness by less than 0.5 percent. For this evaluation, the loading was simple tension until the first element went plastic. In addition, the effects of creep were examined. The data obtained for Elements 1-2, 3-4 and 7-8 are shown in Table 4. In Figure 8, Element 1 is directly below Element 2. The same holds true for element pairs 3-4 and 7-8. In Table 4, Increment 0 is the maximum tension

load (or the start of the creep time) and increment 6 corresponds to the end of the 30-hour creep time. The stresses and strains are listed for the Gaussian integration point in each element. Elements 1 and 2, for example, show that there is essentially no difference in stress or strain at either Increment 0 or 6; i.e., before or after the hold period. The same is true for the other element pairs.

3.3 SUPER-SCALE SPECIMEN

Two finite element models were constructed of the super-scale specimen. Both models have the same geometry. The first utilized solid elements and the second used plate elements. These are illustrated in Figures 9 and 10. Since the structure had two axes of symmetry, only one-fourth of the structure was modeled with the proper boundary conditions. The grid points along Edge A were fixed in the X-direction and similarly those along Edge B in the Y-direction. The results obtained from these models are discussed in Section 6.1.2.

3.4 SIMPLIFIED STRESS CONCENTRATION MODEL

A finite element model was made of the simplified stress concentration specimen shown in Figure 11. The shaded area was modeled as shown in Figure 12. This could be accomplished since there are two axes of symmetry and the nodes along the grip line move uniformly. The boundary conditions consist of the nodes along Axis A fixed in the X-direction, the nodes along Axis B fixed in the Y-direction, and the nodes along the grip line are fixed in the Y-direction. In addition, the X displacements along the grip line were uniformly imposed. The results of this model are discussed in Section 6.1.2.

SECTION IV EXPERIMENTAL PROGRAM

The experimental program conducted here was to develop a sufficient understanding of the time dependent changes in the stress-strain field around central notches such that a creep and stress relaxation module could be developed for the hysteresis analysis. Testing was directed primarily to collecting constitutive data and to verify analytical predictions, both the finite element code simulations and the hysteresis analysis results. Four types of test specimens were used in the program. These include:

- Simple Bar Coupon Specimens

Coupon specimens were used to obtain constitutive data for creep and stress relaxation during sustained hold periods following tensile overloads.

- Simplified Stress Concentration Specimens (SSC)

Unique 3-bar, simplified stress concentration specimens were used to simulate the stress concentration of the center-notched super-scale specimen to evaluate creep, stress relaxation, and load shedding.

- Super-Scale Specimens

Center-notched super-scale specimens were used to verify finite-element code predictions of time-dependent strain variations.

- Center-Notched Coupon Specimens

Circular-notched coupon specimens were flight-by-flight spectrum fatigue tested using loading conditions representative of a transport/bomber wing lower surface. The tests were conducted to verify the predictions from the hysteresis analysis.

Each test series involved complex load-time test sequences representative of typical flight loadings for transport/bomber wing structures. The complexity of the test sequences varied from initial tensile overloads followed by step hold periods for the

simple bar specimens to the spectrum loadings, including hold periods, for the notched coupon specimens. Continuous recordings of load and strain were made during the first three test series. These data were used to determine material constants for the creep formulation and to verify analytical predictions.

Specific details of the four test series are discussed in paragraphs 4.1 through 4.4. The analytical model formulation is discussed in Section V. Data analysis and correlation studies are in Section VI.

4.1 COUPON CREEP TESTS

During the Phase I program it became evident that constitutive data would be necessary in the formulation of a creep-stress relaxation module for the hysteresis analysis model. Consequently, the initial experimental work in Phase II involved testing simple un-notched coupons and continuously measuring strain and/or stress changes during sustained load hold periods.

The specimen used in these tests is illustrated in Figure 13. These specimens and all specimens for subsequent tests in Phase II were made from the same 7075-T651 plates used to fabricate all the Phase I specimens. This was done in order to minimize plate-to-plate material properties variations that might exist. Since the magnitude of the strains being measured is small, care was taken to assure a minimum variation in material.

Twelve coupon specimens were tested. The loading conditions are identified in Table 5. Two replicates were tested for each of the six tests listed in the table for the total of twelve. A schematic of the loading conditions is illustrated in the sketch in the table. Tests 1 through 4 were run under automatic load control and strain data were recorded from extensometers attached to the specimen. Tests 5 and 6 were run in a strain control mode using feedback from the extensometer to automatically control the tests. All tests were run at laboratory ambient conditions.

Each specimen was initially loaded to a positive strain of 0.016 in/in (sufficient to produce plastic deformation) and then unloaded, and reloaded into compression for

three periods of sustained loading. The first sustained load hold was 24-hours. This was followed by two, one-hour periods; the first at a different stress/strain and the second at the initial stress/strain loading. The variation in load in each sequence was to evaluate the "memory" of prior loadings and any subsequent effects. Results from the finite-element analysis of the super-scale specimen were used to establish the test loading conditions. For example, the -50 ksi and -66 ksi are typical of the resultant stress in the most highly loaded element at the hole edge for the -7.9 and -15.8 ksi sustained compression loads.

Table 6 is a summary of the strain creep and stress relaxation data measured during the hold periods for each specimen. The data shown are the total strain or stress measured for each test loading condition. Stress-strain curves and creep/relaxation curves are illustrated in Figures 14 through 37 for these twelve tests. Figure 14, for example, is the stress-strain curve for test 1A and depicts the tensile loading to approximately 0.016 in/in followed by the sustained load hold periods at -50 ksi and -66 ksi. The stress here is calculated based on the specimen net section. During the initial 24-hour hold period at -50 ksi, 1100 μ in/in strain was measured. The stress was then changed to -66 ksi and held for one hour, then returned to -50 ksi for one hour. Approximately 540 μ in/in strain change was measured during the second hold period and zero change was measured during the final hold period at -50 ksi. The creep curves for this test specimen are shown in Figure 15.

Some of the observations that have been made from the data in Table 6 and the accompanying figures are listed below.

1. A significant amount of creep and stress relaxation was measured during these tests.
2. In each case the "primary" creep accounts for the largest percentage of the total measured. Seventy to eighty percent of the total strain or stress change occurs during the first hour of the sustained load hold period.

3. There appears to be a limiting value of creep which occurs, at least at stress levels above -40 ksi. For example, in Sequences 1 through 3, the maximum strain change averages 2000 $\mu\text{in/in}$. There is some variation from test to test which may be attributed to basic differences in specimen and material. The sequence of applied stress does not effect the total strain change measured. In Sequence 2, for example, slightly over 2000 $\mu\text{in/in}$ was measured at the initial -66 ksi hold period, while approximately the same total was measured in Sequence 3 at two stress levels. A similar trend is shown in Sequence 1, although the totals are somewhat less.
4. The creep and stress relaxation seen in these tests does not appear to be sequence dependent. That is, by increasing load, strain and stress changes continue to increase in the classical "primary-secondary" sense. On returning to a lower loading there is little or no change in the measured data.

The creep data from these coupon tests were used to determine the material constants β , p , and n for the creep formulation used in the finite-element analysis. Hoff's creep formulation was used in the MARC analysis; i.e.,

$$\epsilon^C = \beta \sigma^p t^n$$

where:

ϵ^C	= creep strain	
σ	= stress	
t	= time	
β	$= 1.120 \times 10^{-13}$	} material constants for 7075-T6511 plate
p	$= 2.11$	
n	$= 0.065$	

A comparison between predicted and measured creep strain is illustrated in Figures 38 through 40. The analytical curves use Hoff's formulation with the material constants listed above. The experimental data in these figures is the average of the two specimens

tested at each stress level. Agreement between the predictions and the experimental results are considered quite good, especially in the primary creep regime.

The significance of these creep and stress relaxation data is of course, in the formulation of a hysteresis analysis model. The data here do substantiate the hypothesis of the complex, time-dependent relationship between stress and strain and the effect on time to crack initiation which was illustrated previously in Figure 2. Use of these data in the formulation of a creep/stress relaxation module is discussed in Section 5.3.

4.2 SIMPLIFIED STRESS CONCENTRATION SPECIMEN

In all tests during the Phase I program and the coupon and super-scale testing conducted here, strain data could be collected with reasonable ease. However, plastic stress could not be measured directly. Since it has been hypothesized that the time-dependent effects on life are a complex function of both stress and strain, an experiment was necessary to measure stress and stress change. Burski and Hayes, Reference 2, had previously used a unique, three-member specimen to evaluate residual stress relaxation as a function of cyclic loading. This specimen was used here to evaluate the time-dependent changes in plastic stress.

This simplified stress concentration (SSC) specimen is illustrated in Figure 41. The specimen behaves like a notched coupon in that it will have plastic and elastic regions existing simultaneously when loaded axially. It is designed so that the center bar will yield while the two outer bars remain elastic. For this experiment, the specimen geometry is such that the elastic stress concentration is the same as the center notched super-scale specimens tested previously; i.e., approximately 2.43.

4.2.1 Theoretical Analysis

In the analysis of this specimen, it is assumed that the elongation is the same in all three bars when an external load is applied. This allows for the calculation of the plastic stresses in the center bar when only the elastic stresses in the two outer bars and the applied load are known. A schematic representation of the specimen, including a definition of terms used here, is illustrated in Figure 42. The basic assumption here is

$$\delta_1 = \delta_2 = \delta_3 \quad (1)$$

where,

$$\delta_1 = \frac{P_1 L_1}{A_1 E} \quad (2)$$

$$\delta_2 = \frac{P_2 L_2}{A_2 E} \quad (3)$$

$$\delta_a = \frac{P_3 l_a}{A_a E} \quad (4)$$

$$\delta_b = \frac{P_3 l_b}{A_b E} \quad (5)$$

$$\delta_c = \frac{P_3 l_c}{A_c E} \quad (6)$$

Also,

$$P_T = P_1 + P_2 + P_3 \quad (7)$$

or

$$P_3 = P_T - (P_1 + P_2) = P_T - 2P_1 \quad (8)$$

The elongation of the center bar can be written as,

$$\delta_3 = \delta_a + \delta_b + \delta_c \quad (9)$$

and by combining Equations 4, 5, and 9 and assuming $l_a = l_c$ and $A_a = A_c$, then,

$$\delta_3 = \frac{P_3}{E} \left(\frac{2l_a}{A_a} + \frac{l_b}{A_b} \right) \quad (10)$$

Since $P_1 = P_2$, and δ_1 and $\delta_2 = \delta_3$, Equation 10 can be written as,

$$\frac{P_1 l_1}{A_1} = P_T - 2P_1 \left(\frac{2l_a}{A_a} + \frac{l_b}{A_b} \right) \quad (11)$$

also,

$$\frac{P_1}{P_T} = \frac{P_2}{P_T} = \left(\frac{2l_a}{A_a} + \frac{l_b}{A_b} \right) \left(\frac{l_1}{A_1} + \frac{4l_a}{A_a} + \frac{2l_b}{A_b} \right) \quad (12)$$

And then,

$$\frac{P_3}{P_T} = \frac{l_1/A_1}{\left(\frac{l_1}{A_1} + \frac{4l_a}{A_a} + \frac{2l_b}{A_b} \right)} \quad (13)$$

Using the specimen geometry in Figure 41 and the following segment lengths for the center bar,

$$l_a = l_c = 2.09 \text{ in.}$$

$$l_b = 1.82 \text{ in.}$$

results in,

$$\frac{P_3}{P_T} = 0.380 \quad \frac{P_1}{P_T} = \frac{P_2}{P_T} = 0.31 \quad \frac{P_3}{P_1} = 1.23$$

The test sequences utilized in this program were such that the outer two bars did remain elastic and it was possible to calculate the center bar plastic stress from the recorded data. Strain gages and extensometers were used to measure strain in all three bars and the plastic stress calculated using the above relationships.

Stresses σ_1 and σ_2 were calculated directly from the measured elastic strain data. Where multiple measurements were made on any single bar, the average values were used. These outer bar stresses were then converted to loads (P_1 and P_2) and the center bar load calculated from Equation 8; i.e.,

$$P_3 = P_T - (P_1 + P_2)$$

This load was then converted to stress in the center bar by dividing by the center bar area, A_3 . The calculated center bar stress and changes in this stress during hold periods are then plotted versus strain and strain changes measured during the load sequences. A discussion of the loading sequences, instrumentation used, and data collected is in the following section.

4.2.2 Experimental Program

Test sequences for these SSC specimens are illustrated in Figure 43. Four sequences were developed to simulate in part some of the more typical super-scale tests conducted in Phase I. Initially, only creep and stress relaxation data were to be collected; however, the program was modified to also attempt to fatigue test two specimens. In general, the specimen does not lend itself to the magnitude of fatigue loads applied, but the creep and stress relaxation measurements were considered to be very successful.

Sequences 1 and 2 consisted of an initial overload followed by 24-hours of sustained compression loading. This was repeated in each sequence for a total of 48-hours of sustained loading as shown in the figure. Loading conditions for each sequence are included in Figure 43. The loads defined are the total applied loads, P_T discussed in 4.2.1. Sequences 1 and 2 are identical except for the magnitude of the compression load, P_{MIN} . Sequence 3, a fatigue test, consisted of the initial overload followed by a compression underload and then constant-amplitude cycling. The period between tension overloads, N_{OL} , was 1000 cycles of constant amplitude loading. The last sequence, Sequence 4, was identical except that the compression underload was sustained for one hour. Strain data listed in Figure 43 under Loading Conditions are measured center bar data. The variable strains listed for Sequences 3 and 4 illustrate the apparent cyclic strain changes measured during the fatigue cycling. This is discussed in subsequent sections.

Each SSC specimen was instrumented with seven strain gages and two extensometers and tested in a computer-controlled, closed-loop test machine. Instrumentation locations are shown in Figure 44. Specific test procedures and equipment systems are discussed in 4.5.

Two of the SSC specimens were tested under the Sequence 1 loading conditions. Some difficulties were encountered during the first test and, although this was not sufficient to negate the test, it was decided to test a second specimen. Data was recorded from the strain gages on the outer bar as well as the two extensometers shown in Figure 44. A sample of the stress-strain data print-out for the initial ramp to load and the 24-hour

hold period is shown in Table 7. This data is typical of that collected automatically during each SSC test and was used to calculate the center bar plastic stress and stress relaxation. The stress column in the table is the total net section stress across the center of the SSC specimen. These tests were load control and the stress shown is simply the total applied load divided by this total net area ($A_1 + A_3 + A_2$).

The specimen was loaded to the tension peak and unloaded to a predetermined compression stress for the sustained load hold period. Only segments of the data recorded during the hold period are included in the example. However, the time intervals selected were such that the "primary" changes in both strain and stress were documented. These data were then used along with the analysis method outline in 4.2.1 to calculate center bar stress. An average of the two gages, front and rear, was used in the data analysis. For the other two bars, the extensometer data was used directly.

The data from the two Sequence 1 tests are shown in Figures 45 through 48. A stress-strain curve for the elastic-plastic loading on the center bar is illustrated in Figures 45 and 47. Both the first and second load cycle, see Figure 43, and the two sustained load hold periods are shown in these figures. As noted earlier, these tests were considered to be very successful in regard to better defining the relationship between strain creep and stress relaxation. These changes, as measured and as calculated from the measured data, are illustrated in Figures 46 and 48 for these two specimens.

Calculated center bar stress relaxation is plotted versus measured center-bar strain creep in these two figures. Data for both the first and second loading cycles are presented. The initial stress-strain conditions at the start of each hold period (time = 0) are somewhat different for the two specimens. This may be attributed to material or specimen geometry variations, or uniformity in loading through the end grips. The second hold period on the repeat test was only 40-minutes due to a system malfunction; but overall sufficient data was collected to establish trends for the creep/relaxation analysis module development. Additional data was collected from Sequences 2 and 4.

Sequence 2 is a repeat of the first sequence; but the compression stress was less than that used on those tests. Similar data and trends were seen in this loading sequence.

Figure 49 is the stress-strain curve for this specimen. Again, two loading cycles were applied and creep and relaxation data were obtained during the two 24-hour hold periods. The center bar stress relaxation and creep are shown in Figure 50. Less creep was seen in this test than was measured from the Sequence 1 specimens. This follows the trend seen in the simple coupon tests which were discussed in 4.1.; that is, the lower the stress levels the less creep change measured.

Sequences 3 and 4 were intended to be primarily fatigue tests, but, as mentioned earlier, the specimen size, thickness, etc. did not lend itself to fatigue testing at the stress levels used here. This is not to say that the basic specimen should not be used for fatigue testing in general. A considerable amount of useful data was collected from these test sequences, however. Both sequences are defined in Figure 43.

Two test specimens were tested for Sequence 3. It was originally planned to apply 10,000 cycles, N_{OL} , between the tension-compression overloads; however, the first specimen tested failed after 5990 cycles. Only the initial overload cycles were applied. After this test, N_{OL} was reduced to 1000 cycles, primarily to allow for more overloads and subsequent hold periods which were to be applied in Sequence 4. The second specimen was then tested as shown in Figure 43 with 1000 cycles between overloads and failed in the center bar after 10,278 cycles.

The center bar stress-strain curve for the initial load cycles applied on the first Sequence 3 specimen is shown in Figure 51. No hold period was included to measure creep or relaxation, but this figure illustrates the specimen response from the minimum compression stress back to the positive mean stress prior to constant amplitude cycling. Typical data for the first, second, and tenth overload cycles applied on the second specimen are illustrated in Figure 52. These are plots of specimen net section stress versus center bar strain directly from the computer controlled test equipment.

These data plots are included here to illustrate an apparent cyclic creep which was observed during this test (also observed during the Sequence 4 testing). A time history of the center bar strain is illustrated in Figure 53. Both the mean strain and the

minimum strain are plotted versus the N_F/N_{OL} ratio in this figure. The mean strain is the mean value of the constant amplitude loading and the minimum strain is the strain measured during the application of each compression underload. N_F is the total number of applied cycles and N_{OL} is the number of cycles between overloads. The data shown are for the second specimen tested for Sequence 3. These same trends were observed during the super-scale specimen tests in Phase I and are reported in Reference 1. Several investigators have previously reported on cyclic creep and its impact on time to crack initiation. The data here are simply presented as trends in that there is insufficient data to incorporate a cyclic creep module in the hysteresis analysis program.

The final SSC test sequence was Sequence 4 (Figure 43). This was also a fatigue test and was similar to Sequence 3 except that one-hour hold periods were included. One specimen was tested and failed after 9000 cycles of constant amplitude loading during the application of the tenth tensile overload. Figure 54 is an illustration of the center bar strain data for loading cycles 1, 5, and 9. These are typical data plots obtained during the conduct of the test.

Stress relaxation and creep were measured during each sustained load hold period. Figure 55 shows this data for the first, fifth, and ninth hold periods. The trends toward a creep and relaxation saturation limit that was discussed in 4.1 for the coupon tests is also evident here. These data indicate a continuing decrease in both stress and strain change with cumulative time. The stress and strain changes shown in all these SSC specimen tests are sufficient to significantly influence subsequent loadings and the time to crack initiation. The development of the creep and stress relaxation model for the hysteresis analysis is discussed in 5.3.

One final set of data from the Sequence 4 test is illustrated in Figure 56. This is the history of the center bar strain changes which were recorded during the test. Again the mean strain and minimum strain changes are plotted versus the N_F/N_{OL} ratio. The "sawtooth" effect on the minimum strain curve reflects the creep which occurred during each sustained load hold period. This data also illustrates the decrease in the amount of creep per hold period with cumulative time. The discontinuities in the mean strain curve reflect measurements before and after each block of constant amplitude cycles.

4.3 SUPER-SCALE SPECIMEN TESTS

The center circularly notched super-scale test specimen used in Phase I for complex sequence testing was again utilized as a part of the experimental effort in this program phase. This specimen and the instrumentation used for strain measurement (strain gages and strain transducer) are illustrated in Figure 57. Ten loading sequences were included in this program. Tests were conducted to collect strain data in and around the center notch to evaluate the finite-element code predictions and update the constitutive data.

The specimen, as illustrated, is 8.0 inches wide with a 2.0-inch diameter hole in the center. It has a geometric stress concentration factor of 2.43. This scaled-up specimen is used in order to insert the Lockheed-developed strain transducer inside the hole for notch strain measurements. Specific details of the transducer design and function are reported in the Phase I final report, Reference 1. This transducer has a gage length of approximately 0.080-inch and provides a reliable means to continuously monitor notch strain. Four strain gages, as shown, were also installed on each specimen to measure strain immediately adjacent to the hole on the specimen surface.

The ten test sequences, loading conditions and cycles-to-failure are shown in Figure 58. Basically, this group of tests is a continuation of the Phase I super-scale test program and is intended to supplement and expand those data. All test sequences include an initial tensile overload equivalent to a specimen net section stress of 47.3 ksi. All constant amplitude cycling was conducted at a mean stress of 15 ksi and a variable stress of 10 ksi. Both compression underloads and compression underloads with sustained load hold periods follow the initial tensile overloads. Most of the test sequences included underloads at either -7.9 ksi or -4.0 ksi net section stress. The -7.9 ksi was the baseline in Phase I and was carried over here for additional tests. This stress level is typical for transport aircraft wing lower surfaces during on-ground operations as is the -4.0 ksi, which was added to the Phase II test matrix.

The first four sequences had only one tensile overload followed by 24 hours of sustained compression load. Then the specimens were constant amplitude-cycled to failure. Sequences 9 and 10 were similar but only had a single application of the compression

load and did not include the sustained compression load. Cyclic block sizes between overloads (N_{OL}) were 15,000 cycles and 30,000 cycles. The 15,000-cycle block size was the Phase I baseline condition. A maximum of five 24-hour hold periods were included in Sequences 5, 7, and 8. Two specimens were tested for Sequence 7, as shown in Figure 58; only three 24-hour hold periods were applied on the first specimen. Five hold periods were applied on the second.

As noted earlier, both creep and cycles-to-failure were recorded from these tests. The specimen life reported in Figure 58 is the cycles-to-total-specimen separation. No crack initiation times were recorded; however, experience from the Phase I program showed that crack growth under these loading conditions was only 1 - 2 percent of the cycles-to-rupture.

A comparison between these test results and some of the more pertinent and similar tests from the Phase I program is included in Table 8. In the table, Group 1 tests contain only sequences with overloads and underloads, whereas Group 2 includes the addition of the hold period in the sequence. The baseline constant amplitude cycling only is also included for comparison. The data illustrate the following basic points:

- o Tension overloads produce a life lengthening effect over the basic constant amplitude loading.
- o Compression underloads reduce the effect of the tension overload.
- o Cyclic block size affects the induced beneficial residual stresses which produce the life-lengthening effect.
- o Time dependent creep does alter the residual stress-strain state and significantly affects time to failure.

The overload/underload effects can be seen by comparing Sequence 1-6 with 1-8 and 11-6. In all three of these sequences, the overload and underload is repeated every 15,000 cycles. The repeated overloads tend to greatly extend the specimen life. A single

initial overload/underload (Sequences II-9 and II-10) does extend the life of the baseline but the overload contribution of extending life decays with continued constant amplitude cycling. This is illustrated by comparing Sequences II-9 and II-10 with I-8 and II-6, respectively. Another indication of this decay in the overload-induced beneficial residual stress/strain state can be seen in comparing the Group 2 specimens tested with N_{OL} 15,000 and 30,000 cycles. In each case, the 30,000-cycle block size results in the shorter specimen life.

In general, the compression hold periods at 0, -7.9, and -15.8 ksi net section stress follow expected trends. The data for the -4.0 ksi compression stress do not, however. For example, Sequence II-5 should have resulted in a shorter life than Sequence II-6, but the two specimens failed at approximately the same lifetime. Sequence II-5, with the compression hold period, actually had a longer life. It may be that the -4.0 compression stress does not sufficiently overcome the beneficial residual stresses induced by the tension overload at least within the 15,000 cycle block size between overloads. For example, in Sequence II-8 with $N_{OL} = 30,000$ cycles, there is a significant reduction in time-to-failure as compared to II-5 and II-6.

A similar trend has been observed in evaluating the effects of underloads on crack growth, Reference 3. In that program, various levels of compression underloads were applied immediately following tensile overloads, similar to the sequences here. Crack growth is retarded after the tensile overload due to the plastic zone formed at the crack tip. Compression load application tends to negate this effect; however, the magnitude of the compression load affects reinitiation of the crack growth. The lower the compression load, such as the -4.0 ksi here, the more constant amplitude cycles that must be applied in order to overcome the plastic zone effects. It appears that this same effect may hold true for crack initiation.

Creep strain was recorded during the compression hold periods from the transducer inside the hole and from the four strain gages immediately adjacent to the hole on the specimen surface. A summary of the measured strains is shown in Table 9. Plots of creep strain versus time are illustrated in Figures 59 through 61 for Sequences II-5, II-7, and II-8. The data shown are from the strain transducer located inside the hole.

In general, the measured data follow expected trends; however, the magnitude of the measured strain was less than expected. The transducer measurements for Sequence 4 appear to be high, but the instrumentation was checked and no problems were found. Sequence 1 also appears not to follow the expected trends in that the change inside the hole is in the positive sense where the changes on the specimen surface were negative (decreasing strain). This phenomena was also observed during the Phase I testing, however. Variations in the material properties and homogeneity from specimen to specimen apparently have a significant impact on the stress-strain history variability.

4.4 SPECTRUM FATIGUE TESTS

A series of flight-by-flight spectrum fatigue tests was run on center circularly notched coupon specimens to evaluate the data developed from the previously described tests and to evaluate the hysteresis fatigue analysis. The test sequences developed were representative of transport wing lower surface loadings. Typical flight and ground loads data were used to develop logistics and low-level mission spectra for these test sequences. Three test sequences were developed for this study. One sequence included periodic overloads followed by sustained load-hold periods. The test specimen used was a simple center notched (open hole) coupon with a geometric stress concentration of 2.43. Three specimens were tested for each loading sequence.

Typical mean and alternating flight and ground loads for a transport wing lower surface which were used in the test sequence development are included here in Figures 62 through 66. Three typical flights were developed from the loads data and then mixed to form the three test spectra. Flight A was derived from the logistics mission data. For each flight segment, four loads equal to the five times per flight load and one load equal to the once per flight load are included. Only one taxi load was included to obtain a compression unloading in the flight. Flight B was derived in the same manner, except that the low-level mission data was used. Flight C is identical to Flight B except for the inclusion of the tensile overload. The mean loads were selected to bracket the loadings used in the previous Phase I and II loading sequences, and the applied flight loads were adjusted for the appropriate means. The resulting flights are illustrated in Figure 67. The stresses shown in these figures are net section stresses.

These flights were combined into the following three spectra:

Spectrum 1

$$50 [C + 150(2A + B)] + N(2A + B)$$

That is, Flight C
A
A
B } Repeat 150 Times

Repeat 50 Times

⋮

Then,

A
A
B } Until Failure

Spectrum 2

$$50 [(C + \text{Hold}) + 150 (2A + B)] + N(2A + B)$$

This spectrum included a 1-hour sustained load hold period at -9.4 ksi after each application of Flight C.

Spectrum 3

$$50 [B + 150 (2A + B)] + N(2A + B)$$

A sample of a strip chart recording for Spectrum 2 is illustrated in Figure 68. The specimen life, in flights-to-failure, of each specimen tested is tabulated in Table 10.

These test results tend to confound this study in that no discrimination can be made between the results of the Spectrum 1 and 2 tests. It was anticipated, based on all the preceding tests, that there would be distinguishable differences between the spectra with and without the hold periods included. The periodic overloads included in Spectra 1 and 2 do result in a life extension effect, as compared to Spectrum 3 which does not have the overload. But, at least for the transport spectra used here, the compression load and hold period do not show the life reduction that was previously demonstrated.

This is not a conclusive test since it was limited to one typical spectrum for a transport aircraft, but the results are discouraging. The data base developed in the preceding tests and in Reference 3 did show a pronounced effect of underload and sustained load hold periods on both cycles to crack initiation and crack growth. It is possible that these effects do show up on an individual cycle; but, for a transport spectrum, the effect is not that significant and can be accounted for by a statistical smear across the spectrum.

4.5 GENERAL TEST METHODS

Four different types of tests were performed during the Phase II experimental evaluations; namely, superscale, single-bar coupon, three-bar simplified stress concentration, and spectrum fatigue tests. All testing was performed in modern electrohydraulic servo-controlled test systems manufactured by MTS Systems Corporation. Each test system was interfaced to a digital computer which was used to command the different profiles, monitor loads and strains, and perform failsafe functions. Additionally, the computer was used to store, reduce, tabulate, and plot data in a reportable format. The data collection, reduction and display functions operated in near real time as the test progressed. The heart of each system was a PDP-11 computer and a modified BASIC programming language which was interactive. Inputs and outputs were managed through a teletype terminal and CRT supported by a hard-copy unit. A typical system containing a three-bar SSC specimen is shown in Figure 69.

A user program was written in BASIC for each different test type, and each program contained options to accommodate the different profiles required. The superscale tests were performed under load control, and outputs from the load and strain transducers were recorded at programmed time intervals throughout the tests. The time intervals were controlled by a programmable clock which resulted in precise load-strain-time history data.

The single-bar and three-bar SSC tests were also performed under load control; however, the test system computer program was written so that loading was reversed once a desired value of strain was reached during the initial tensile loading. For the SSC tests, the load reversal was based on strain in the center bar. All subsequent events in the profiles were applied under unconditional load control. Single bar strains were measured using an extensometer. Strains for the three bar tests were measured using an extensometer on the center bar and one outer bar, and strain gages on the other outer bar. (As was done for the superscale tests, load and strain data were recorded at programmed time intervals throughout the tests.) Load and strain history data for a typical three-bar test are contained in Table 7. A summary plot of these data is shown in Figures 52 and 54. Both the data tabulations and plots represent copies from the test system CRT.

The spectrum fatigue tests were performed under load control with peak-to-peak load transition. An analog representation of the three different flights contained in the baseline spectrum is shown in Figure 68. Tests for two different variations of the baseline were also performed. One variation was the inclusion of 50, one-hour hold periods at the minimum load in Flight C. The other variation was to replace Flight C with Flight B which eliminated the high tensile overload.

SECTION V

HYSTERESIS ANALYSIS PROGRAM DEVELOPMENT

The experimental data collected during both Phases I and II have been used to formulate a rather comprehensive hysteresis type of an analysis. This analysis contains the following elements:

- (1) Notch Stress-Notch Strain Algorithm
 - o Locus Curves
 - o Branch Curves
 - o Neuber Analysis (Modified)
- (2) Material Hardening or Softening
- (3) Creep/Stress Relaxation
- (4) Fatigue Damage Computation

A detailed discussion of Items 1, 2, and 4 is included in the Phase I final report, Reference 1. The creep and stress relaxation module was developed in Phase II and a discussion of this formulation is included here. Specifics of the analysis program sub-routines, program input, etc. are included in Appendix A, Hysteresis Analysis Computer Program Description, and Appendix B, Hysteresis Analysis User's Guide.

5.1 NOTCH STRESS-NOTCH STRAIN ANALYSIS

The basic algorithm for the notch stress-notch strain calculations is illustrated in Figure 70. Phase I test data were used to formulate the locus curves and branch curves for the 7075-T651 material used here. For a material that has been cyclically hardened or softened it can be said that:

- (1) Stress and strain will always expand along the cyclic locus curve.
- (2) Excursions from the cyclic locus curve will follow branch curves.

The formulation and use of the locus curves and branch curves are discussed in detail in Reference 1. In Figure 70, a specimen can be loaded along the tension locus curve to a point A which is located at the intersection of the locus curve and the modified Neuber hyperbola. When the loading is reversed, the stress and strain will follow the branch curve, as illustrated, to the next loading condition.

Figure 71 is a typical example of the notch stress-notch strain calculations from the hysteresis analysis and its plotter option. The input loading sequence and the resultant stress-strain plot are illustrated here. These data are then used in the damage calculations within the program to predict structural life.

5.2 CYCLIC HARDENING AND SOFTENING

The rationale and data used in the development of the cyclic hardening and softening module are included in Reference 1 and in Appendix A. The hypothesis is based on the premise that an excursion along the tension locus or branch curves will harden the compression locus and branch curve. The cycles necessary to harden the material must be determined experimentally and will depend in part on the material, alloy, and strain level. For the 7075 used here, the number of cycles to fully harden the material varied from 4 cycles at a strain of 0.035 in/in to 50 cycles at 0.010 in/in. An example of the material hardening module in the analysis program is illustrated in Figure 72.

5.3 CREEP AND STRESS RELAXATION

The following two phenomenon were observed in the simple bar creep tests and the simplified stress concentration tests:

- (a) When a material is loaded to a local stress σ_A and allowed to creep and then further loaded to a local stress σ_B the creep at σ_B is less than if the material were loaded directly to σ_B and allowed to creep. This is illustrated in Figure 73.
- (b) When a material is loaded to a local stress σ_A and allowed to creep, and then the cycle is completely reversed and the material reloaded to σ_A and again allowed to creep, for equal hold periods the two creep levels will be the same, see Figure 74.

These observations form the basis for the creep algorithm used in the hysteresis stress-strain analysis.

The creep formulation credited to Nutting, Hoff, and Scott-Blair is used in the algorithm:

$$c_\epsilon = K \sigma_t^{p,n} \quad (1)$$

where

c_ϵ is the creep strain

σ is the current stress

t is the time in hours

$K, p, \& n$ are material constants.

Based on observations of the simplified stress concentration tests, the relationship between the creep strain and the stress relaxation R_σ is assumed to be proportional to the current stress:

$$\frac{R_\sigma}{c_\epsilon} = \sigma \bar{E} \quad (2)$$

where

$$\sigma = \sigma^0 - R_\sigma$$

σ^0 is the stress at time zero

Hence, given σ and t , equations (1) and (2) are used to find the creep strain c_ϵ and the stress relaxation R_σ .

H_ϵ , a creep history value, and H_t the corresponding time, are used to simulate the preceding observations (a) and (b). Initially H_ϵ and H_t are zero. After a sustained load hold period H_ϵ is set to c_ϵ and H_t to the time for the hold period. The next time a hold period is reached H_ϵ effects the amount that the material can creep. This is done by ratioing the values obtained from equations (1) and (2) by an amount that makes c_ϵ at time H_t plus H_ϵ equal to the creep that would have been experienced if H_ϵ were zero. I.e.,

$$\text{Ratio} = \frac{c_\epsilon(H_t) - H_\epsilon}{c_\epsilon(H_t)}$$

If the ratio is < 0 then no creep is allowed. This satisfies (a).

After a hold period any plastic strain involved in the cycles that are reversals to the cycle immediately preceding the hold period will subtract from H_ϵ . Hence a fully reversed cycle following a hold period will make $H_\epsilon = 0$ which satisfies (b).

To check the algorithm three sequences shown in Figures 75, 76, and 77 were analyzed using the following creep parameters.

E 58.436
K $3.085 \times 10^{-13.44}$
P 2.11
n .065

The first sequence, Figure 75, considers a single hold period. The second sequence, Figure 76, considers two hold periods where there is a fully reversed cycle following the first hold negating any effect on the second hold. The third sequence, Figure 77, considers two hold periods where the first hold decreases the amount of creep during the second hold. These analysis results correlate well with the coupon creep test results and the SSC specimen data discussed in 4.1 and 4.2. The analysis with the creep and stress relaxation module included was used in the data correlation studies reported in Section VI.

SECTION VI

DATA ANALYSIS AND CORRELATION

The experimental creep and stress relaxation data collected from the simple coupon, the simplified stress concentration specimen, and the super-scale specimens have been used to establish the necessary material constants, to formulate analysis algorithms for both the finite-element studies, and develop the hysteresis analysis program.

Finite-element simulation studies were conducted for the SSC and the super-scale specimen tests using the creep constants determined from the simple coupon tests. The analysis results using the constants from the coupon tests do not correlate well with the observed experimental data; however, the SSC test data was used to update the constitutive data and much better correlation was obtained. These constitutive data plus the observed data from the SSC experiments were used in formulating the creep/stress relaxation module in the hysteresis analysis. The hysteresis analysis was used, in turn, to predict times to failure for the super-scale test sequences and the spectrum fatigue tests. In addition, a linear cumulative damage (Miner's) analysis method was used to predict the life of these two groups of specimens for comparison with the hysteresis analysis predictions. This data analysis and correlation is discussed in detail in the following sections.

6.1 FINITE ELEMENT ANALYSIS CORRELATION

6.1.1 Simplified Stress Concentration Specimen

A discussion of the SSC specimen finite-element analysis and an illustration of the model details are included in 3.4 and Figures 11 and 12. This model was used in the simulation of the four SSC specimen test sequences (Figure 43) discussed earlier. The tension overloads and sustained compression loadings used in the analysis were representative of the test conditions for each sequence. Element stresses for the maximum tension overload and for the -20 ksi net section sustained compression load ($\sigma_b \approx -50$ ksi) are illustrated in Figures 78 and 79.

In this model, the stresses and strains in each element are calculated at four Gaussian integration points. The data listed were calculated at the first integration point. Element No. 1 in the center bar plastic zone and Element No. 71 in the elastic outer bar are identified in these two figures. The data for these two elements were used in the correlation studies with the experimental results. The analytical stress-strain curves for these two elements are illustrated in Figure 80.

Three finite-element model runs were made, one each at a sustained net section compression stress of -12 ksi, -20 ksi, and -30 ksi. These are identified in the figure for both Element Nos. 1 and 71. A tension overload preceded each sustained underload hold period and creep data were calculated at each compression load. As can be seen in the figure, the outer bar remains elastic during the tension loading whereas the center bar does go plastic and has a maximum strain of approximately 0.016 in/in. A comparison between these analytical stress-strain curves and those from the SSC test sequences is shown in Figures 81 and 82. The agreement between the analytical results and the tests is quite good. Although these plots are labeled center bar stress and strain, the measured data for the elastic outer bars is included with this data for comparison. This elastic response of the outer bar is practically identical to the analysis. Differences in the center bar data can be attributed to the dependency of the method of calculation of the center bar stress on the measured data in all three bars (see Section 4.2.1, for example). Slight variations in material properties, dimensions, strain gage or extensometer data, etc. can account for the differences shown here.

Creep data, as measured on the center bar during SSC test Sequences 1 and 2, are illustrated in Figures 83 and 84 with the creep strains predicted from the MARC finite-element analysis. Initially, the analytical predictions were made using the material constants derived from the simple coupon data, as discussed in 4.1. The basic trends in this analysis and the tests are similar; however, as shown in the figures the correlation is rather poor. At first it was thought that the coupon creep constants would be applicable to the SSC and super-scale specimen analysis, but, as shown here, the analysis is very conservative as far as the amount of creep that is predicted. Since the SSC and super-scale specimens are more complex than the simple coupons, it was assumed that load redistribution in these two configurations (which did not occur in the coupon) could affect the magnitude of creep.

Based on this, the experimental data collected in the SSC specimen tests were used to derive a new set of material constants for the analysis. These constants are listed below along with those derived from the coupon tests for comparison.

<u>Coupon Tests</u>	<u>SSC Tests</u>
$\beta = 1.120 \times 10^{-13}$	$\beta = 1.851 \times 10^{-13}$
$p = 2.11$	$p = 2.01$
$n = 0.065$	$n = 0.067$

The correlation between the test and the analysis is, of course, greatly improved by using these SSC specimen derived constants as illustrated in Figures 83 and 84. The variation between the analysis and test is due to the calculated stresses being somewhat different from the experimental data. This again points out the extreme sensitivity of stress and strain measurements to variations in material, specimen geometry, and loadings.

6.1.2 Super-Scale Specimen

Two finite-element models were used in the analysis of the stress-strain distribution and creep around the center notch of the super-scale specimen. These two models were previously illustrated in Figures 9 and 10. One of the models used the solid, three-dimensional elements in the MARC program and the second model was constructed using the two-dimensional plate elements. The solid elements were used to attempt to determine if there is a through-the-thickness variation in the stress-strain field around the stress riser. In addition, the layered plate model discussed in 3.2 was also run to evaluate this through-thickness variation. No effect could be identified using either the 3-D elements or the layered model. The solid elements are also more expensive to run than the 2-D plate elements; therefore, only the 2-D plate elements were used throughout this analysis. A comparison of a typical element using the 2-D and 3-D elements is illustrated in Figure 85. The solid elements are much stiffer, as illustrated, and do not correlate as well as the plate element analysis with the experimental data.

Figure 86 is an enlarged view of the plate elements immediately adjacent to the stress riser. The figure also contains a listing of the elements that go plastic along with the

far-field tension stress level at which each element exceeds the proportional limit. Elements 8 and 7 are the most highly stressed elements in the model, as expected. The plastic zone adjacent to the hole, for the maximum tensile loading of 47.3 ksi net section stress, is highlighted in this figure.

Typical stress-strain curves from the finite-element analysis are illustrated in Figure 87 for Elements 8, 7, 16, and 15. The different symbols used locate the initial values, prior to creep, of stress and strain for each element at four different compression stress levels. These curves vividly illustrate the multi-linear stress-strain curve representation capabilities incorporated into the MARC analysis program as well as the different compression yield points for the elements.

Creep analyses were run at the four compression loadings listed in Figure 87 using the creep subroutine in the MARC program. Table 11 illustrates the analysis results for the -7.9 ksi loading. The analysis results for each of the four Gaussian integration points in each element are listed in the table. This analysis utilizes the creep constants from the simple coupon tests. Figures 88 through 92 are comparison plots of the analytical creep predictions and the measured data from the super-scale specimen tests.

The material constants used in these creep analyses include those developed from both the simple coupon tests and the simplified stress concentration specimen (SSC) tests. In general, the analyses using the SSC material constants agree better with the experimental data than the analyses using the coupon test constants. The experimental data for the -15.8 ksi compression loading do not agree with either analysis. This loading condition has been repeated and similar results were obtained. No explanation for this particular set of data is offered.

6.2 FATIGUE LIFE CORRELATION

Fatigue tests involving the super-scale specimens and simple, center notched coupons were conducted and the results from these tests have been compared with analysis predictions using both the hysteresis analysis and a linear damage analysis. The super-scale specimen tests included combinations of overloads, hold periods, and constant amplitude cycling, as defined in Figure 58; whereas, the notched coupons were tested under the flight-by-flight spectra described in 4.4. In addition, super-scale test data from Phase I have been included in the correlation studies discussed here.

Table 12 is a comparison of the Phase II super-scale test results with analytical predictions using both a linear damage analysis and the hysteresis analysis. The test life listed for each specimen is the number of cycles to total specimen rupture. It was demonstrated earlier that cycles to crack initiation are approximately 90-95 percent of the cycles to rupture at the stress levels used in these tests; therefore, cycles to rupture was used throughout the program as the failure criteria.

The cycles to failure from the various test sequences is variable and depends on the applied loading sequence. For example, magnitude of underload, hold period, number of overloads and number of cycles between overloads, each effect cycles to failure. The linear analysis, as shown, cannot discriminate among the different loading conditions and predicts approximately the same life (97,000 cycles) regardless of the applied loading. The hysteresis analysis on the other hand, with the capabilities for residual stress accountability and creep, does account for the differences in the loading sequences.

The results of the hysteresis analysis for these sequences are quite encouraging. At first glance, the correlation may be considered as less than anticipated; but, after a detailed study of the data, the trends are as expected but point out a need for additional parameter studies necessary for the hysteresis analysis which were not a part of the research conducted here.

The first observation from these results is the influence of the notch-root residual stress, following the overload-underload combination, on the fatigue life. That is, the magnitude of the underload is a definite factor effecting the life. Also, the effect of the

creep and stress relaxation during the hold periods is accounted for with the analysis. For example, the trends in Sequences 1 through 4 and 9 and 10 illustrate the effect of underload magnitude on the residual or mean stress during constant amplitude cycling. Sequences 5 and 6 illustrate the creep and stress relaxation accountability. Although not exact, the hysteresis analysis-to-test correlation for Sequences 5, 6, 7, and 8 is considered quite good. More on this will be discussed later.

The correlation for the sequences with only an initial overload-underload combination (Sequences 1 through 4 and 9 and 10) is not as good as that for Sequences 5 through 8 although the trends are as expected. The reason for this can be (and quite probably is) associated with micro crack growth through the induced plastic zone around the notch as well as a cyclic change in the notch stress and notch strain. The cyclic change in the notch stress and notch strain was observed during the simplified stress concentration specimen tests and is illustrated in Figures 53 and 56. The hysteresis analysis as currently formulated does not consider either of these phenomena.

When the initial overload-underload combination is applied, a plastic zone is established immediately adjacent to the notch. If no other overloads were applied, such as in Sequence 1, and the specimen is only constant amplitude cycled; micro crack coalescing and growth progresses slowly through this zone and can then accelerate once it passes through the plastic zone. The hysteresis analysis assumes a constant micro crack growth, for example, or a continuous plastic zone and does not consider any possible acceleration. This is one reason the analysis predicts a longer life than demonstrated in these test sequences with the initial overload-underload only. When there are periodic overloads in the sequence this would tend to retard micro crack growth by enlarging or reinforcing the initial plastic zone and resulting in a more constant rate of micro crack growth. The number of cycles between overloads would also be an influence on specimen life. Both of these can be seen by comparing Sequences 5, 7, and 10, for example. This is one reason why better correlation between analysis and test was shown for those test sequences which included multiple overload-underload combinations (either with or without hold periods).

The cyclic creep, as discussed earlier, from an initial favorable compression mean stress to a more positive mean stress value, can also reduce the specimen life.

The experiments conducted here were limited and were not designed to evaluate these possible effects of the plastic zone and subsequent micro crack coalescing and growth through this zone, or the effect of the cyclic changes in residual stress. Therefore, the current hysteresis analysis does not include either of these phenomena in its present form. If sufficient data were collected, however, additional modules can be added to the current analysis program to perhaps include the accuracy for these specific loading conditions.

Table 13 illustrates the test-to-analysis correlation for additional sequences, some of which are from the Phase I program. Again, the correlation is considered quite good. One surprise was the excellent correlation for Sequence 1. Here, only an initial tension overload was applied, no underload, and the correlation is quite good. This is contrary to the trends just discussed for those sequences which have the initial overload-underload combination. The initial notch mean stress was more compressive than any of the sequences with an initial overload-underload combination and this may have effected the micro crack growth through the plastic zone or the cyclic stress changes.

A final comparison of the test results and the analytical predictions is included in Table 14. Here, the results of the spectrum fatigue tests are compared with analytical predictions for both a linear damage analysis and the hysteresis analysis. Once again the linear analysis predicts approximately the same life for each of the three test spectra. The three test spectra were described in 4.4 and are typical of a transport wing lower surface loading conditions. The hysteresis analysis does distinguish between the three different spectra and follows the test trends quite well.

It was pointed out earlier that the hold periods included in these spectra did not demonstrate the significant reduction in life shown in the other sequence tests; however, the analytical predictions show the same results. These spectra may not have been a good combination of loading conditions to demonstrate this phenomena. Certainly, other

load combinations, type of aircraft, and spectra should be evaluated before definite conclusions can be drawn.



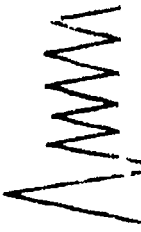
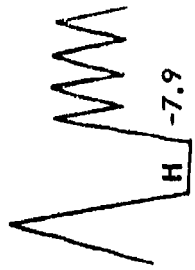
SECTION VII

OBSERVATIONS AND CONCLUSIONS

The following observations and conclusions are based on the results of both the Phase I and Phase II program:

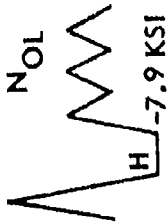
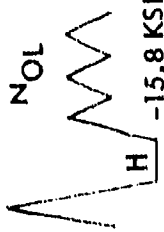
1. Overloads and underloads induce a plastic zone around a stress concentration. The plasticity induced by overloads results in a life lengthening effect in structures. Underloads can reduce life lengthening effects produced by overloads.
2. Creep and stress relaxation occur in this plastic zone at the stress concentration during sustained load hold periods. This creep and relaxation is a complex function of both notch stress and notch strain.
3. Finite element methods of analysis can be used to successfully model the elastic-plastic stress-strain state around stress concentrations.
4. The cyclic period between overload-underload combinations has a significant impact on specimen life. Multiple overload-underload combinations result in a longer life than a single application of an overload-underload.
5. There is evidence of a cyclic dependent change in mean stress which may affect specimen life.
6. A hysteresis type of stress-strain analysis more closely represents complex loadings and specimen fatigue life than does a linear analysis method.
7. Spectrum fatigue tests conducted here are inconclusive as to effect of sustained load hold periods on time to crack initiation.
8. The loading events included in the test sequences do occur in service and may effect component and full scale tests, test spectra development, and interpretation of data.

TABLE 1 - SUMMARY OF PHASE I EXPERIMENTAL RESULTS

TEST SEQUENCE DEFINITION			
			
SEQUENCE	HOLD PERIOD (H - HOURS)	CYCLES BETWEEN OVERLOADS (N _{OL} CYCLES x 10 ⁻³)	CRACK INITIATION (CYCLES x 10 ⁻³)
A	-	-	48
B	-	15	2,000 NF
B	-	1	1,000
C	-	15	715
C	-	1	309
D	24	15	141
D	1	15	337
D	24	1	101
D	1	1	154

NF = No Failure

TABLE 2 - TEMPERATURE AND UNDERLOAD EFFECTS FROM PHASE I TEST SEQUENCES

SEQUENCE	TEST CONDITIONS	CRACK INITIATION (CYCLES $\times 10^{-3}$)
	H = 24 HR. - $N_{OL} = 15,000$ CYCLES	141
	H = 1 HR. - $N_{OL} = 15,000$	337
16	H = 24 HR. - $N_{OL} = 1,000$	101
	H = 1 HR. - $N_{OL} = 1,000$	154
{ 20 21	H = 24 HR. - $N_{OL} = 15,000$	114
	H = 1 HR. - $N_{OL} = 15,000$	149
	H = 24 HR. - $N_{OL} = 15,000$	114
	H = 24 HR. - $N_{OL} = 1,000$	130
	H = 1 HR. - $N_{OL} = 1,000$	60

H = Sustained Load Hold Period, Hours

N_{OL} = Cycles Between Overloads

TABLE 3 - SUMMARY OF CREEP FORMULATIONS

Equation	Investigator	Comments
$\dot{\epsilon} = k\sigma^p$	Bailey, Norton	1. Good for low stresses
k, p are		2. Represents only secondary creep
material constants		3. Uniaxial stress
		4. Tension
		5. Small $\dot{\epsilon}$
$\dot{\epsilon} = k\rho \frac{\sigma}{\sigma_c}$	Ludwitt	1. Represents only secondary creep
k, σ_c are		2. Uniaxial stress
material constants		3. Is not zero for zero stress
		4. Tension
		5. Small $\dot{\epsilon}$
$\dot{\epsilon} = c(\rho \frac{\sigma}{\sigma_c} - 1)$	Soderberg	1. Represents only secondary creep
c, σ_c are		2. Uniaxial stress
material constants		3. Gives $\dot{\epsilon} = 0$ for $\sigma = 0$
		4. Tension
		5. Small $\dot{\epsilon}$
$\dot{\epsilon} = D \sinh \frac{\sigma}{\sigma^+}$	Nadai, Eyring	1. Represents only secondary creep
D, σ^+ are		2. Nearly linear for low stresses and nonlinear for higher stresses
material constants		3. Tension
		4. Small $\dot{\epsilon}$
		5. Uniaxial stress
$\epsilon^c = k\sigma^n t^n$	Nutting, Hoff, Scott-Blair	1. Primary and most of secondary creep
k, p, n are		2. n is almost always less than 0.5
material constants		3. Uniaxial stress

$\dot{\epsilon}$ = Creep Strain Rate

ϵ^c = Creep Strain

σ = Stress

t = Time

TABLE 3 - SUMMARY OF CREEP FORMULATIONS (Cont'd)

Equation	Investigator	Comments
$\dot{\epsilon}_{ij} = \frac{3}{2} c \sigma_e^{p-1} S_{ij}$ <p> c, p material constants σ_e - equivalent stress S_{ij} - deviatoric stress </p>	Odqvist	1. Multi-axial case 2. Secondary creep only
$\dot{\epsilon}_{ij} = \frac{3}{2} c n \sigma_e^{p-1} S_{ij}^{n-1}$ <p> c, p, n are material constants σ_e - equivalent stress S_{ij} - deviatoric stress </p>	Lal, Findley, Onaran	1. Multi-axial case 2. Primary and most of secondary creep

TABLE 4 - LAYERED PLATE MODEL ANALYSIS RESULTS

INCREMENT 0 1st YIELD	ELEMENT NO. 1		ELEMENT NO. 2	
Gaussian Integration Point	$\sigma(10^4 \text{ psi})$	$\epsilon(10^{-3} \text{ in./in.})$	$\sigma(10^4 \text{ psi})$	$\epsilon(10^{-3} \text{ in./in.})$
1	7.674	7.452	7.675	7.452
2	7.458	7.452	7.458	7.452
3	5.663	5.814	5.663	5.814
4	5.699	5.814	5.701	5.814
5	7.674	7.452	7.675	7.451
6	7.459	7.452	7.461	7.451
7	5.663	5.814	5.662	5.813
8	5.700	5.814	5.704	5.813
Avg.	6.623	6.633	6.625	6.633

INCREMENT 6 CREEP 30 HR.	ELEMENT NO. 1		ELEMENT NO. 2	
Gaussian Integration Point	$\sigma(10^4 \text{ psi})$	$\epsilon(10^{-3} \text{ in./in.})$	$\sigma(10^4 \text{ psi})$	$\epsilon(10^{-3} \text{ in./in.})$
1	7.261	7.915	7.261	7.915
2	6.688	7.915	6.690	7.915
3	5.870	6.140	5.870	6.140
4	5.877	6.140	5.879	6.140
5	7.261	7.915	7.264	6.914
6	6.690	7.915	6.697	7.914
7	5.870	6.140	5.871	6.139
8	5.878	6.140	5.884	6.139
Avg.	6.424	7.028	6.427	7.027

TABLE 4 - LAYERED PLATE MODEL ANALYSIS RESULTS (Cont'd)

INCREMENT 0 1st YIELD	ELEMENT NO. 3		ELEMENT NO. 4	
Gaussian Integration Point	$\sigma(10^4 \text{ psi})$	$\epsilon(10^{-3} \text{ in./in.})$	$\sigma(10^4 \text{ psi})$	$\epsilon(10^{-3} \text{ in./in.})$
1	4.965	4.741	4.965	4.741
2	3.102	4.741	5.102	4.741
3	3.349	3.448	3.349	3.448
4	3.409	3.448	3.409	3.448
5	4.965	4.741	4.965	4.741
6	5.102	4.741	5.102	4.741
7	3.349	3.448	3.349	3.448
8	3.409	3.448	3.410	3.448
Avg.	4.206	4.095	4.206	4.095

INCREMENT 6 CREEP 30 HR.	ELEMENT NO. 3		ELEMENT NO. 4	
Gaussian Integration Point	$\sigma(10^4 \text{ psi})$	$\epsilon(10^{-3} \text{ in./in.})$	$\sigma(10^4 \text{ psi})$	$\epsilon(10^{-3} \text{ in./in.})$
1	5.233	4.977	5.233	4.976
2	5.400	4.977	5.401	4.976
3	3.454	3.573	3.454	3.573
4	3.521	3.573	3.522	3.573
5	5.233	4.977	5.233	4.976
6	5.401	4.977	5.402	4.976
7	3.454	3.573	3.455	3.572
8	3.522	3.573	3.523	3.572
Avg.	4.402	4.275	4.403	4.274

TABLE 5 - SIMPLE BAR CREEP TESTS

TEST	INITIAL STRAIN	APPROX. FIRST LEVEL CREEP STRESS	APPROX. FIRST LEVEL CREEP STRAIN	APPROX. SECOND LEVEL STRESS/STRAIN	COMMENTS
1	0.016	-50	-	-66	From MARC Analysis, Element No. 8 at -7.9 ksi
2	0.016	-66	-	-50	Typical for notch stress analysis from Phase I at -15.8 ksi
3	0.016	-40	-	-66	Additional stress levels to solve for creep constants
4	0.016	-35	-	-50	
5	0.016	-	+0.0025	- 0.003	Repeat No. 1 with strain control
6	0.016	-	-0.0040	- 0.001	Repeat No. 2 with strain control

- NOTES: 1. Tests 1 - 4 were run under load control and creep strain was measured during hold periods.
2. Tests 5 and 6 were run under strain control and load relaxation was measured during hold period.
3. First and Second Level creep defined in sketch below.

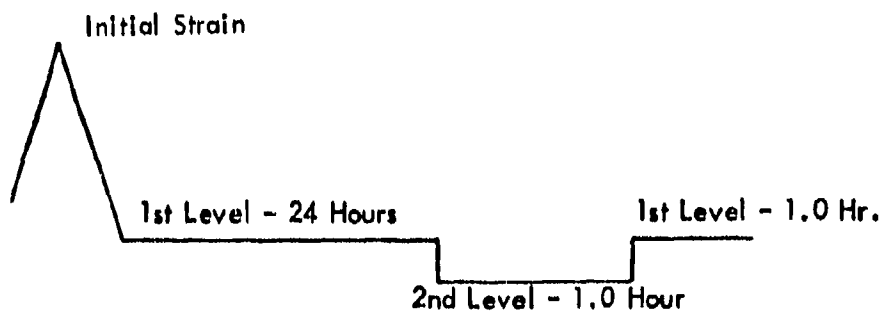


TABLE 6 - SIMPLE BAR CREEP/STRESS RELAXATION DATA

SPECIMEN NUMBER	HOLD PERIOD DEFINITION		MEASURED CREEP STRAIN OR STRESS RELAXATION
	Applied Stress or Strain	Time	
1A	-50 ksi -66 -50	24 hrs 1.0 1.0	1100 μ in/in 540 0
1B	-50 ksi -66 -50	24 hrs 1.0 1.0	1035 μ in/in 850 0
2A	-66 ksi -50 -66	24 hrs 1.0 1.0	2300 μ in/in 0 0
2B	-66 ksi -50 -66	24 hrs 1.0 1.0	2015 μ in/in 0 0
3A	-40 ksi -66 -40	24 hrs 1.0 1.0	600 μ in/in 1550 0
3B	-40 ksi -66 -40	24 hrs 1.0 1.0	650 μ in/in 1350 80
4A	-35 ksi -50 -35	24 hrs 1.0 1.0	300 μ in/in 100 0
4B	-35 ksi -50 -35	24 hrs 1.0 1.0	380 μ in/in 85 0
5A	0.0025 in/in -0.0030 0.0025	24 hrs 1.0 1.0	6.10 ksi 7.60 1.10
5B	0.0025 in/in -0.0030 0.0025	24 hrs 1.0 1.0	5.20 ksi 6.28 0.68
6A	-0.0040 in/in -0.0010 -0.0040	24 hrs 1.0 1.0	10.70 ksi 0.50 0.50
6B	-0.0040 in/in -0.0010 -0.0040	24 hrs 1.0 1.0	10.50 ksi 0.64 0

TABLE 7 - SAMPLE OF TABULATED STRAIN DATA FOR SIMPLIFIED
STRESS CONCENTRATION SPECIMEN TESTS

THREE BAR TEST

PASS NO. 1

SEQUENCE NO. 1--REPEAT

RAMP THRU PEAK STRESS TO START OF 24 HR. HOLD

STRESS, KSI	MTS EXTENSOMETERS		STRAIN GAGES OUTER BAR	
	OUTER BAR STRAIN, MICRO. IN/IN	CENTER BAR STRAIN, MICRO. IN/IN	FRONT STRAIN, MICRO. IN/IN	BACK STRAIN, MICRO. IN/IN
2.58	167	353	147	161
4.75	349	733	308	328
6.93	506	1086	469	495
9.15	688	1484	627	668
11.38	841	1863	786	830
13.50	998	2262	947	1000
15.73	1166	2650	1111	1170
17.95	1299	3039	1275	1337
20.13	1471	3437	1436	1504
22.30	1619	3817	1597	1671
24.53	1782	4234	1759	1838
26.70	1935	4641	1917	2002
28.98	2097	5066	2081	2172
31.10	2240	5473	2251	2333
33.38	2403	5907	2406	2503
35.55	2551	6332	2571	2673
37.78	2699	6766	2739	2840
39.90	2861	7237	2908	3013
42.13	3014	7861	3092	3198
44.30	3234	9082	3321	3435
46.48	3468	10828	3576	3696
48.65	3707	12809	3840	3960
50.72	3941	15224	4098	4221
50.87	3955	15342	4109	4235
51.03	3960	15451	4127	4250
51.08	3965	15550	4136	4259
51.23	3974	15668	4150	4274
51.28	3994	15785	4165	4288
51.43	4013	15903	4177	4303
0.00	0	0	0	0
46.58	3659	15170	3810	3928
32.16	2637	12411	2708	2808
17.70	1648	9643	1630	1706
3.29	611	6875	577	616
-11.13	-468	3953	-466	-498
-20.08	-1170	1701	-1126	-1246

TABLE 7 (Continued)

24 HOUR HOLD

STRESS, KSI	EXTENSOMETERS		STRAIN GAGES		TIME INTERVAL	ELAPSED TIME
	OUTER STRAIN, MICRO. IN/IN	CENTER STRAIN, MICRO. IN/IN	FRONT STRAIN, MICRO. IN/IN	REAR STRAIN, MICRO. IN/IN		
-20.18	-1175	1601	-1146	-1257	0.2 SEC.	0.2 SEC.
-20.18	-1185	1565	-1149	-1260	0.2 SEC.	0.4 SEC.
-20.18	-1194	1538	-1152	-1263	0.2 SEC.	0.6 SEC.
-20.18	-1199	1520	-1155	-1266	0.2 SEC.	0.8 SEC.
-20.18	-1194	1511	-1152	-1266	0.2 SEC.	1.0 SEC.
-20.18	-1194	1511	-1152	-1263	0.2 SEC.	1.2 SEC.
-20.18	-1218	1330	-1172	-1284	5.0 SEC.	55.0 SEC.
-20.18	-1223	1312	-1172	-1284	5.0 SEC.	60.0 SEC.
-20.18	-1218	1321	-1170	-1284	5.0 SEC.	65.0 SEC.
-20.18	-1223	1312	-1172	-1284	5.0 SEC.	70.0 SEC.
-20.18	-1218	1312	-1170	-1284	5.0 SEC.	75.0 SEC.
-20.18	-1223	1303	-1172	-1287	5.0 SEC.	80.0 SEC.
-20.18	-1218	1312	-1172	-1284	5.0 SEC.	85.0 SEC.
-20.18	-1232	1212	-1181	-1298	5.0 MIN.	25.0 MIN.
-20.18	-1232	1203	-1181	-1298	5.0 MIN.	30.0 MIN.
-20.23	-1237	1194	-1181	-1301	5.0 MIN.	35.0 MIN.
-20.23	-1232	1203	-1181	-1301	5.0 MIN.	40.0 MIN.
-20.18	-1237	1194	-1181	-1301	5.0 MIN.	45.0 MIN.
-20.33	-1223	1122	-1196	-1316	60.0 MIN.	19.0 HRS
-20.33	-1223	1122	-1199	-1316	60.0 MIN.	20.0 HRS
-20.33	-1242	1158	-1199	-1319	60.0 MIN.	21.0 HRS
-20.28	-1223	1158	-1199	-1316	60.0 MIN.	22.0 HRS
-20.28	-1242	1086	-1199	-1319	60.0 MIN.	23.0 HRS

TABLE 8 - PHASE I AND PHASE II SUPER-SCALE
TEST SPECIMEN COMPARISON

Sequence Number	Test Conditions	Cycles To Failure
I-1	Constant Amplitude Baseline $F_m = 15, F_v = \pm 10$	47,647
<u>Group 1</u>		
I-6	+47.3, $N_{OL} = 15,000$, CA	1,977,450 (NF)
I-8	+47.3, -7.9, $N_{OL} = 15,000$, CA	712,000
II-6	+47.3, -4.0, $N_{OL} = 15,000$, CA	1,269,770
II-9	+47.3, -7.9, CA	286,838
II-10	+47.3, -4.0, CA	131,956
<u>Group 2</u>		
I-13	+47.3, 24 Hr. @ -7.9, $N_{OL} = 15,000$, CA	141,329
II-7	+47.3, 24 Hr. @ -7.9, $N_{OL} = 30,000$, CA	88,000
II-5	+47.3, 24 Hr. @ -4.0, $N_{OL} = 15,000$, CA	1,375,000
II-8	+47.3, 24 Hr. @ -4.0, $N_{OL} = 30,000$, CA	581,557

NOTE: Roman numerals refer to Phase I or II
in the sequence identification.

TABLE 9 - CREEP STRAINS FROM SUPER-SCALE SEQUENCE TESTS

Sequence Number	Maximum Measured Strain Change (μ in/in)				
	Transducer	S. G. No. 1	S. G. No. 2	S. G. No. 3	S. G. No. 4
1	+131	-209	-258	-149	-177
2	-248	-352	-305	-229	-213
3	-203	-200	-190	-120	-150
4	-509	-122	-125	- 86	- 86
5	-187	-140	-160	- 80	- 80
7	-300	-200	-200	-130	-100
8	-350	-190	-270	-130	-110

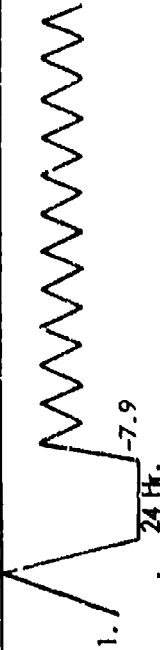
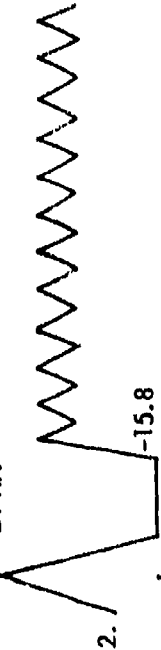
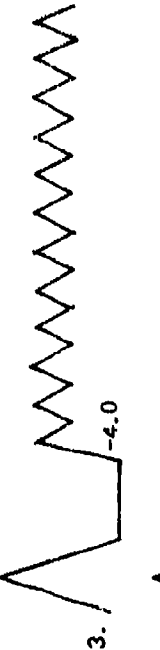
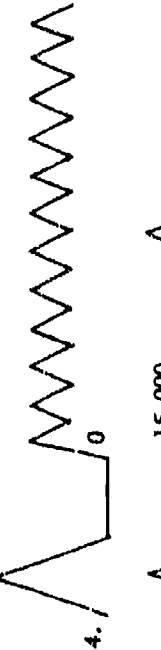
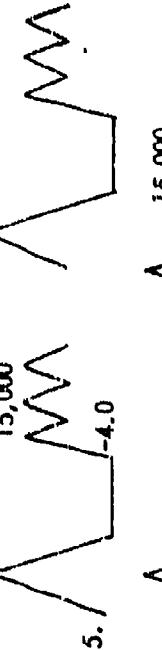
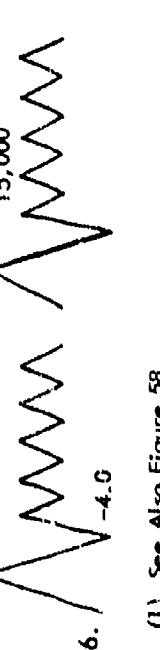
TABLE 10- SPECTRUM FATIGUE TEST RESULTS

SPECTRUM 1	SPECTRUM 2	SPECTRUM 3
30,710 Flights	29,861 Flights	8,458 Flights
29,858	31,932	8,245
<u>30,218</u>	<u>30,020</u>	<u>8,233</u>
Average: 30,262 Flights	30,604 Flights	8,312 Flights

TABLE 11 - FINITE-ELEMENT CREEP ANALYSIS
SUPER-SCALE SPECIMEN, -7.9 KSI COMPRESSION

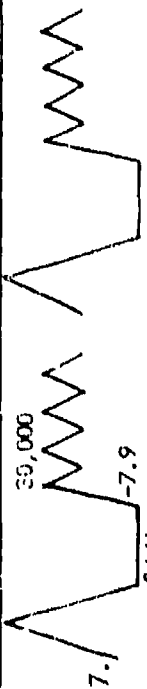

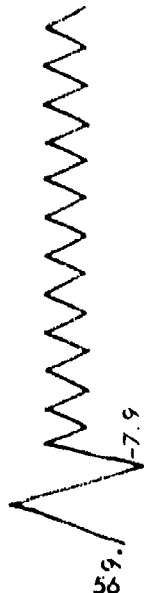
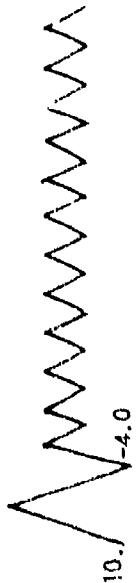
Time (Hours)	Gaussian Integration Point	Element 8	
		σ_r (ksi)	ϵ^c (10^{-4} in/in)
Time = 0	1	-47.48	0
	2	-41.34	0
	3	-47.85	0
	<u>4</u>	<u>-41.78</u>	<u>0</u>
	Avg.	-44.61	
Time = 0.5	1	-46.47	- 9.104
	2	-40.59	- 7.029
	3	-46.82	- 9.147
	<u>4</u>	<u>-41.01</u>	<u>- 7.064</u>
	Avg.	-43.72	- 8.086
Time = 3.0	1	-45.88	- 9.989
	2	-40.14	- 7.742
	3	-46.22	-10.04
	<u>4</u>	<u>-40.56</u>	<u>- 7.781</u>
	Avg.	-43.20	- 8.888
Time = 13.7	1	-45.34	-10.80
	2	-39.73	- 8.398
	3	-45.68	-10.85
	<u>4</u>	<u>-40.14</u>	<u>- 8.442</u>
	Avg.	-42.72	- 9.622
Time = 24.5	1	-45.12	-11.13
	2	-39.57	- 8.664
	3	-45.46	-11.18
	<u>4</u>	<u>-39.97</u>	<u>- 8.708</u>
	Avg.	-42.53	- 9.921

TABLE 12 - COMPARISON OF EXPERIMENTAL DATA WITH ANALYTICAL LIFE PREDICTIONS
FOR SUPER-SCALE TEST SPECIMENS

TEST SEQUENCE DEFINITION ⁽¹⁾	TEST LIFE (CYCLES)	ANALYTICAL LIFE $\Sigma \frac{1}{N}$	ANALYTICAL LIFE HYSTERESIS
1. 	80,018	97,955	562,000
2. 	49,867	97,920	334,400
3. 	44,277	97,972	695,000
4. 	135,801	97,987	841,300
5. 	1,375,000	97,604	653,200
6. 	1,269,770	97,604	712,000

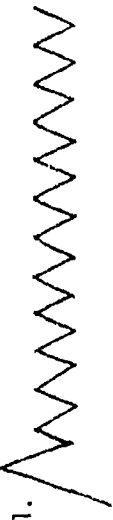

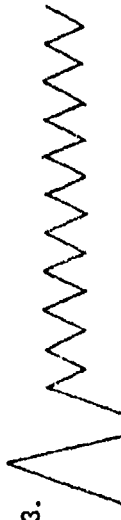
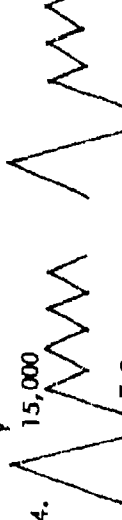

(1) See Also Figure 58

TABLE 12 - COMPARISON OF EXPERIMENTAL DATA WITH ANALYTICAL LIFE PREDICTIONS
FOR SUPER-SCALE TEST SPECIMENS (Continued)

TEST SEQUENCE DEFINITION ⁽¹⁾	TEST LIFE (CYCLES)	ANALYTICAL LIFE ΣN	ANALYTICAL LIFE HYSTERESIS
 <p>7.</p>	214,546 88,000	97,765	532,000
 <p>8.</p>	581,557	97,821	666,700
 <p>9.</p>	286,838	97,955	950,500
 <p>10.</p>	131,956	97,987	771,000

(1) See Also Figure 58

TABLE 13 - ANALYSIS AND EXPERIMENTAL RESULTS FOR TEST SEQUENCES INCLUDING OVERLOADS AND UNDERLOADS

TEST SEQUENCE DEFINITION	TEST LIFE (CYCLES)	ANALYTICAL LIFE $\sum \frac{1}{N}$	ANALYTICAL LIFE HYSTERESIS
1. 	910,000 (1)	$\sim 97,000$	978,000
2.  15,000	1.98×10^6 N.F.		950,500
3.  57 -7.9	286,800		646,300
4.  15,000 -7.9	715,000		615,800
5.  1,000 -7.5	308,000		364,400

(1) Test data from Reference 4.

TABLE 13 - ANALYSIS AND EXPERIMENTAL RESULTS FOR TEST SEQUENCES INCLUDING OVERLOADS AND UNDERLOADS (Continued)

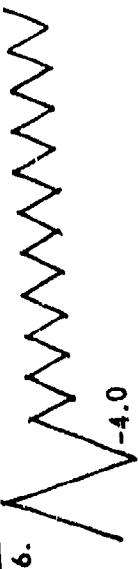
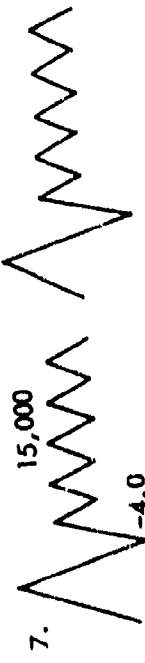
TEST SEQUENCE DEFINITION	TEST LIFE (CYCLES)	ANALYTICAL LIFE $\sum \frac{n}{N}$	ANALYTICAL LIFE HYSTERESIS
6.  -4.0	132,000	~97,000	771,000
7.  15,000 -4.0	1,269,770		712,200

TABLE 14 - EXPERIMENTAL AND ANALYTICAL LIFE COMPARISON
FOR SPECTRUM FATIGUE TESTS

TEST (1) SPECTRUM	TEST LIFE (2) (FLIGHTS)	ANALYTICAL LIFE LINEAR DAMAGE $\sum \frac{n}{N}$	ANALYTICAL LIFE HYSTERESIS ANALYSIS
1	30,262	15,460	43,282
2	30,604	15,460	46,106
3	8,312	15,920	8,681

(1) Spectrum 1: 50 [Flt. C + 150 (2 Flt. A + Flt. B)] + N (2 Flt. A + Flt. B)

Spectrum 2: 50 [(Flt. C + Hold Period) + 150 (2 Flt. A + Flt. B)]
+ N (2 Flt. A + Flt. B)

Spectrum 3: 50 [Flt. B + 150 (2 Flt. A + Flt. B)] + N (2 Flt. A + Flt. B)

(2) Average of Three Tests

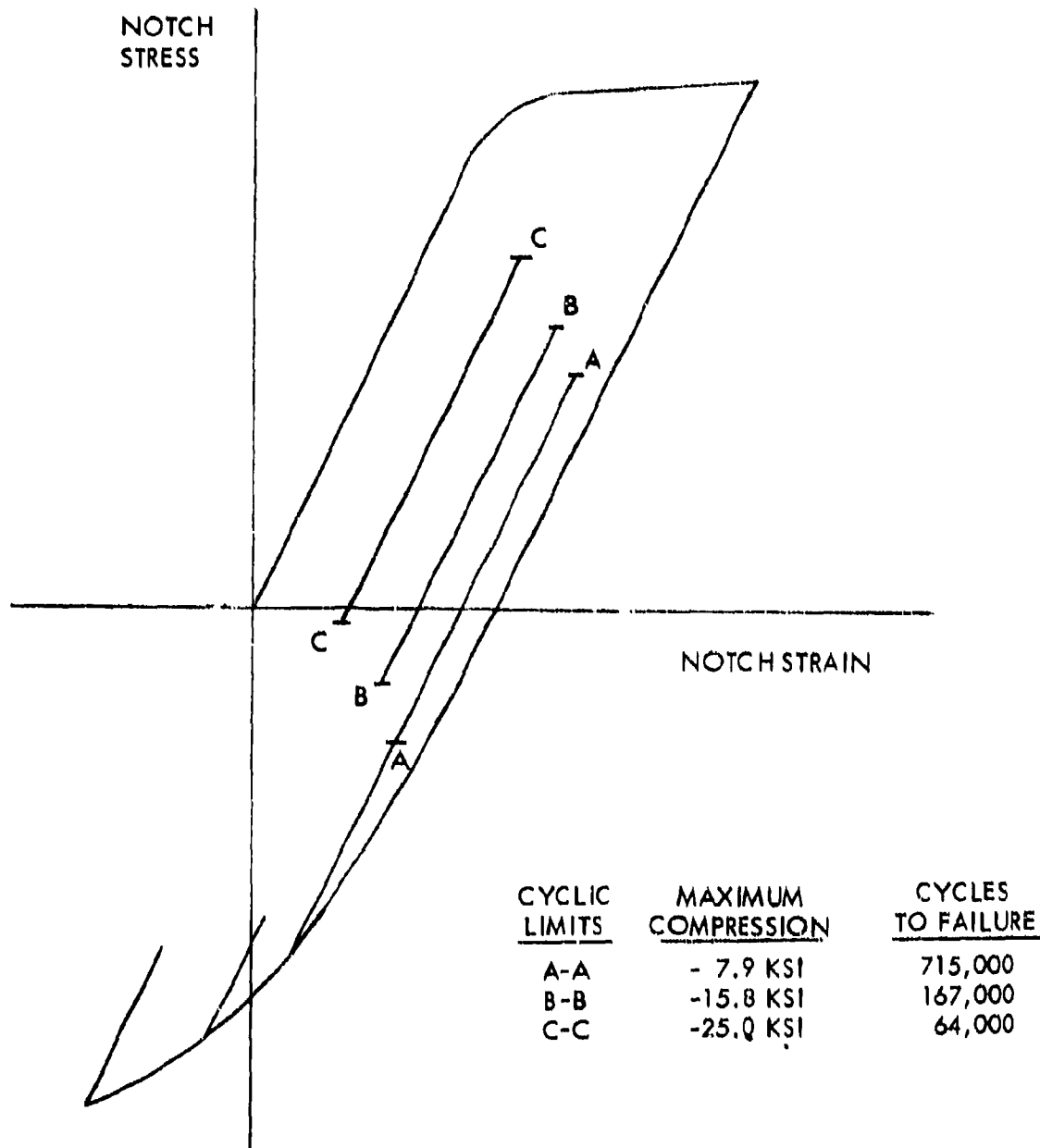


Figure 1. Underload Effects on Cycles to Crack Initiation

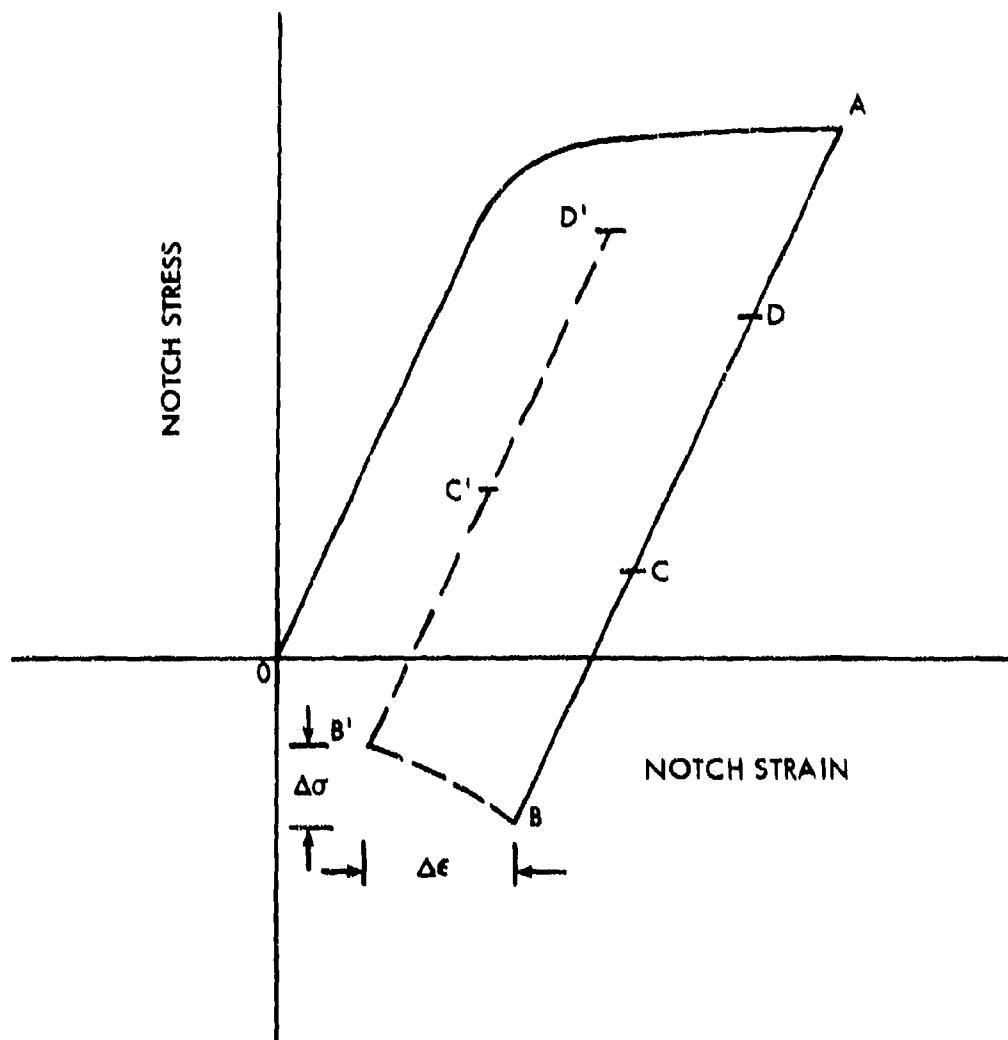


Figure 2 . Hypothesis of Complexity of Time Dependent Stress-Strain Change

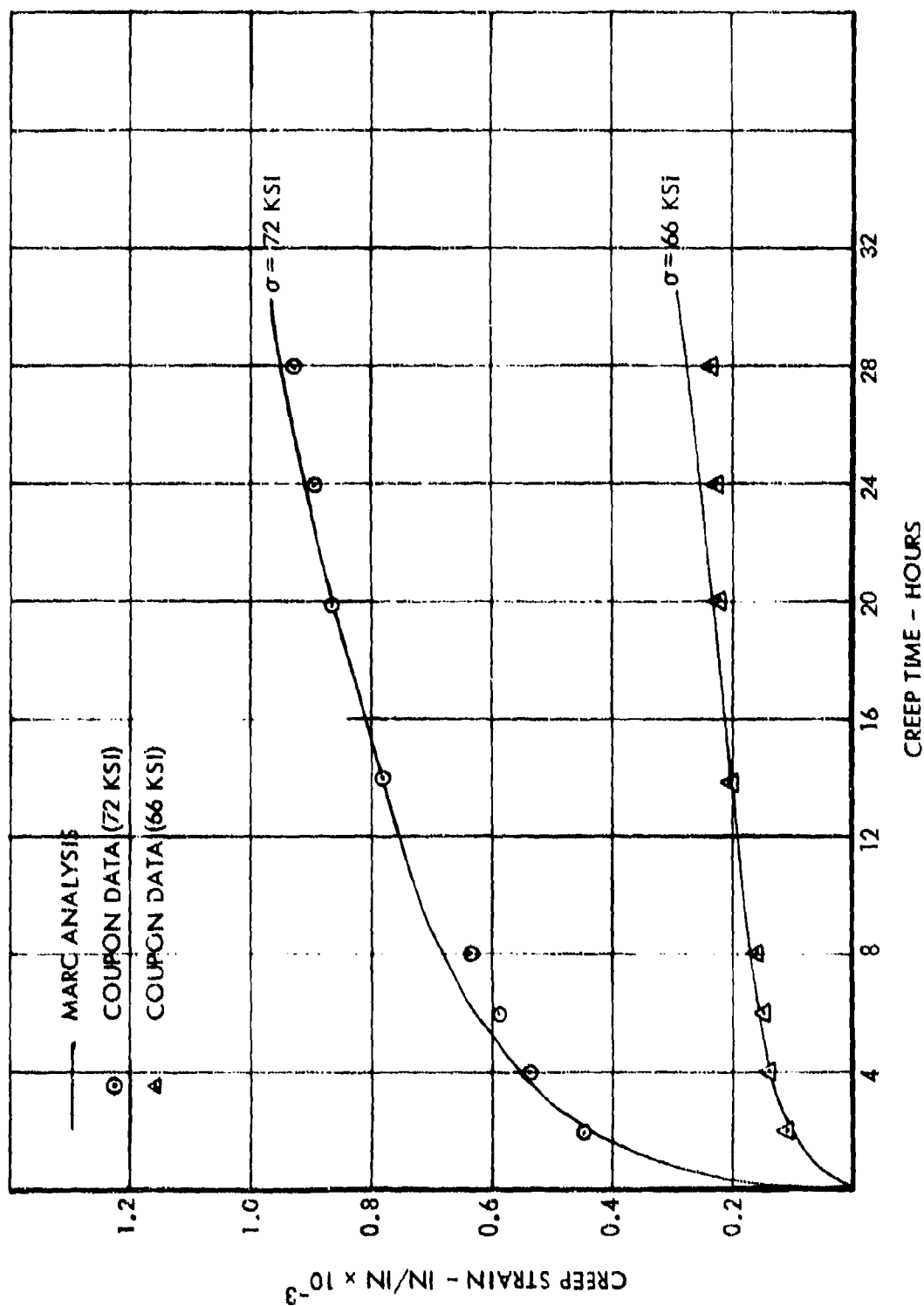


Figure 3. Comparative Creep Data - Finite Element Simulation Versus Coupon Test Data

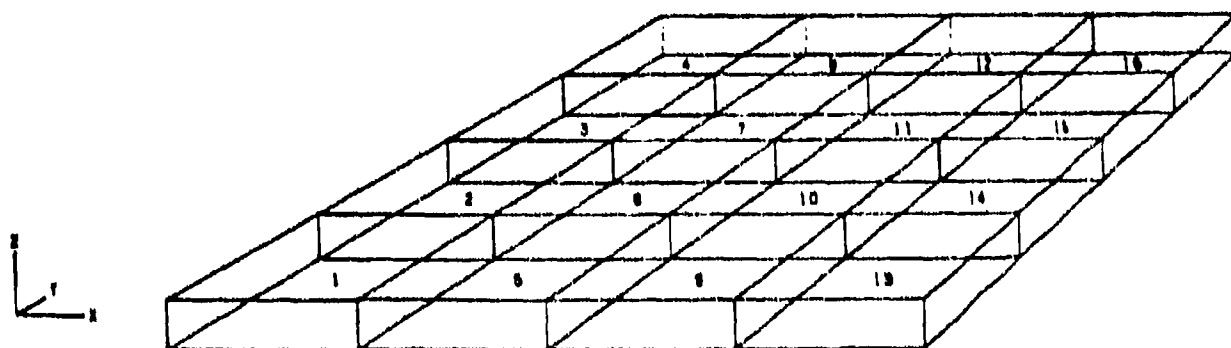


Figure 4. Simple Plate Finite Element Model

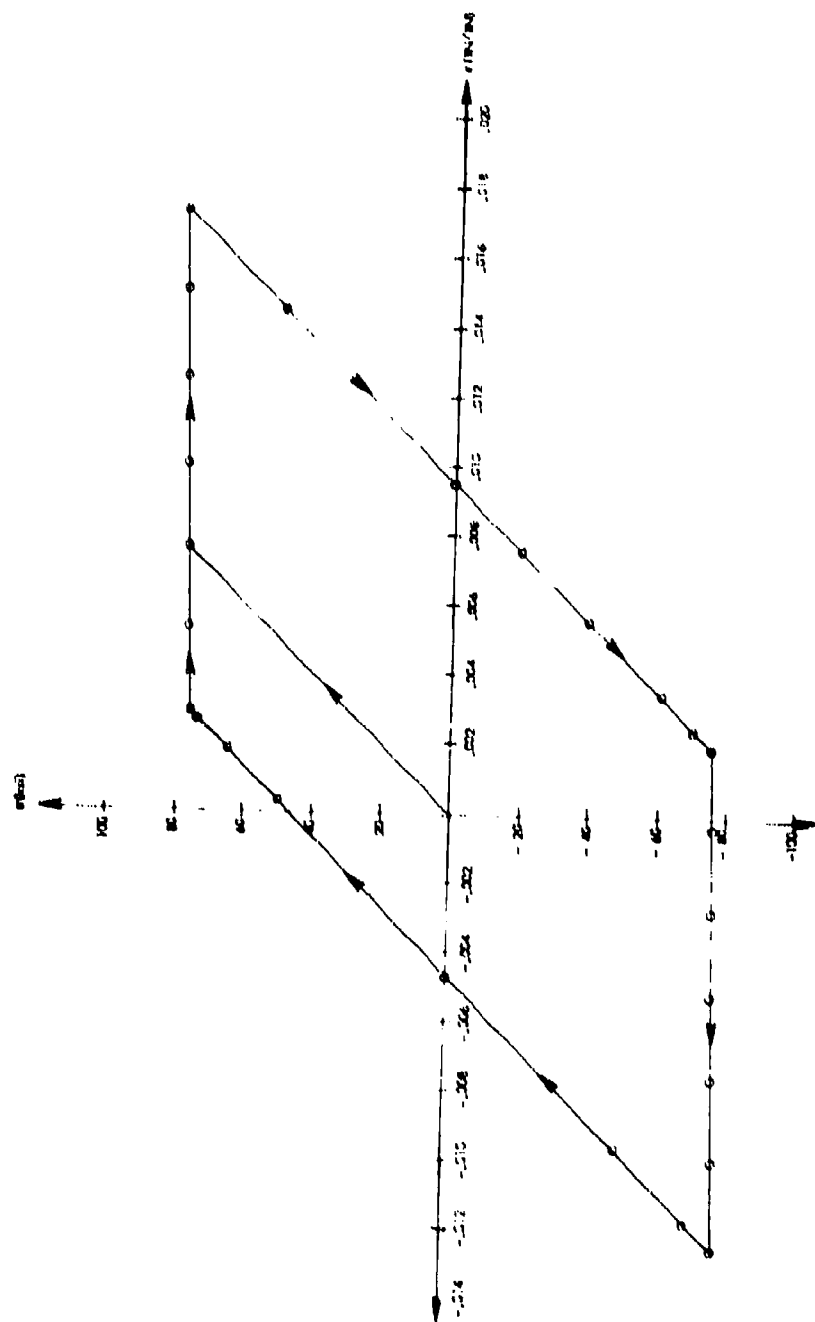


Figure 5. MARC Bilinear Stress-Strain Representation

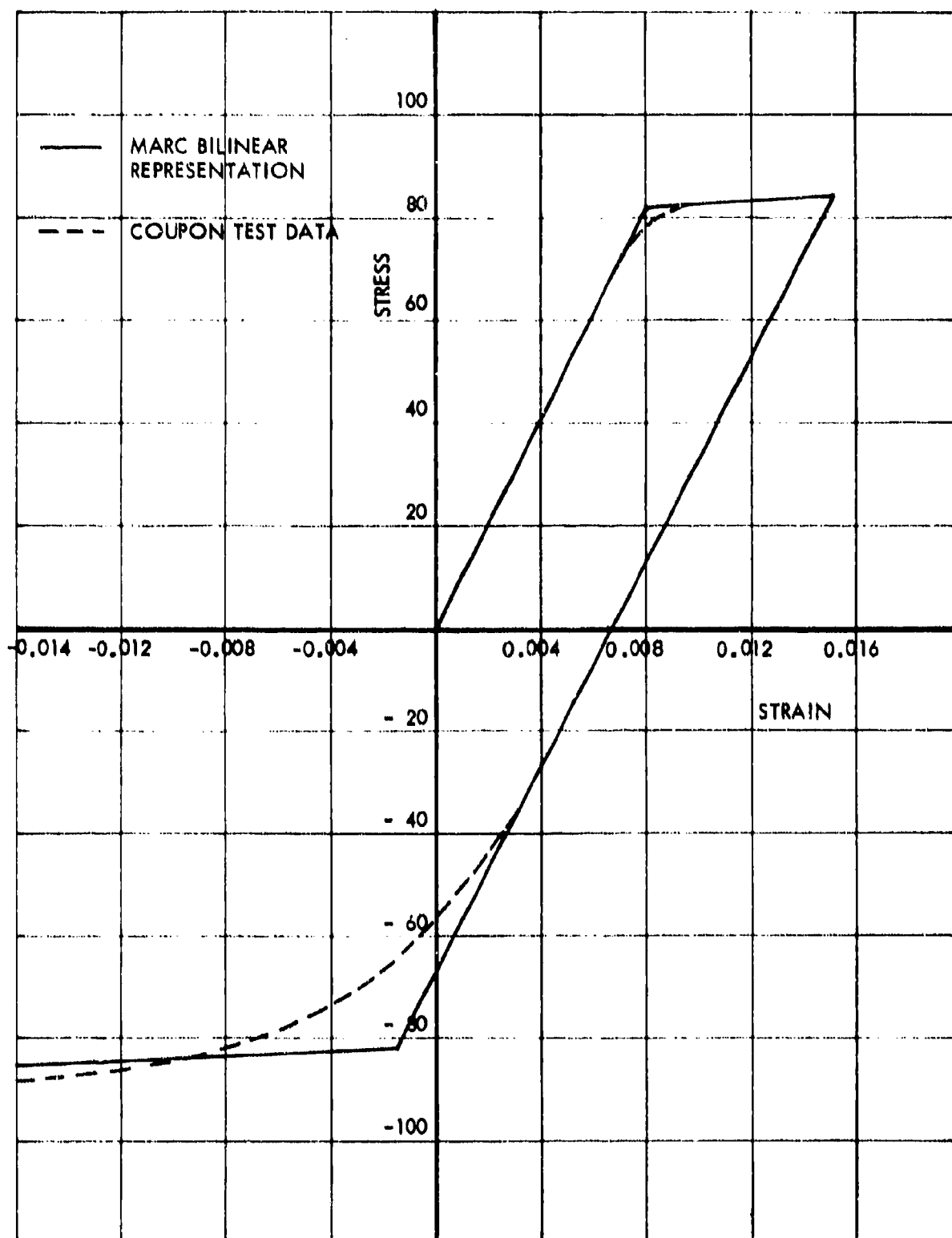


Figure 6. Comparison of Coupon Test and MARC
Representation of Stress-Strain Data

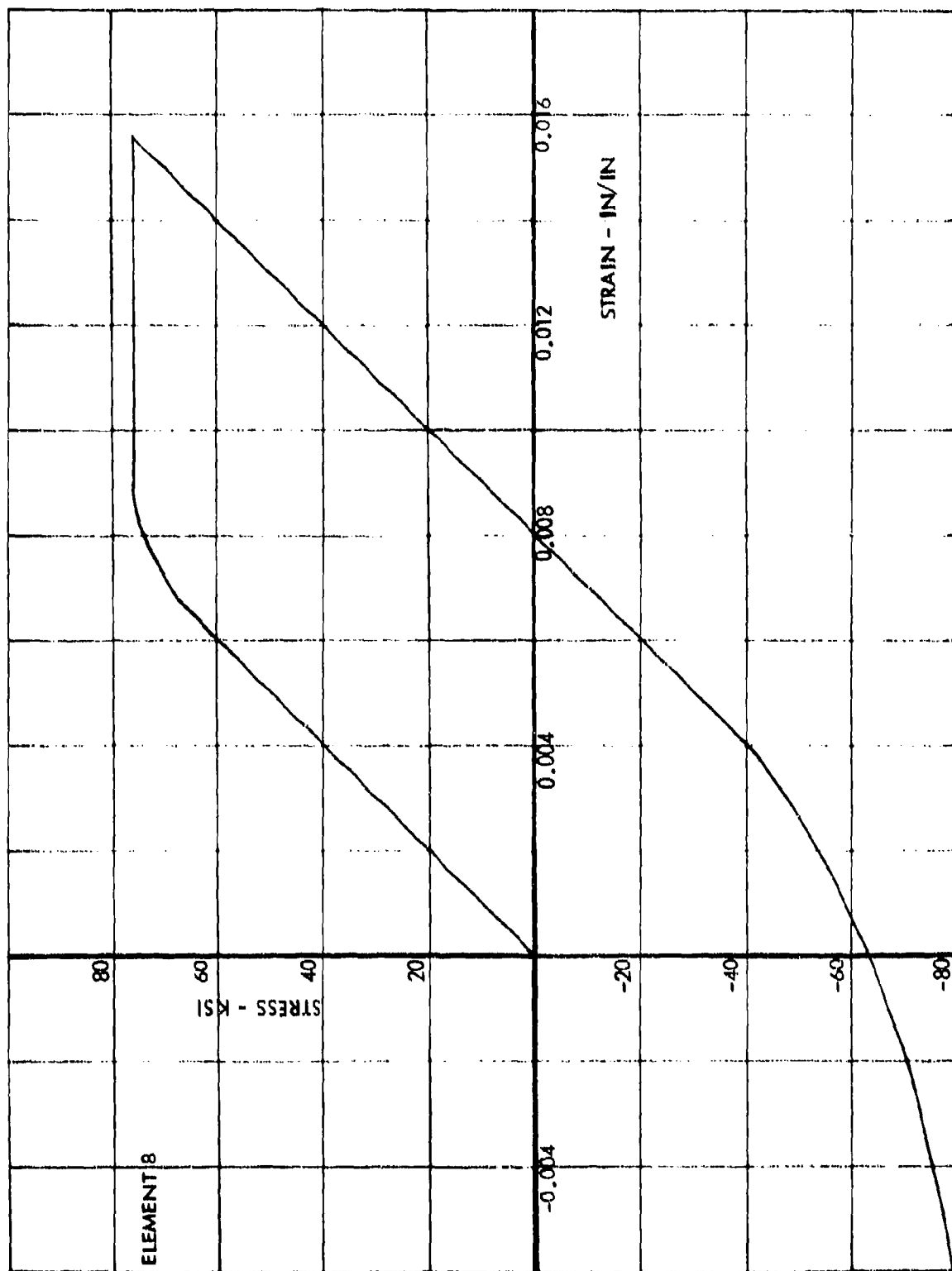


Figure 7. Tension-Compression Stress-Strain Curve
Element No. 8

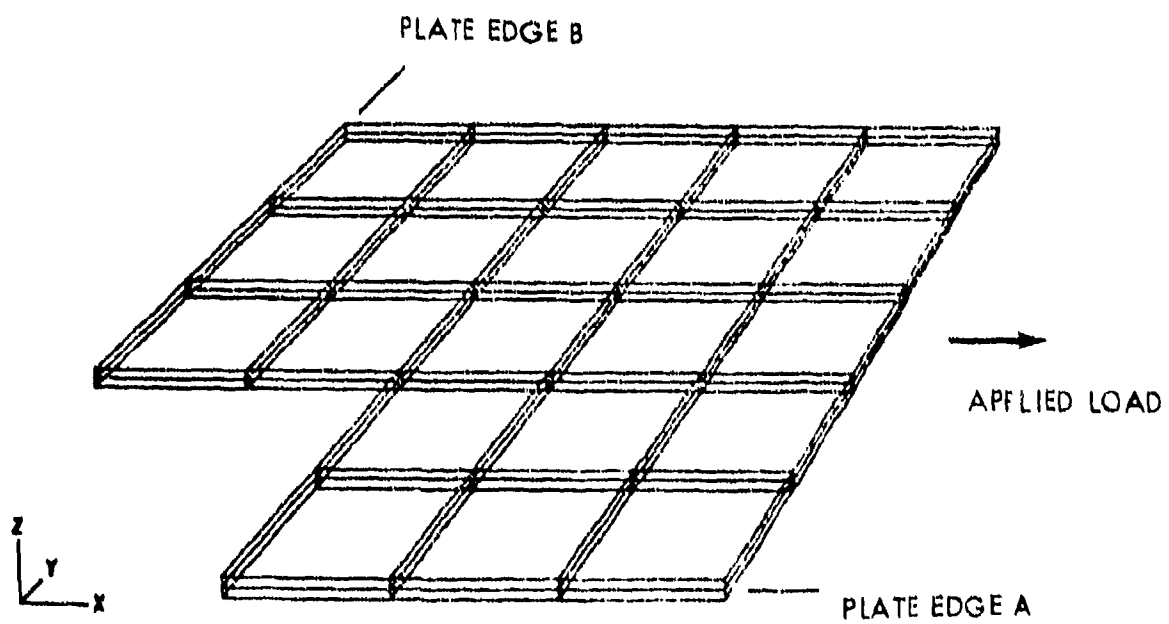


Figure 8. Layered Finite Element Model

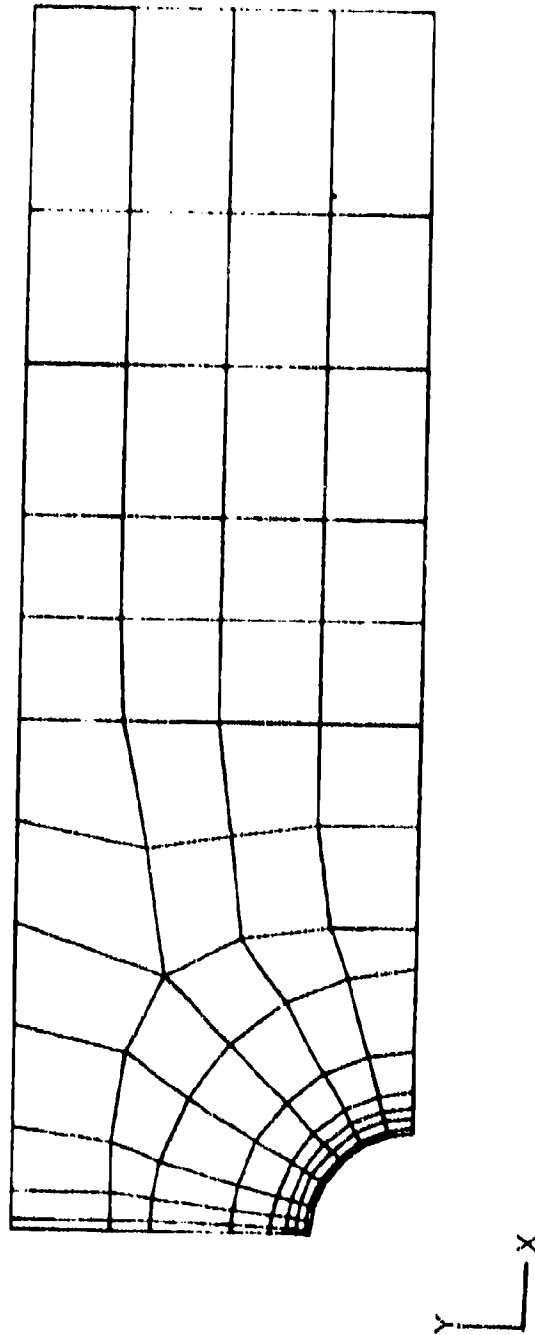


Figure 9. Super-Scale Finite Element Model

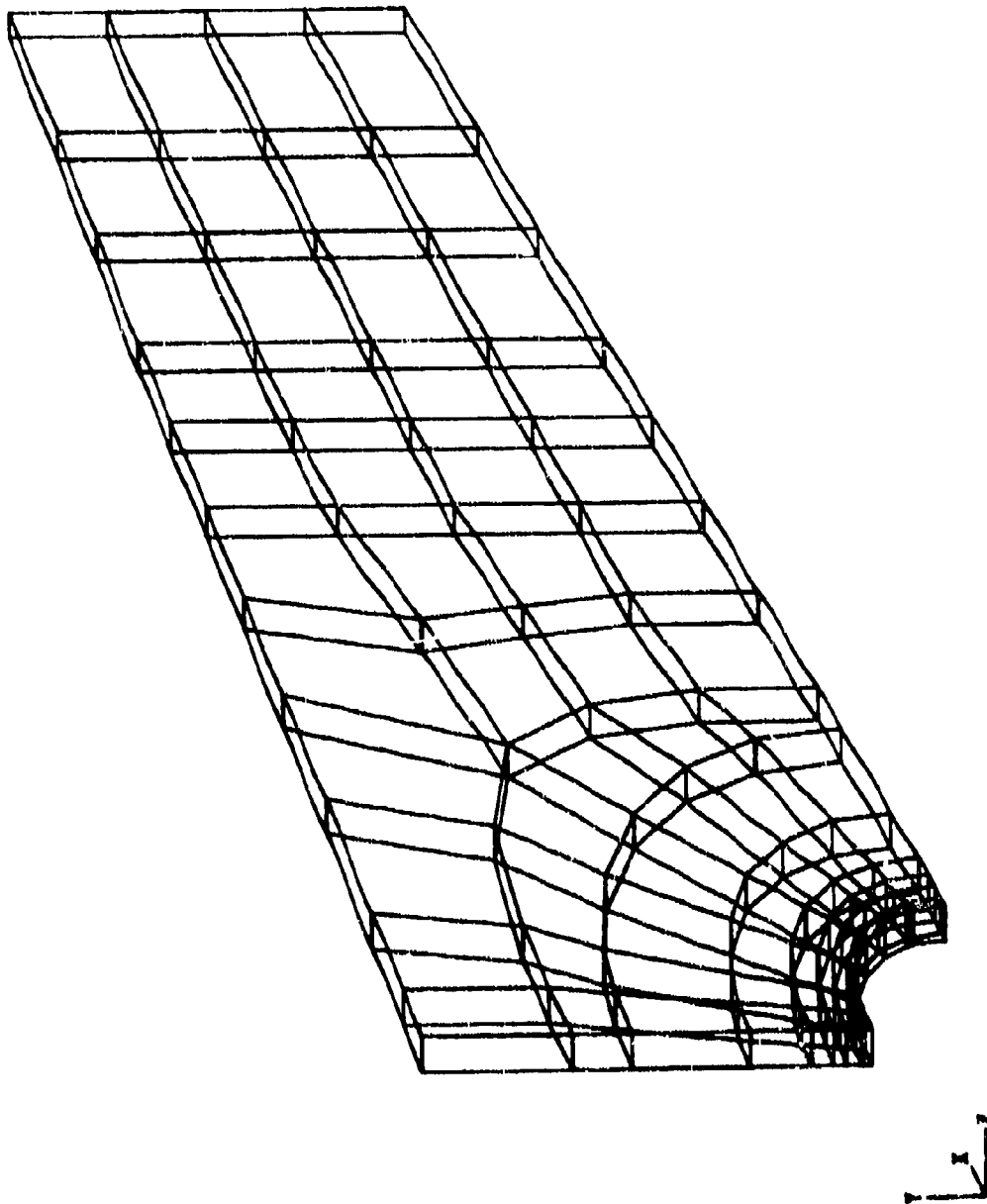


Figure 10. Three-Dimensional Super-Scale Model

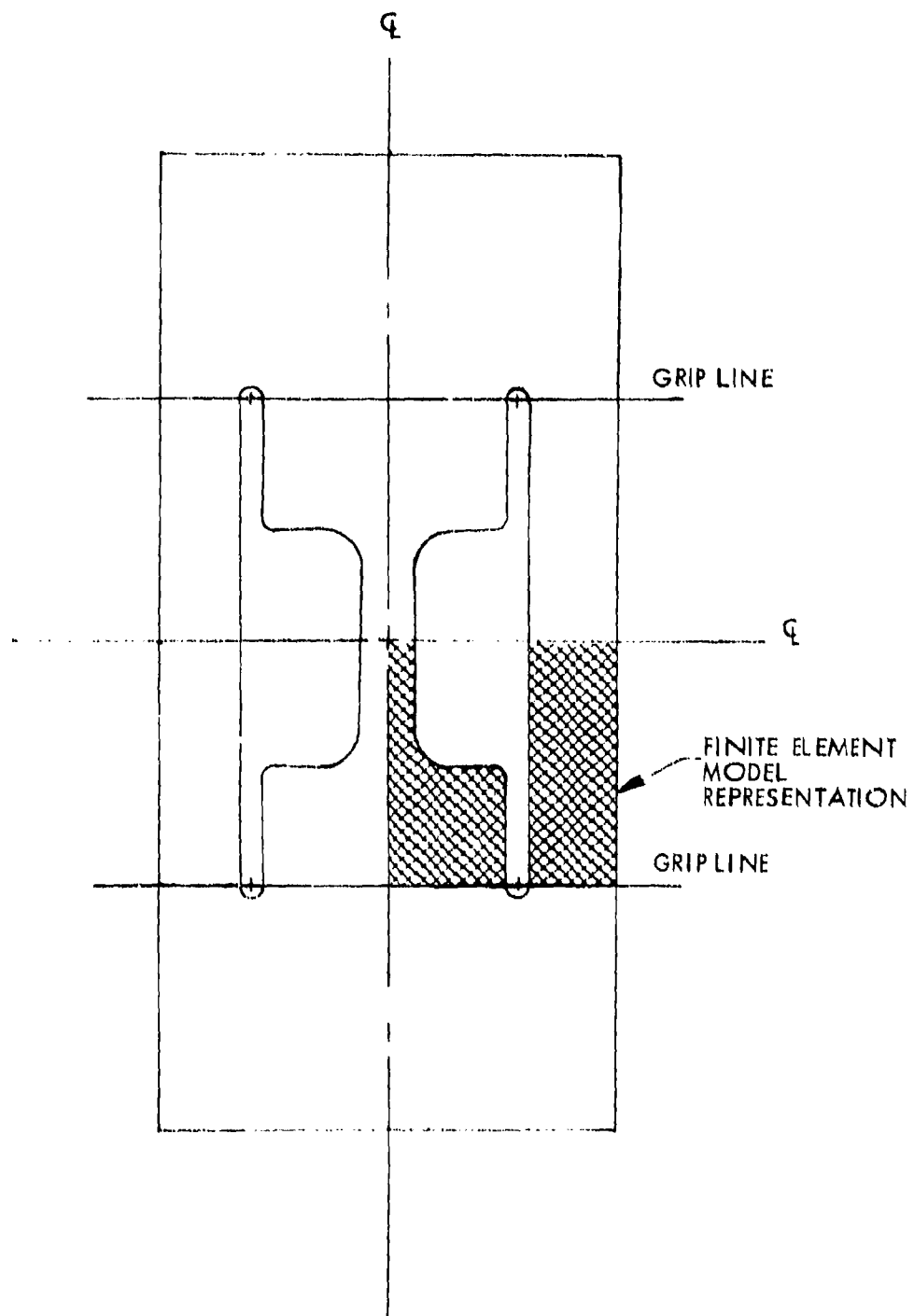
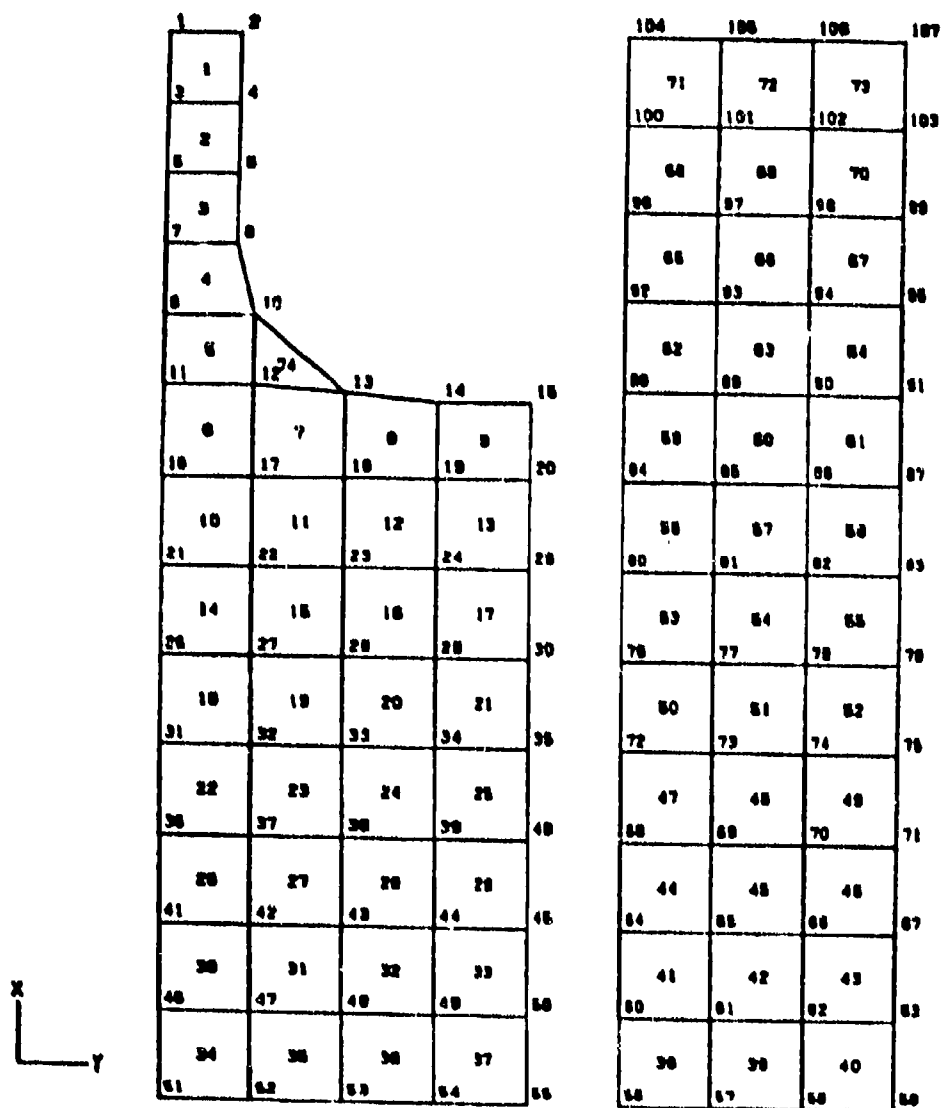


Figure 11. Area of Simplified Stress Concentration Specimen
To Be Represented by Finite Element Simulation



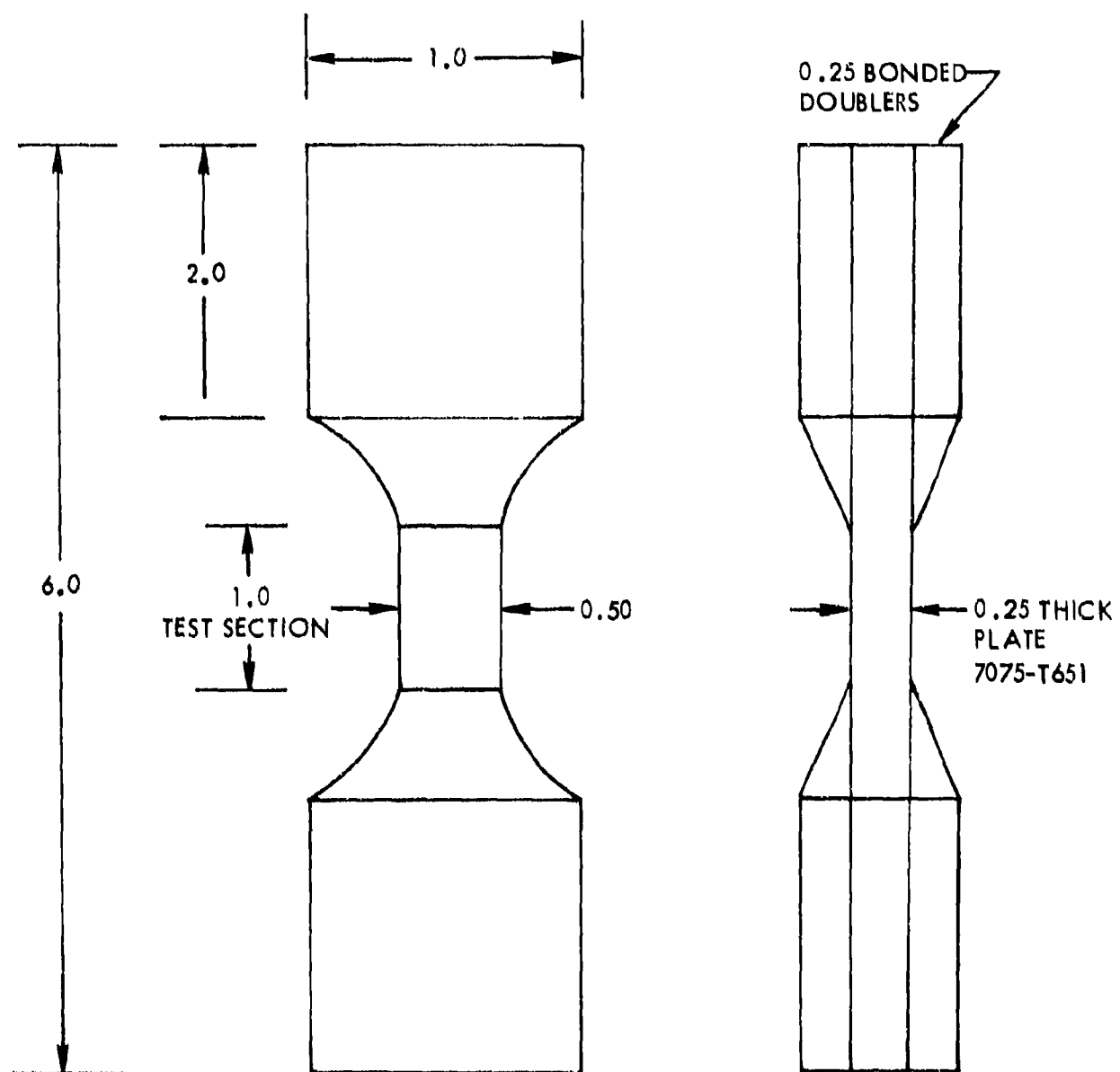


Figure 13. Coupon Specimen for Creep and Stress Relaxation Tests

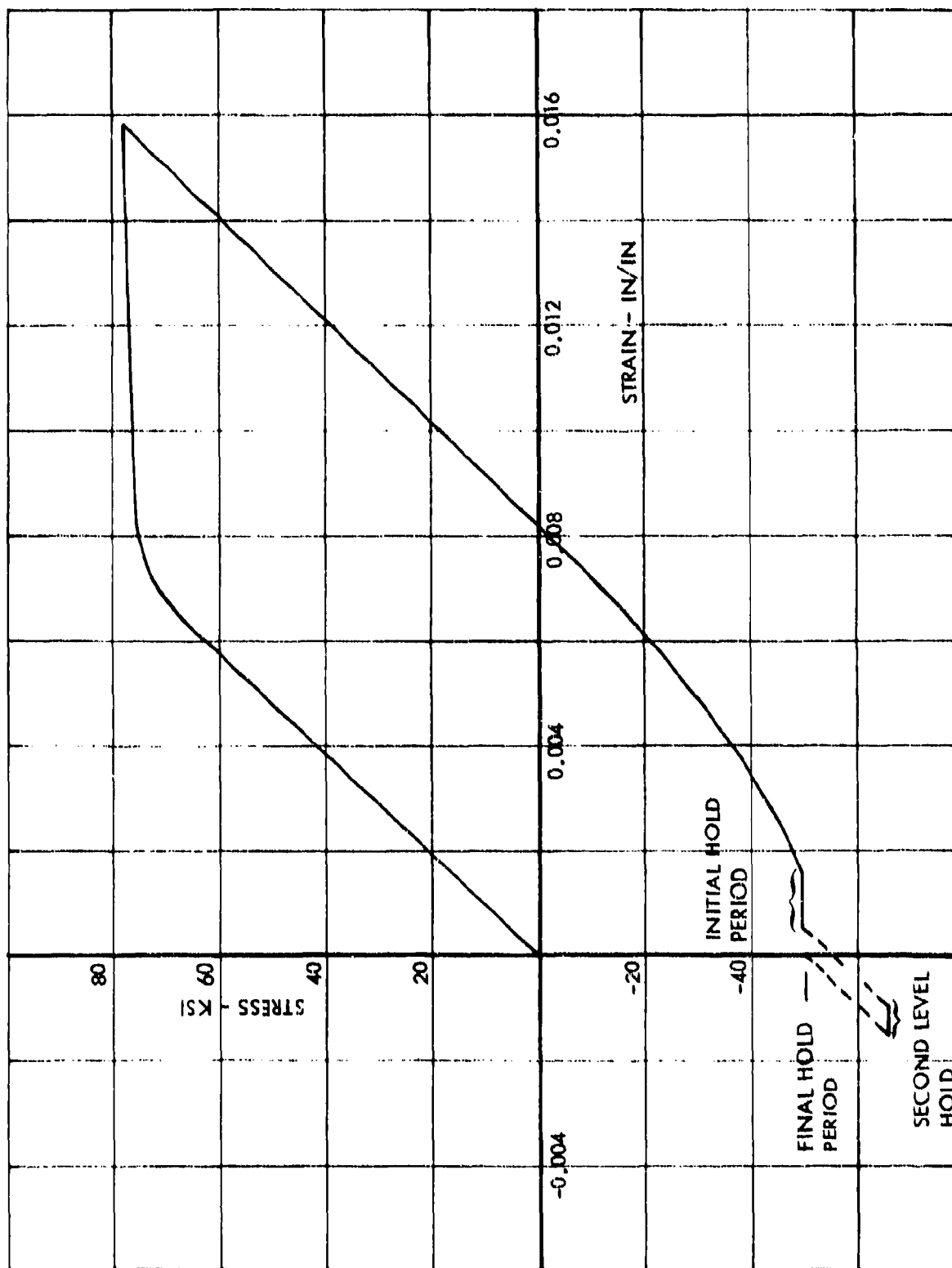


Figure 14. Stress-Strain Curve, Coupon Specimen 1A
Load Control Test

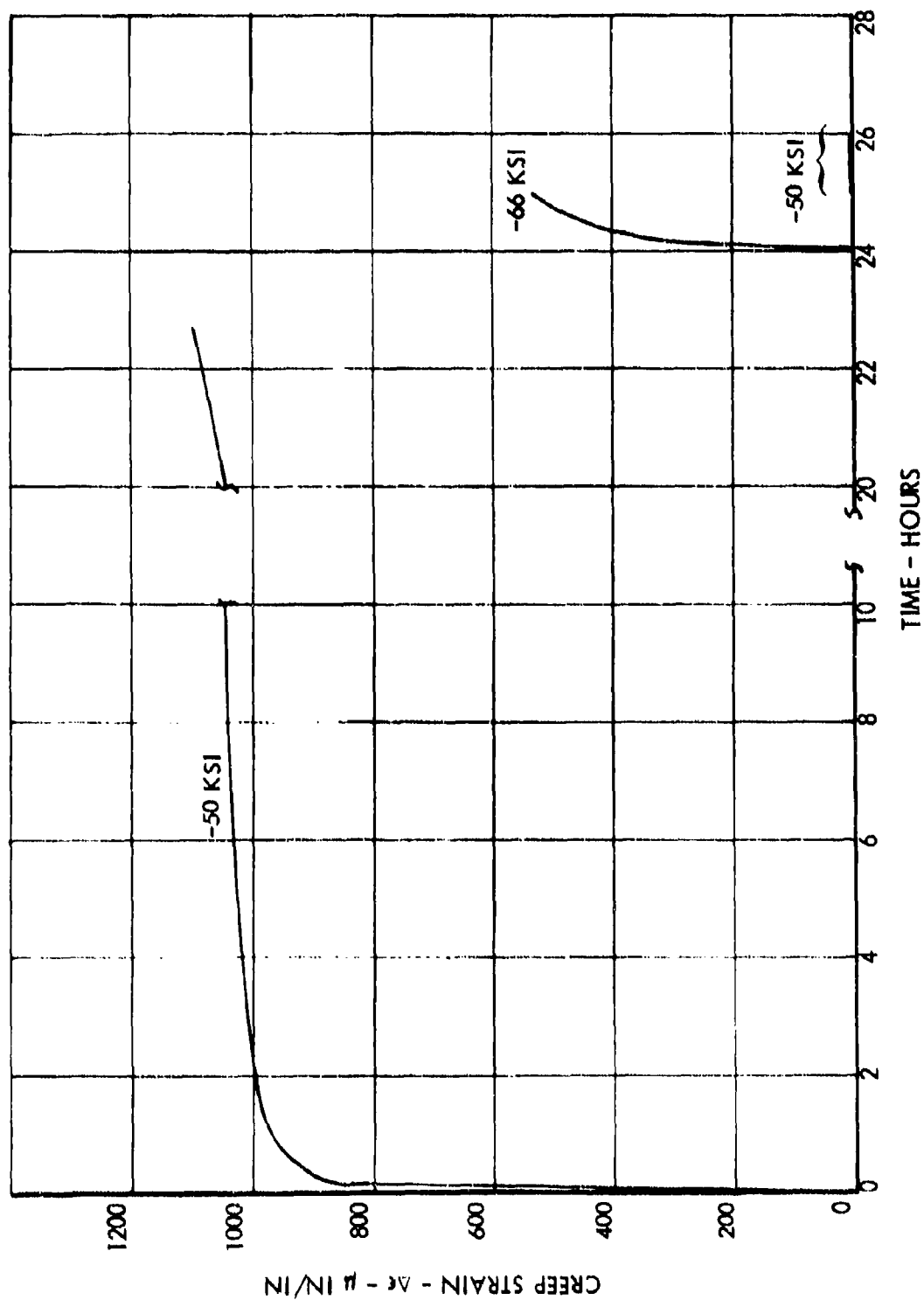


Figure 15. Coupon 1A Creep Strain

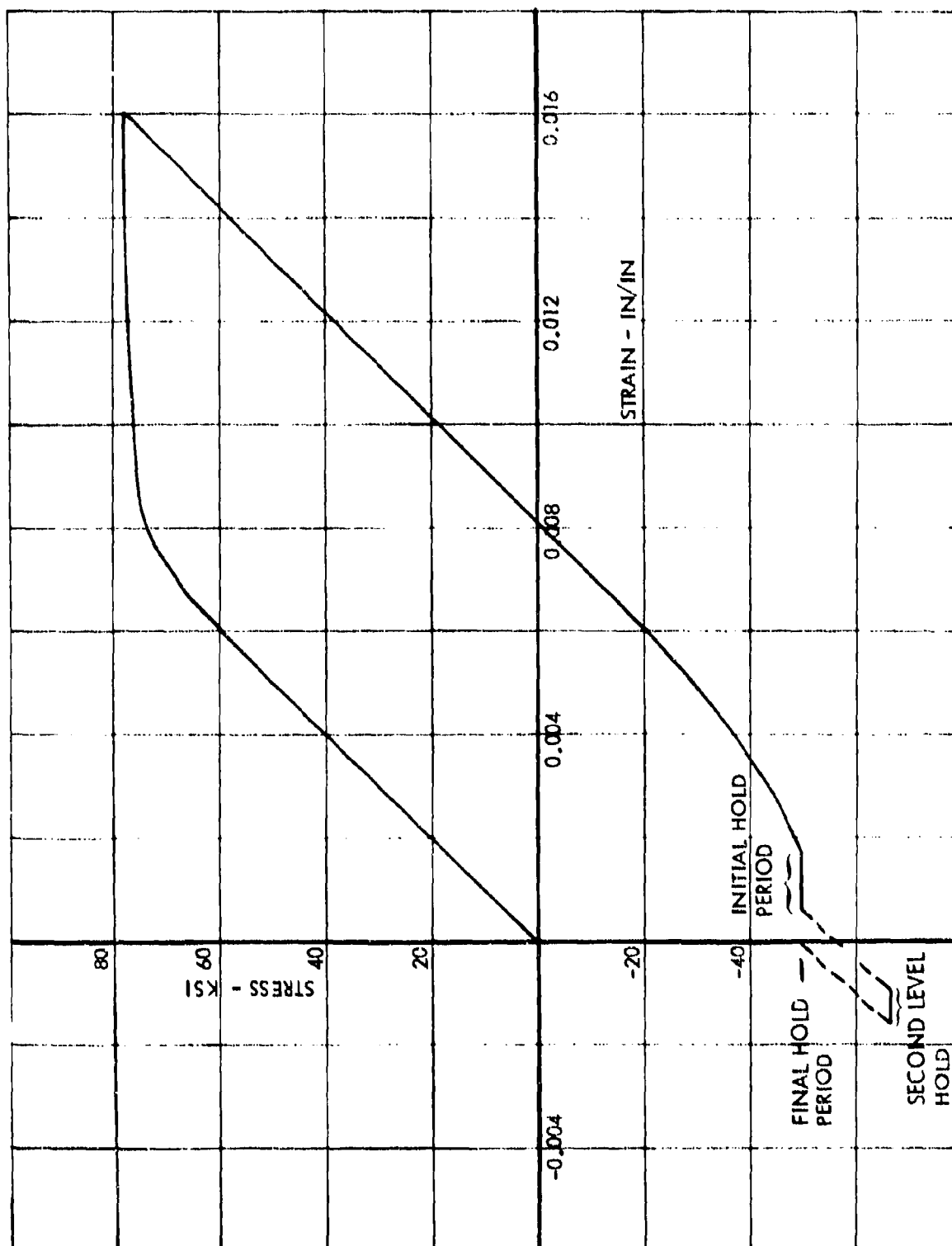


Figure 16. Stress-Strain Curve, Coupon Specimen 1B
Load Control Test

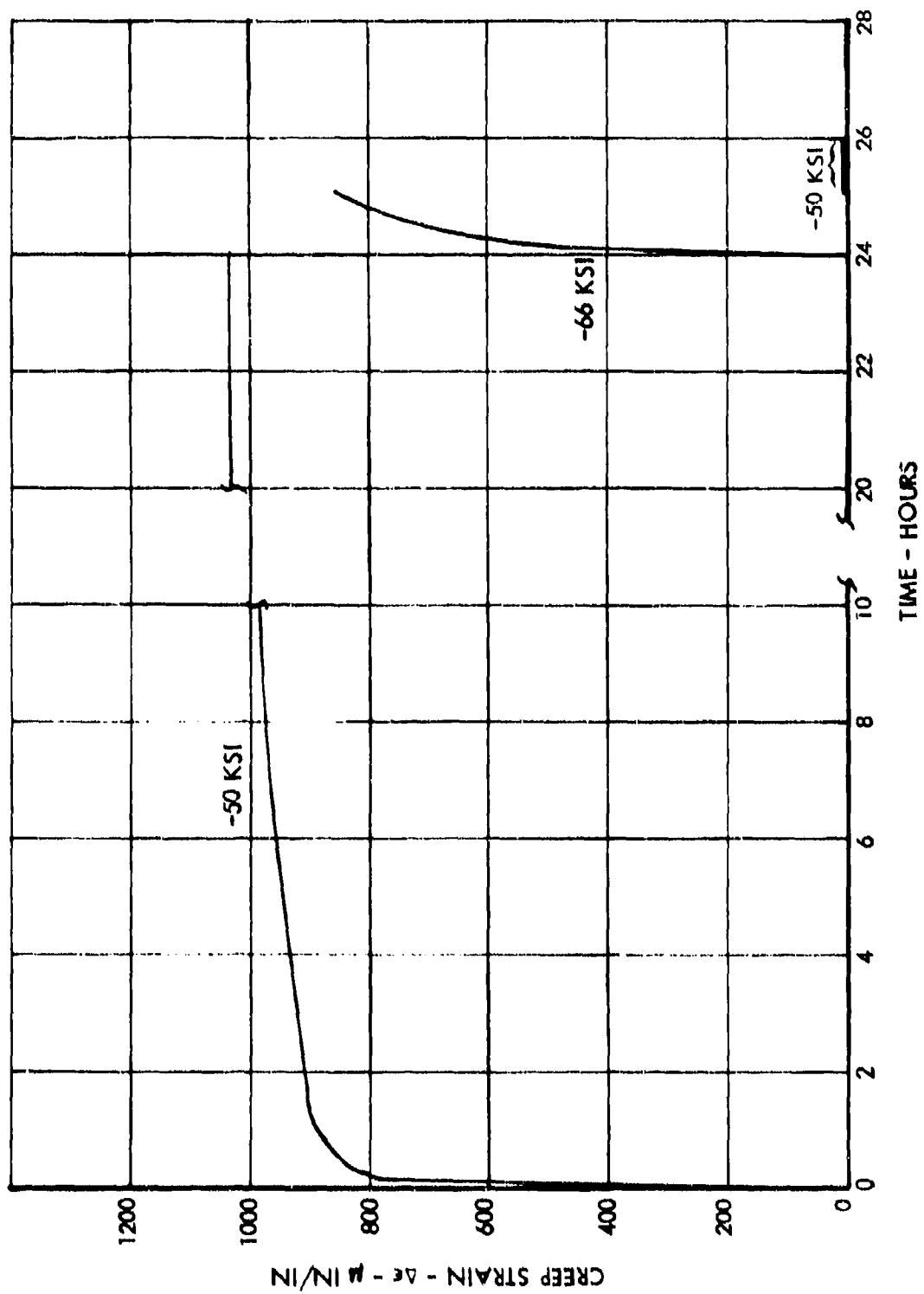


Figure 17. Coupon 18 Creep Strain

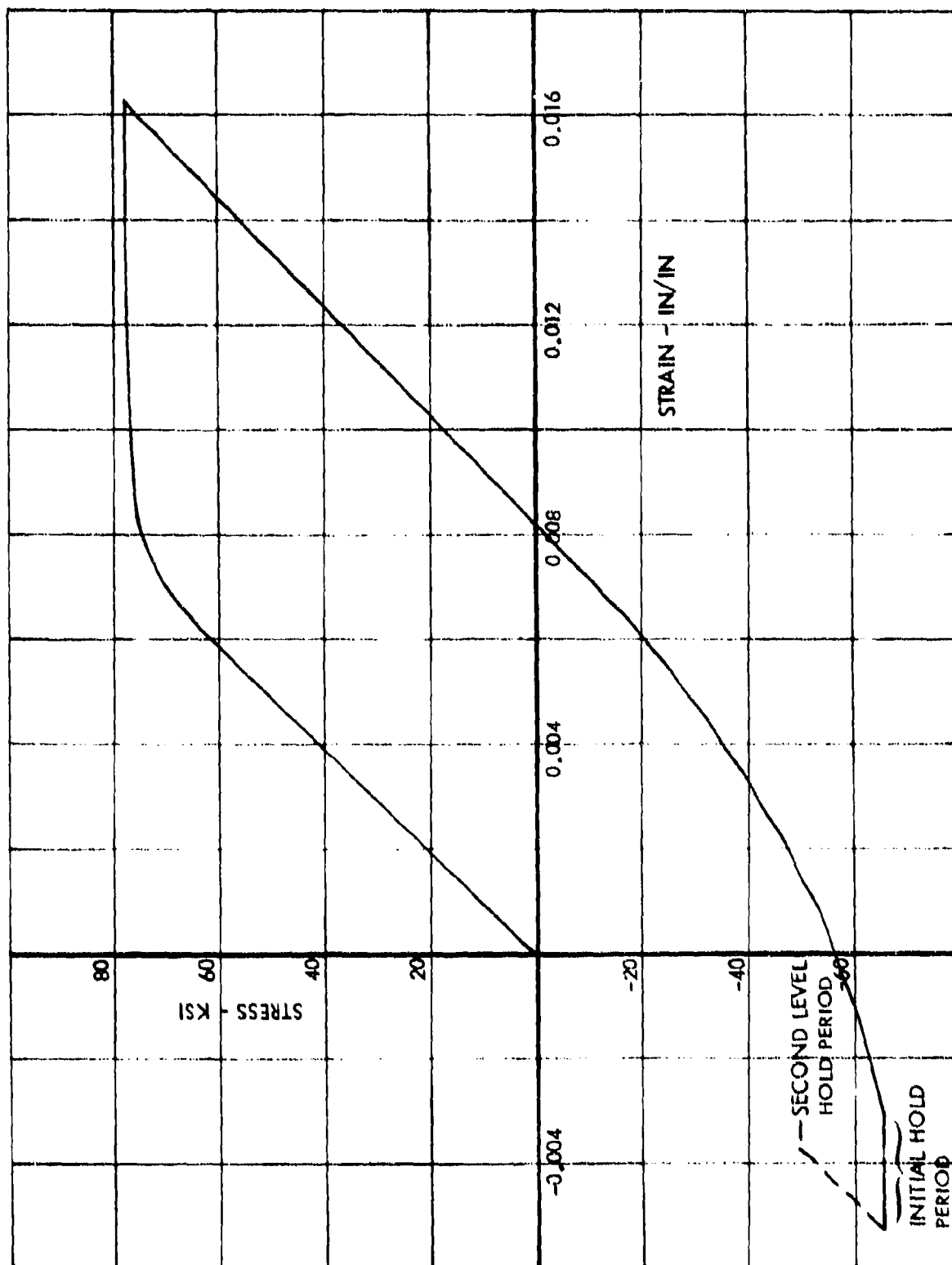


Figure 18. Stress-Strain Curve, Coupon Specimen 2A
Load Control Test

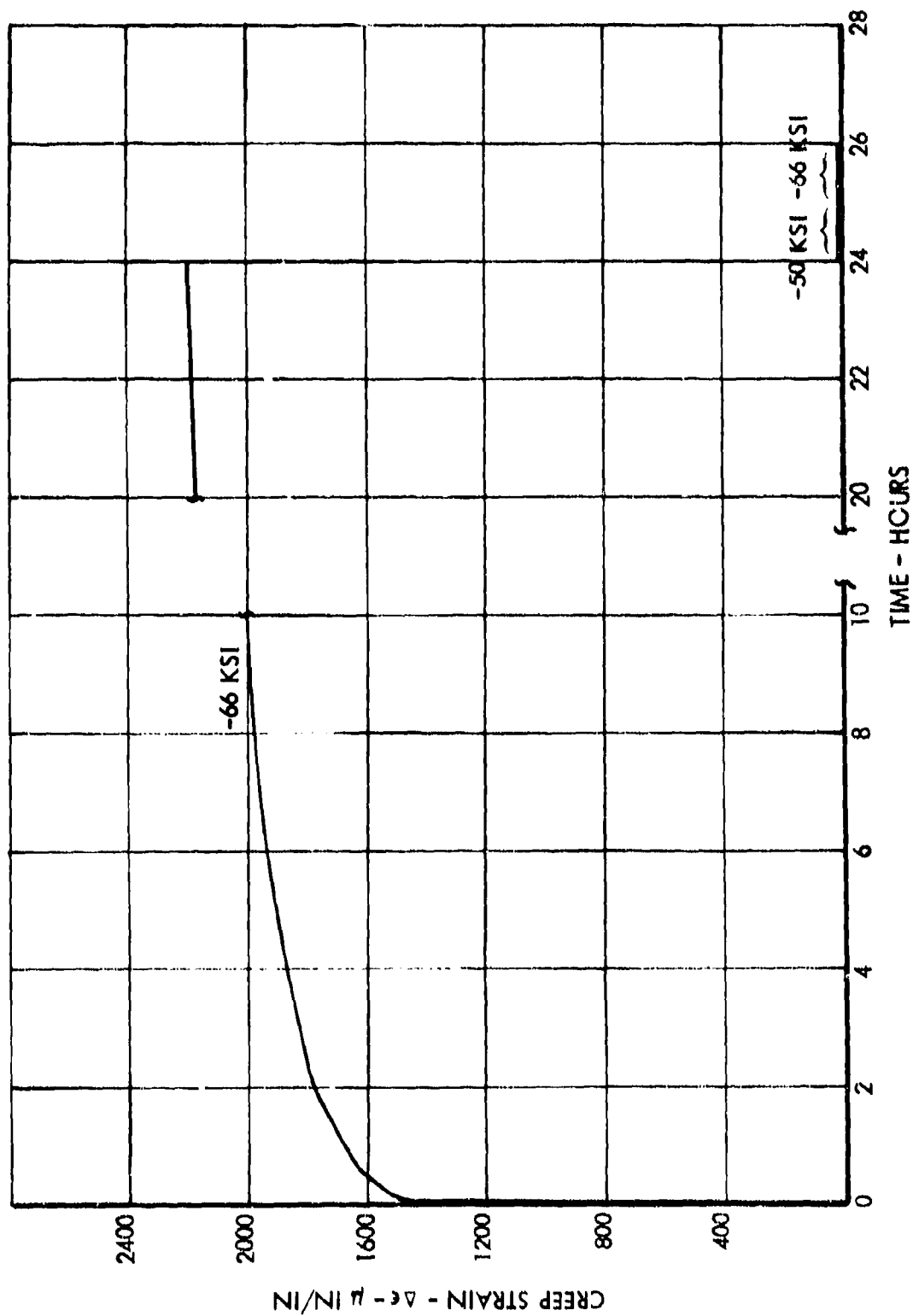


Figure 19. Coupon 2A Creep Strain

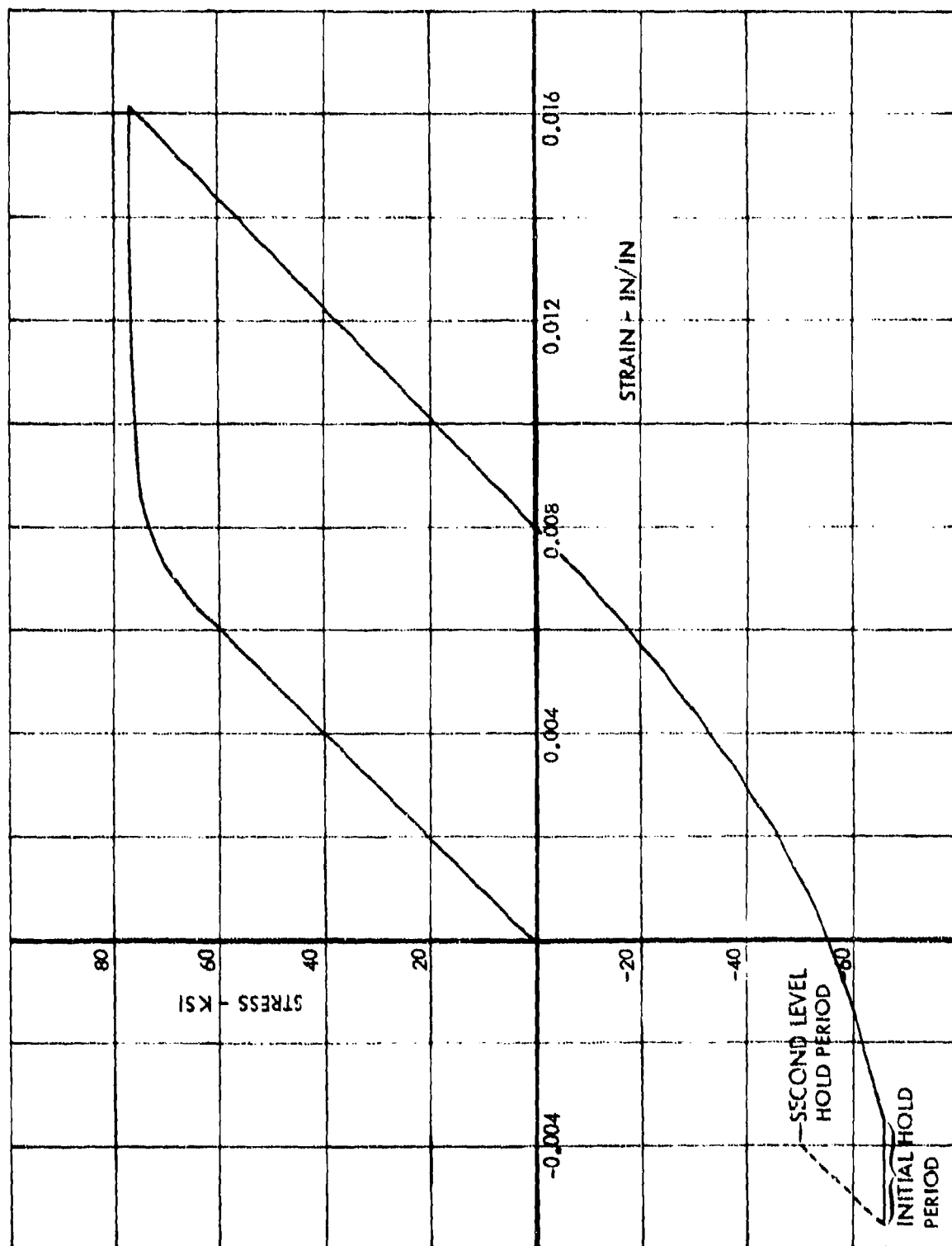


Figure 20. Stress-Strain Curve, Coupon Specimen 2B
Load Control Test

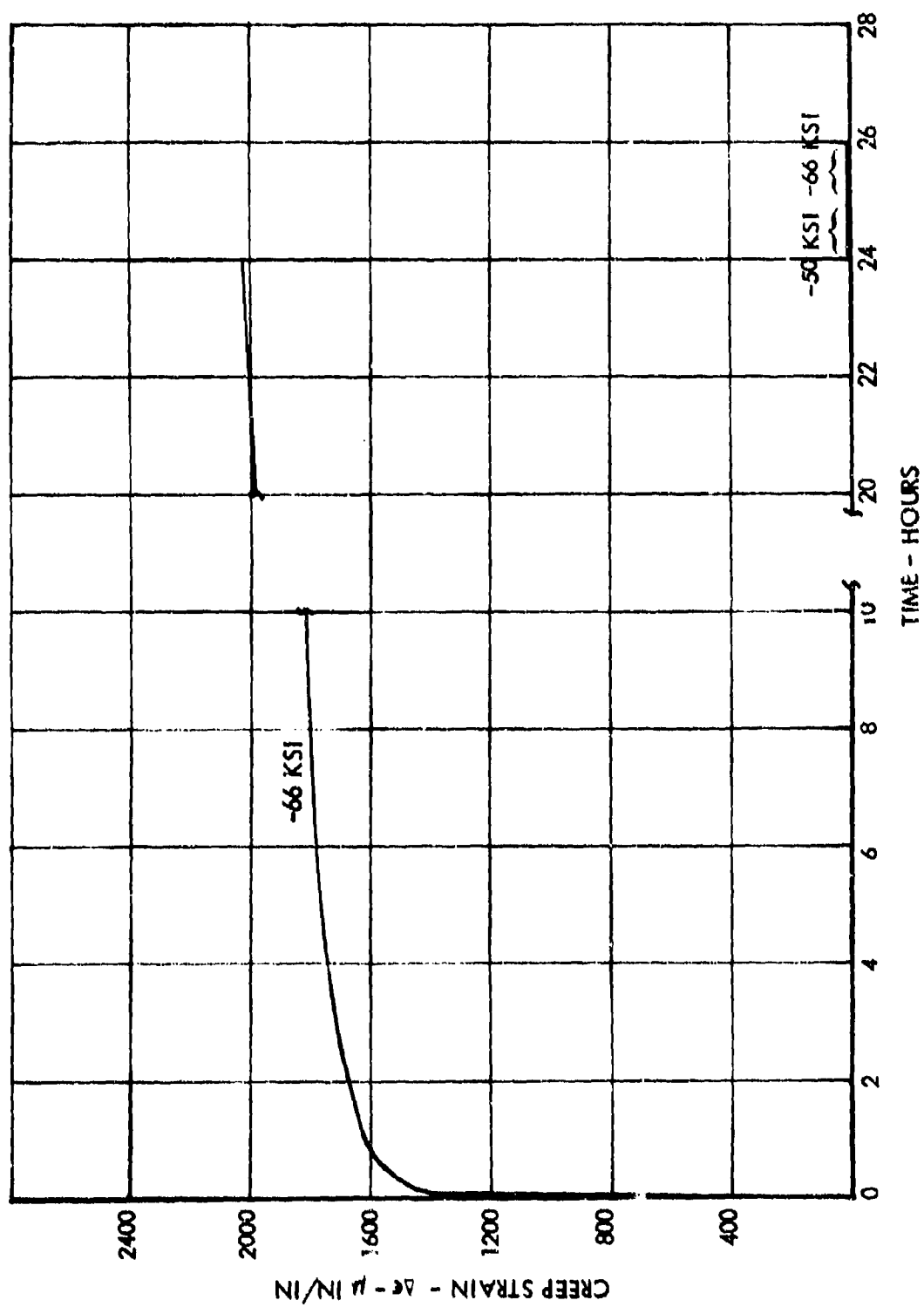


Figure 21. Coupon 2B Creep Strain

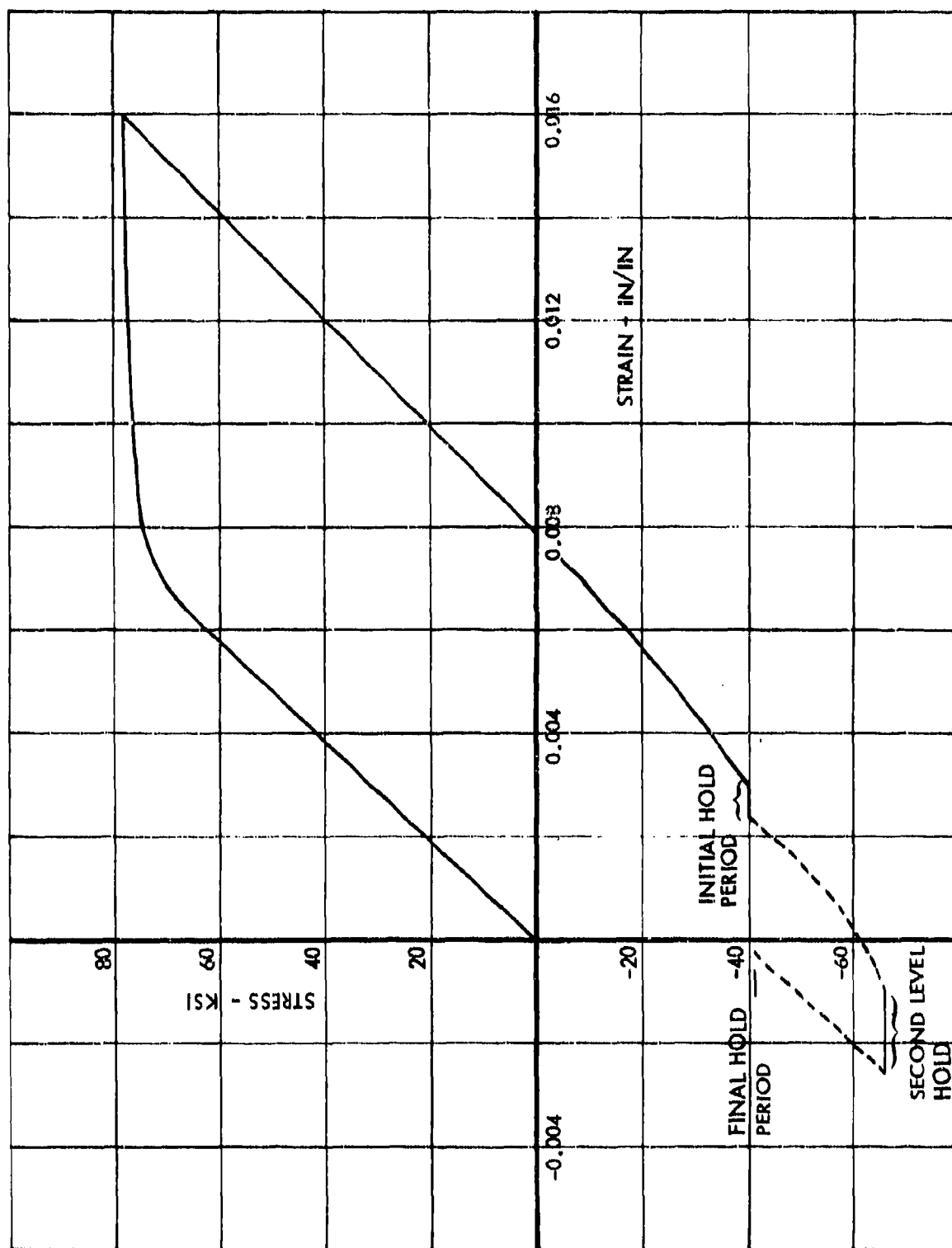


Figure 22 . Stress-Strain Curve, Coupon Specimen 3A
Load Control Test

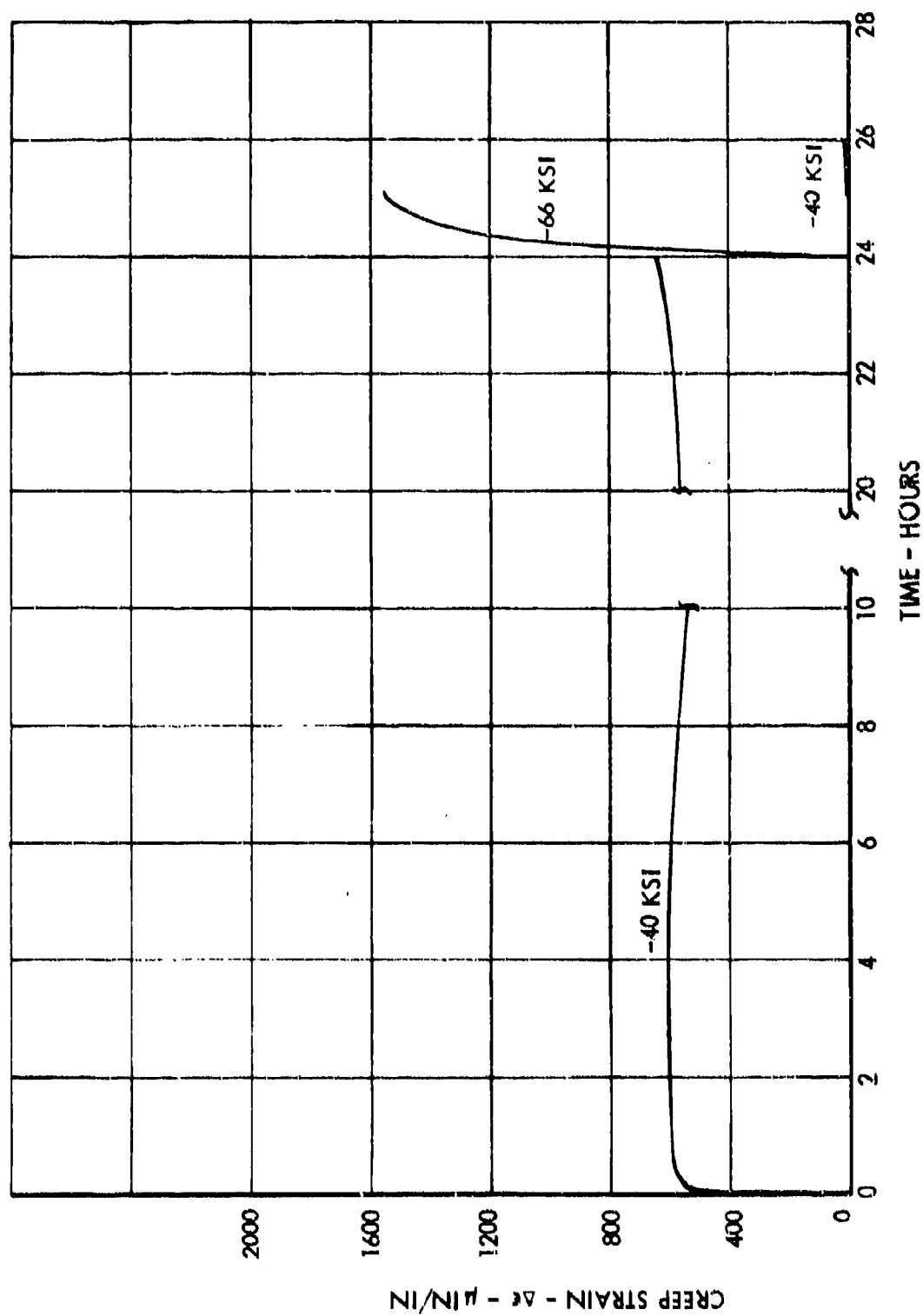


Figure 23. Coupon 3A Creep Strain

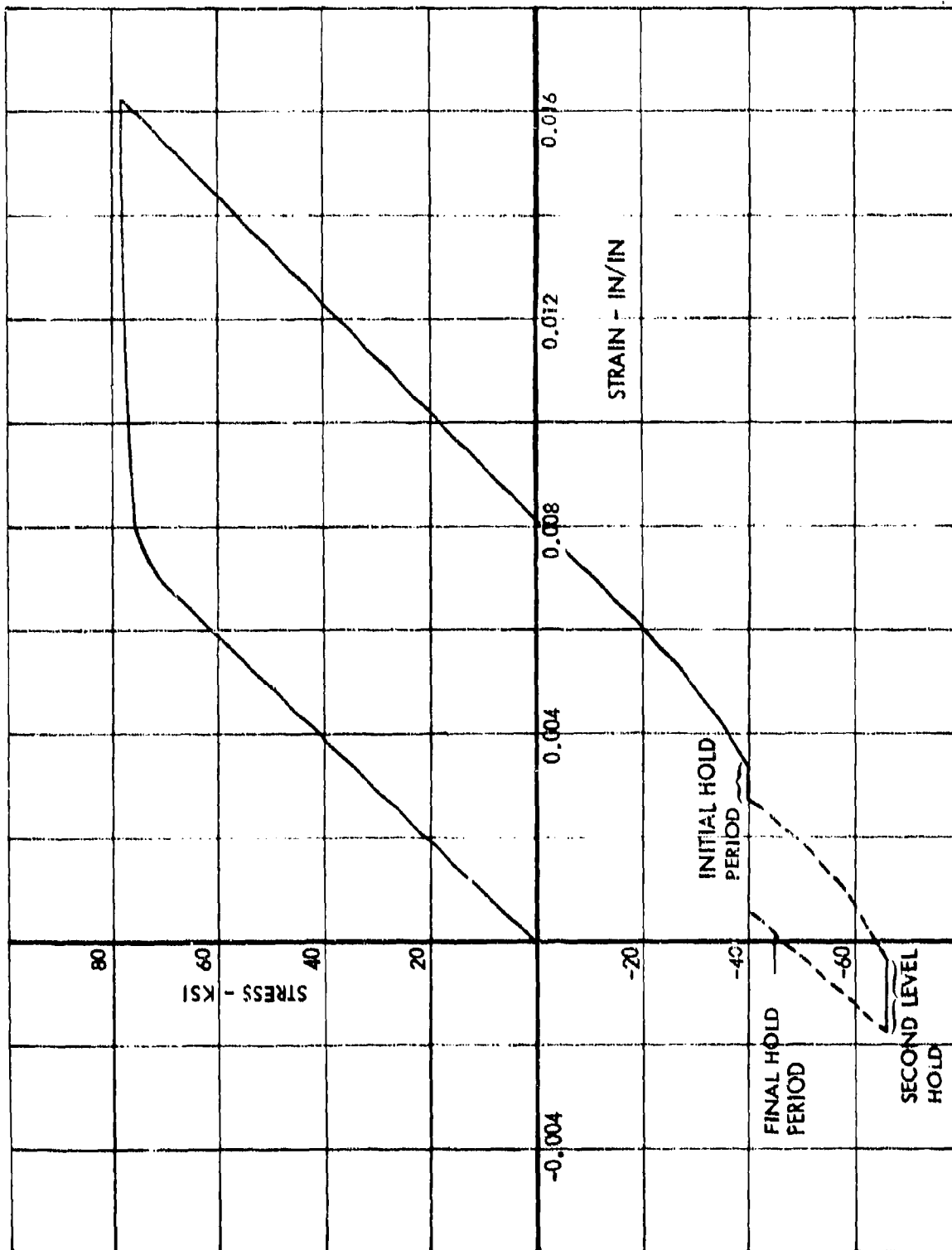


Figure 24. Stress-Strain Curve, Coupon Specimen 38
Load Control Test

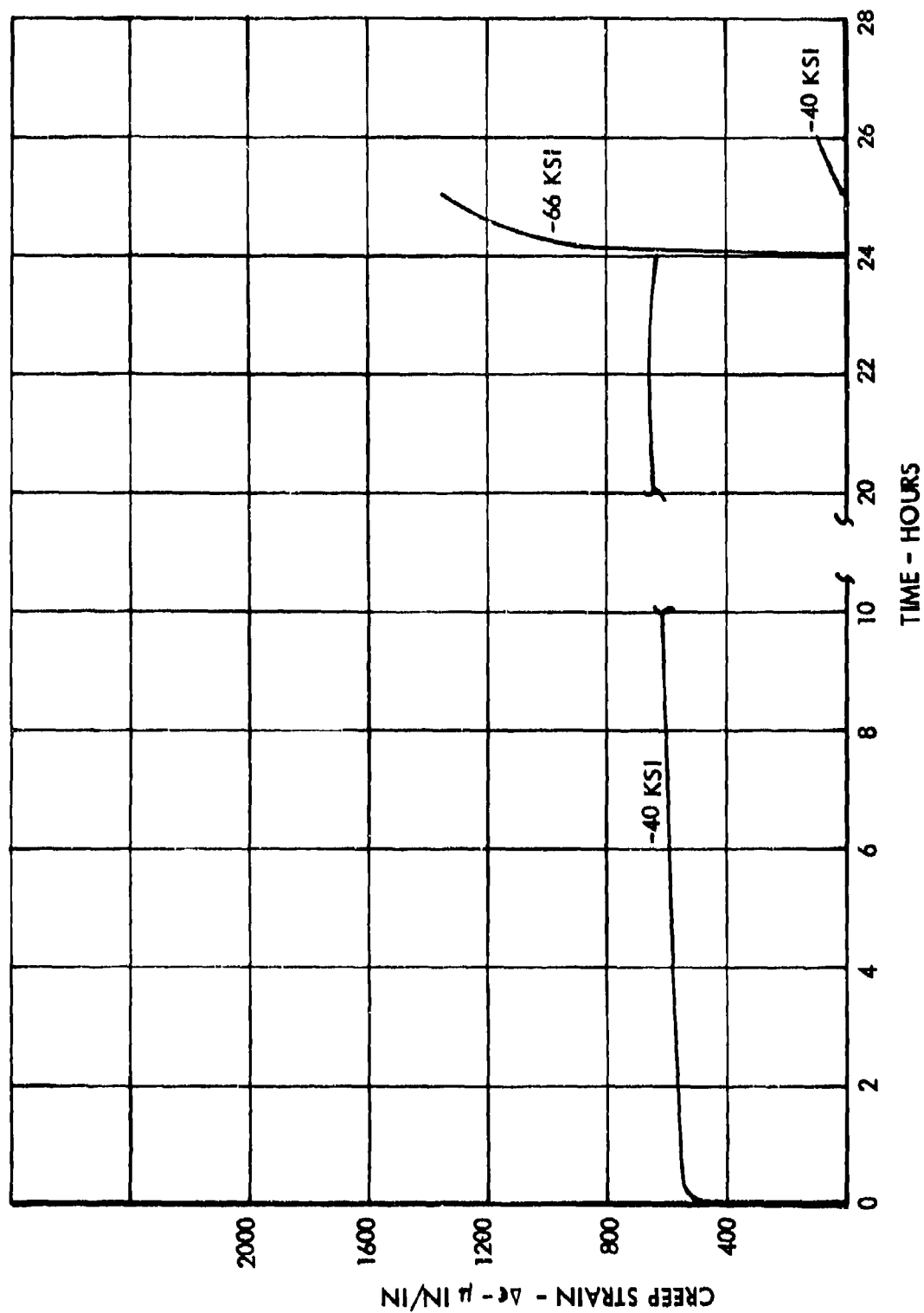


Figure 25. Coupon 38 Creep Strain

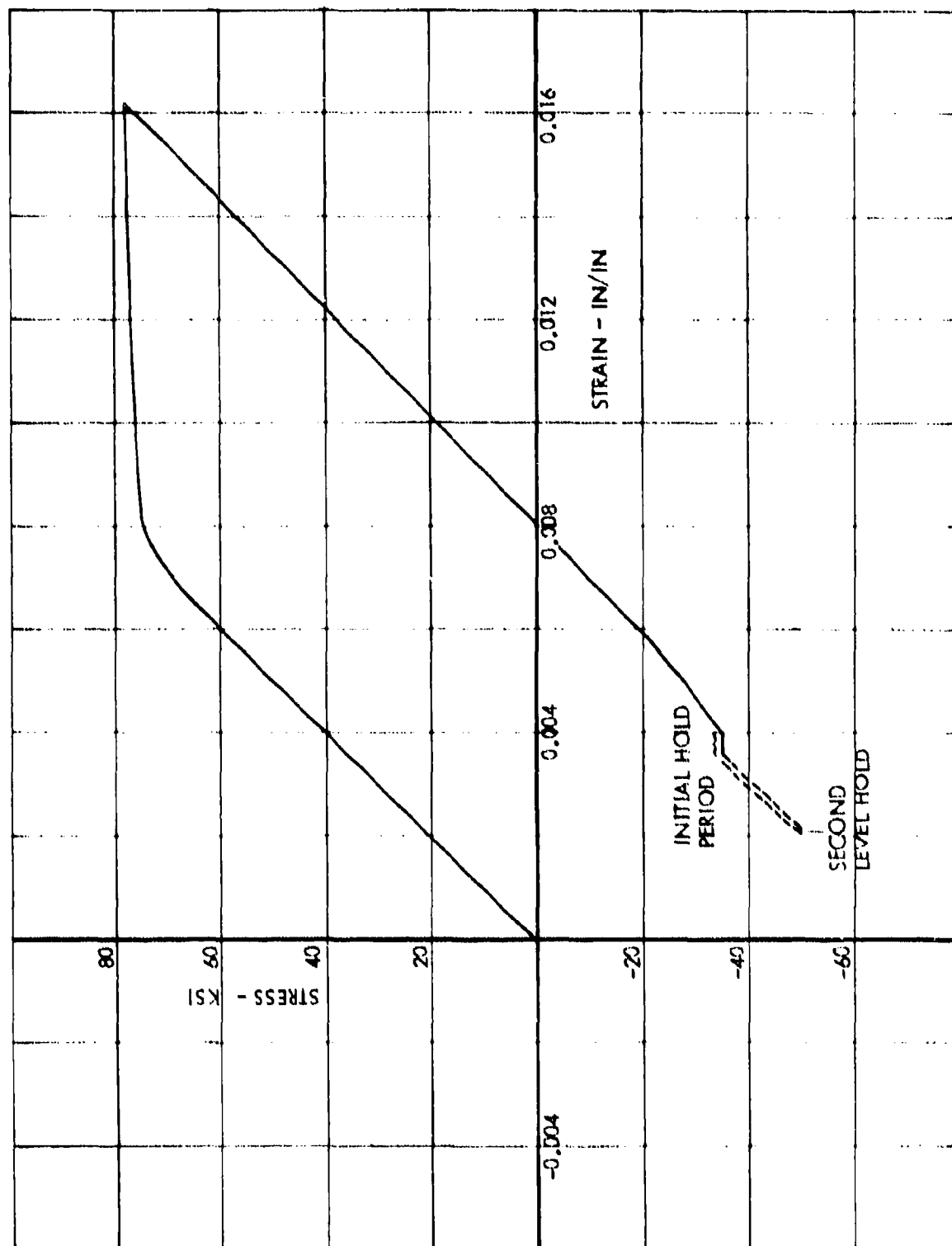


Figure 26. Stress-Strain Curve, Coupon Specimen 4A
Load Control Test

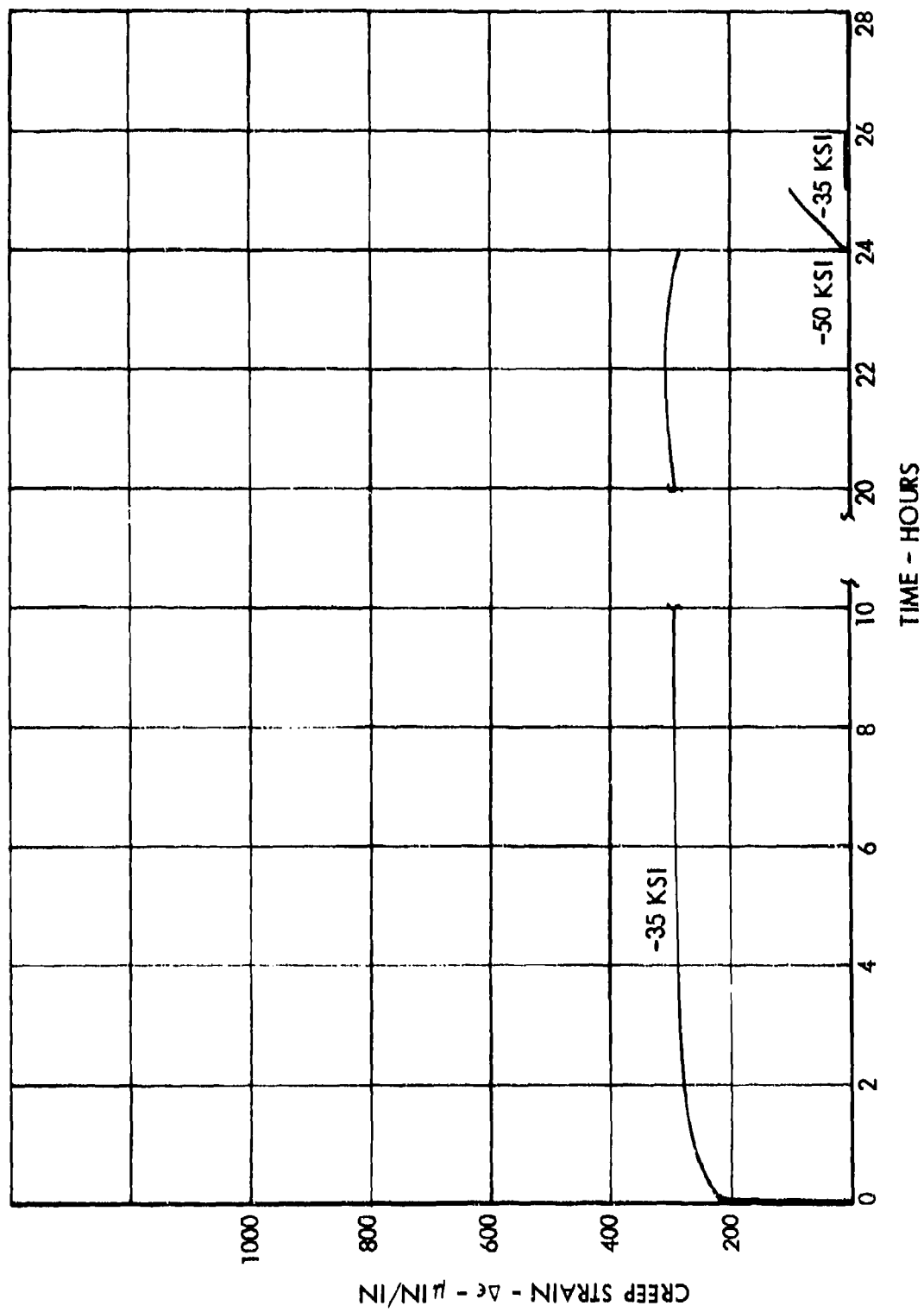


Figure 27. Coupon 4A Creep Strain

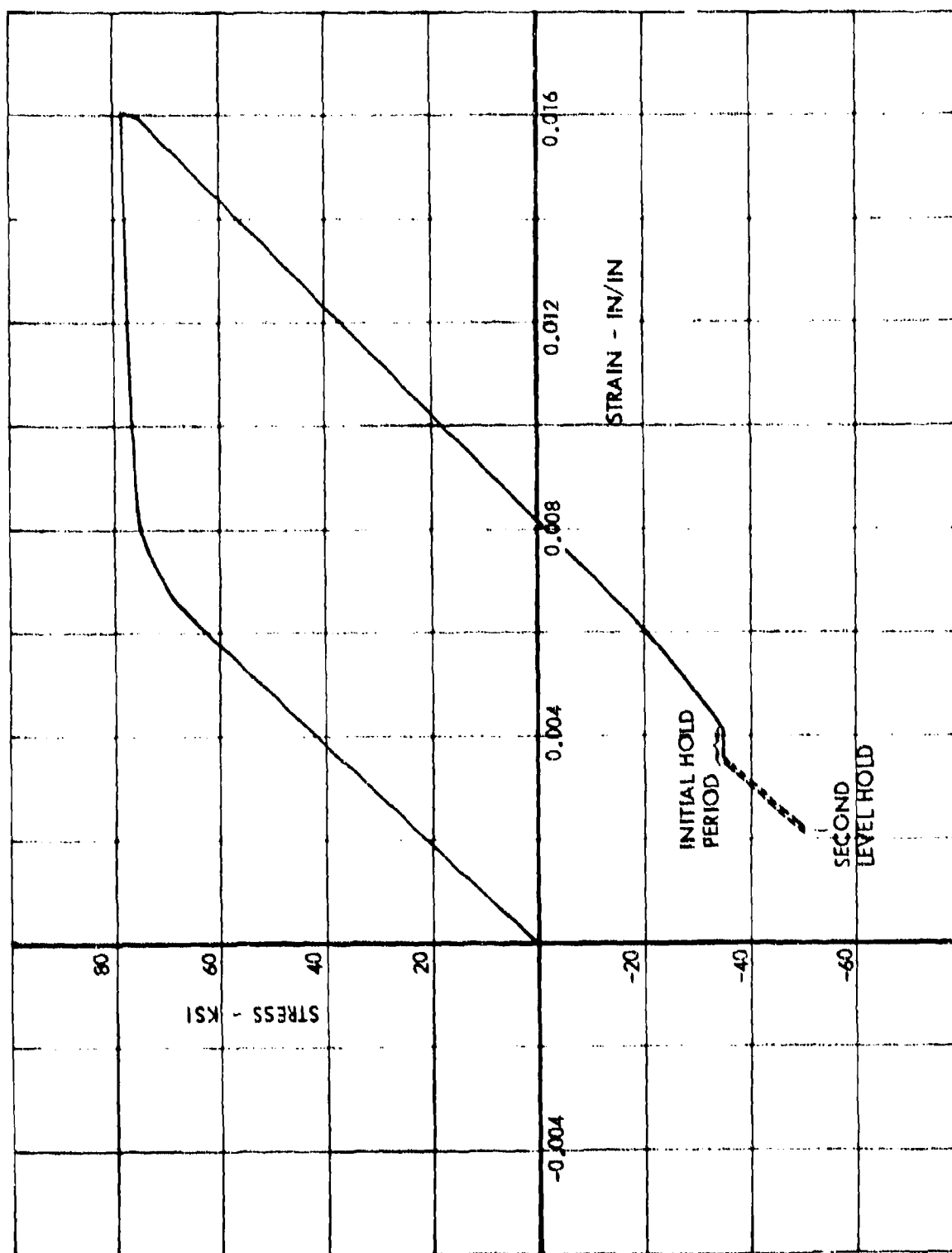


Figure 28. Stress-Strain Curve, Coupon Specimen 4B
Load Control Test

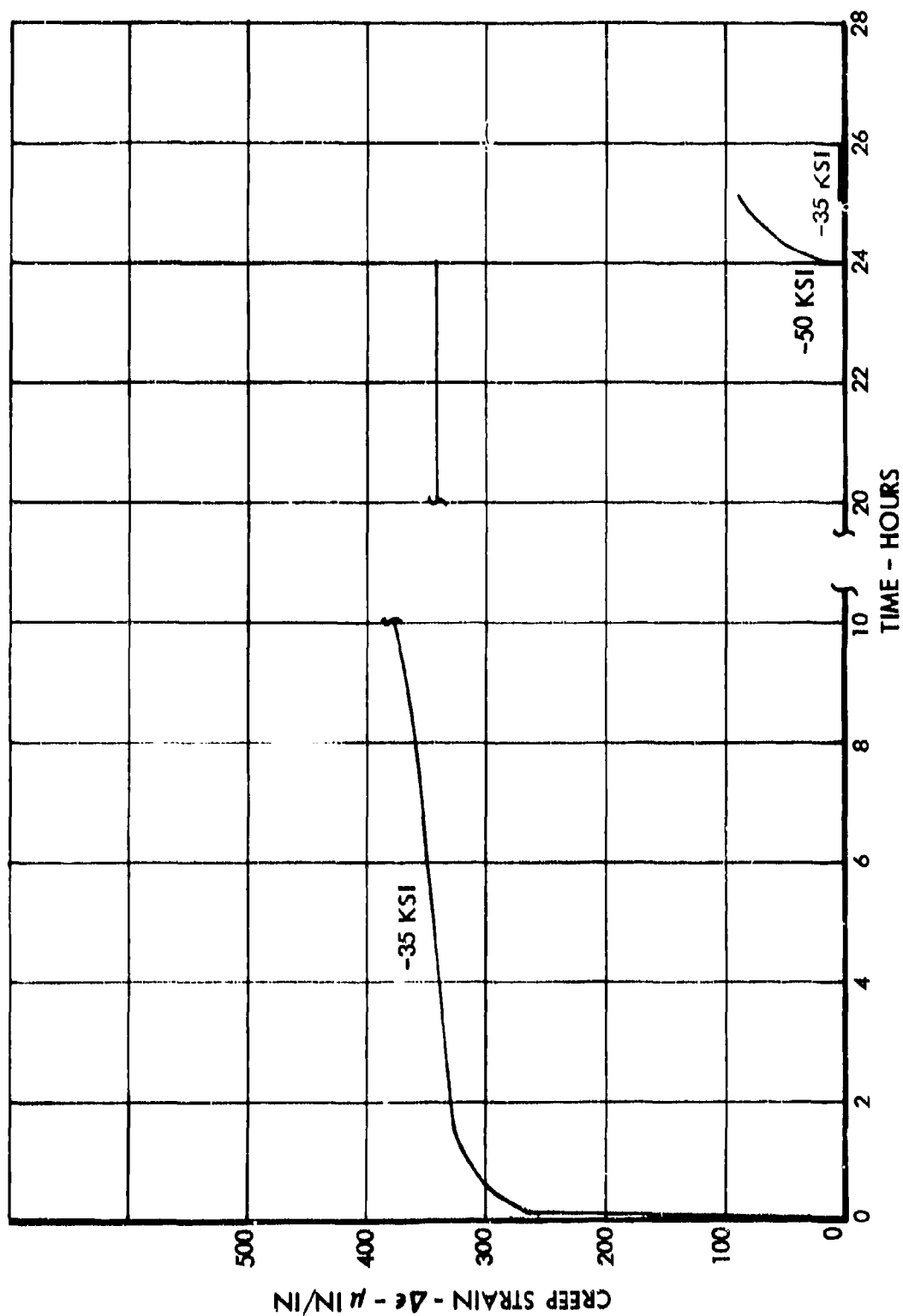


Figure 29. Coupon 4B Creep Strain

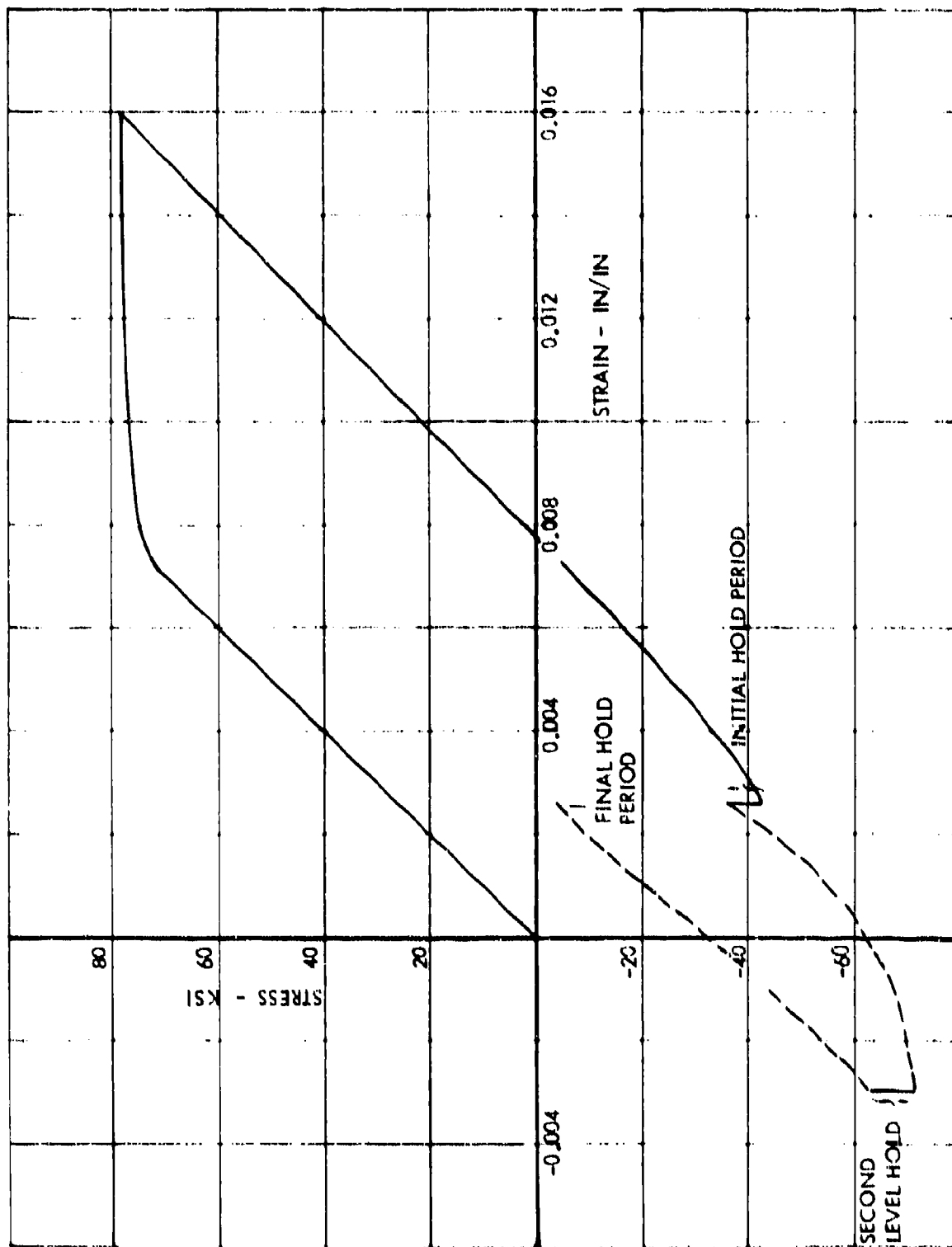


Figure 30. Stress-Strain Curve, Coupon Specimen 5A
Strain Control Test

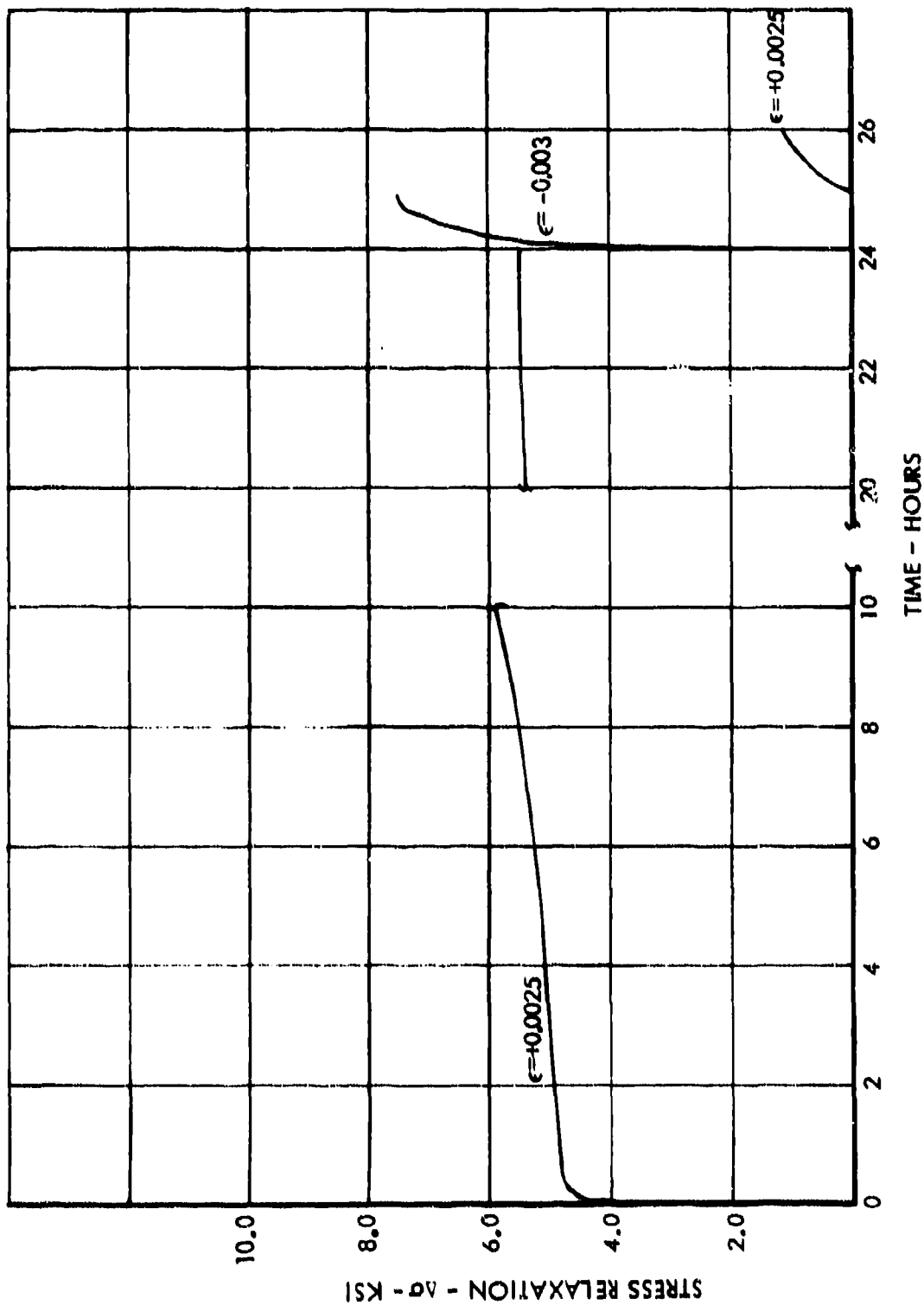


Figure 31 . Coupon 5A Stress Relaxation

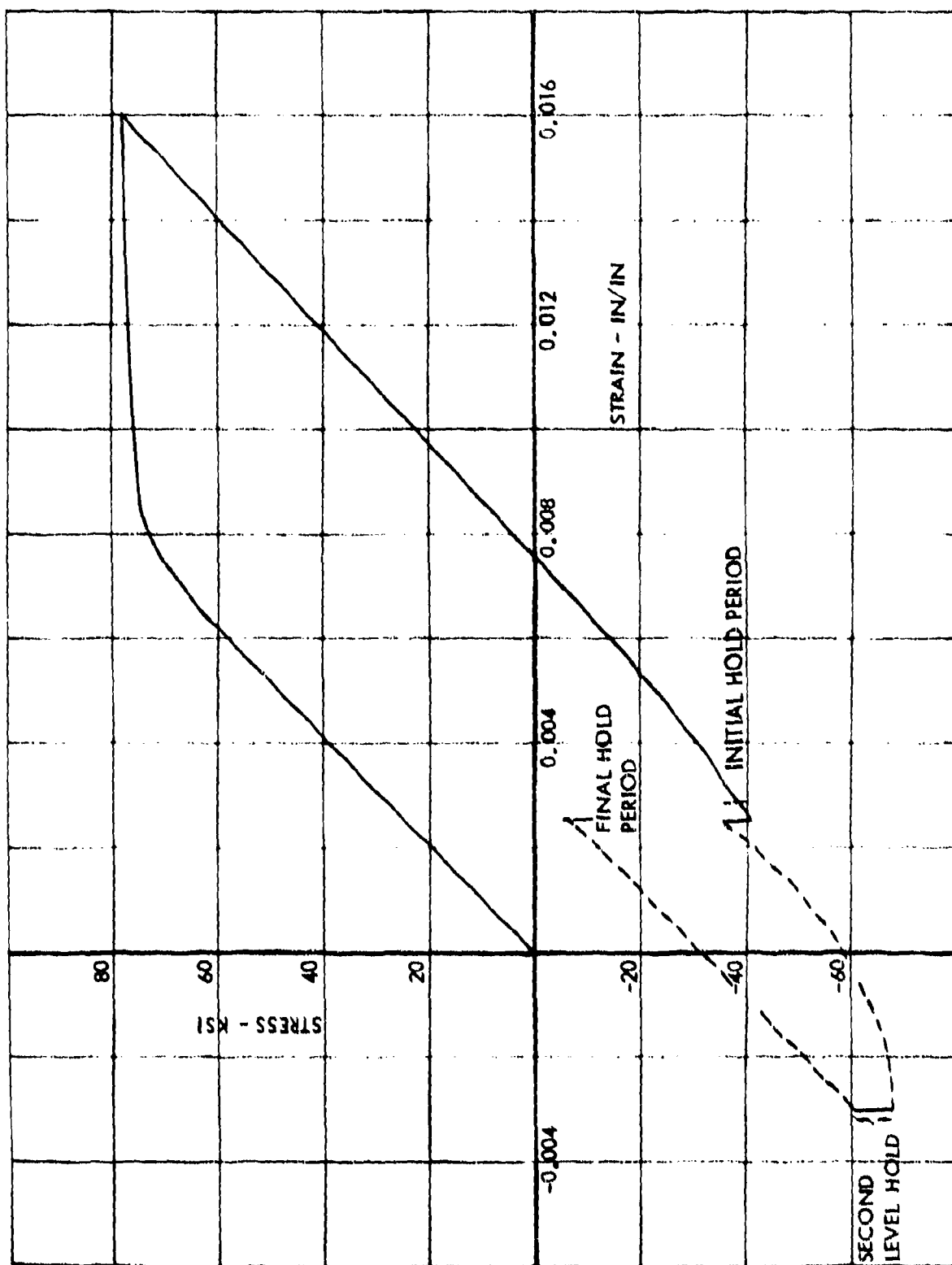


Figure 32. Stress-Strain Curve, Coupon Specimen 58
Strain Control Test

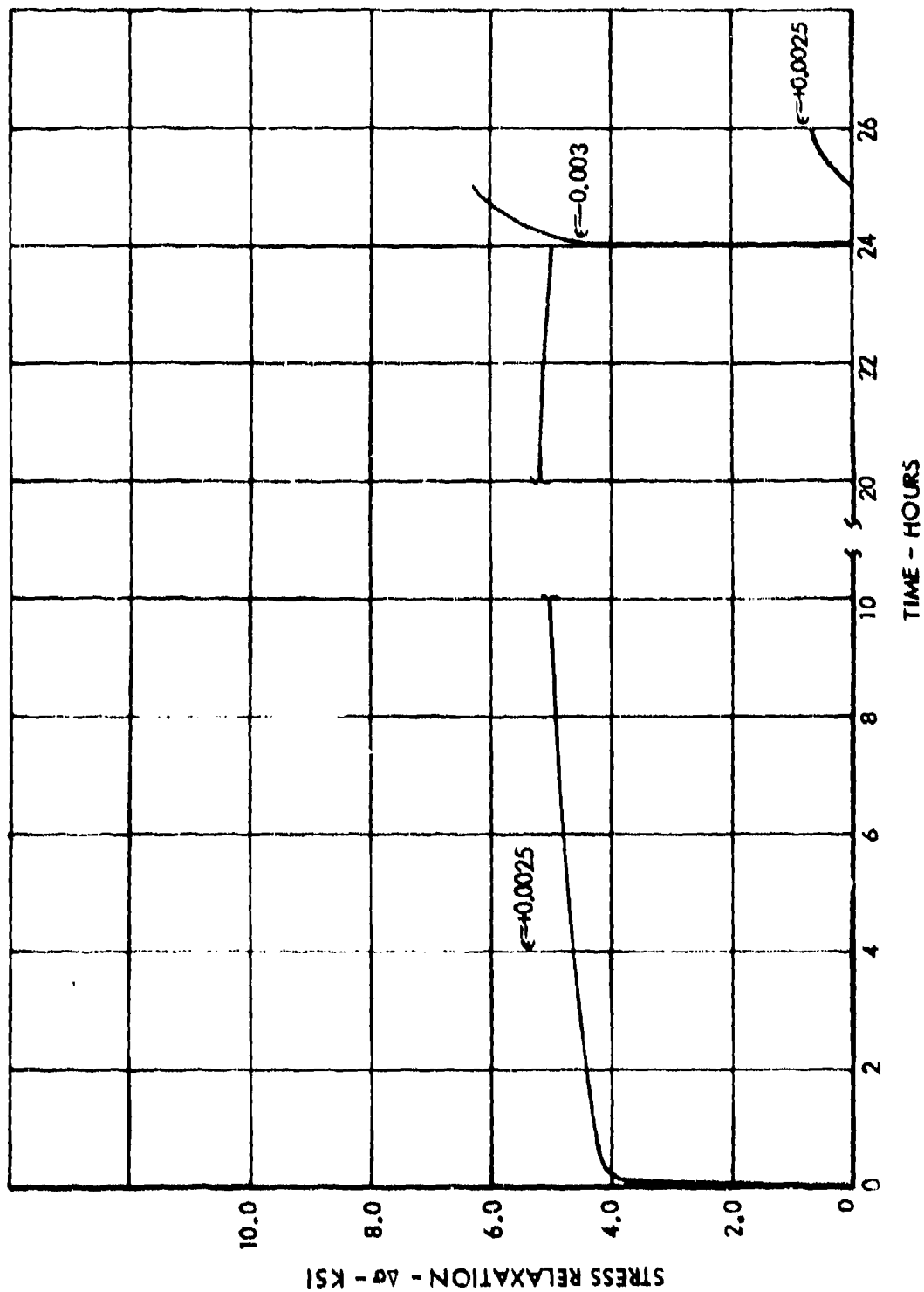


Figure 33. Coupon 5B Stress Relaxation

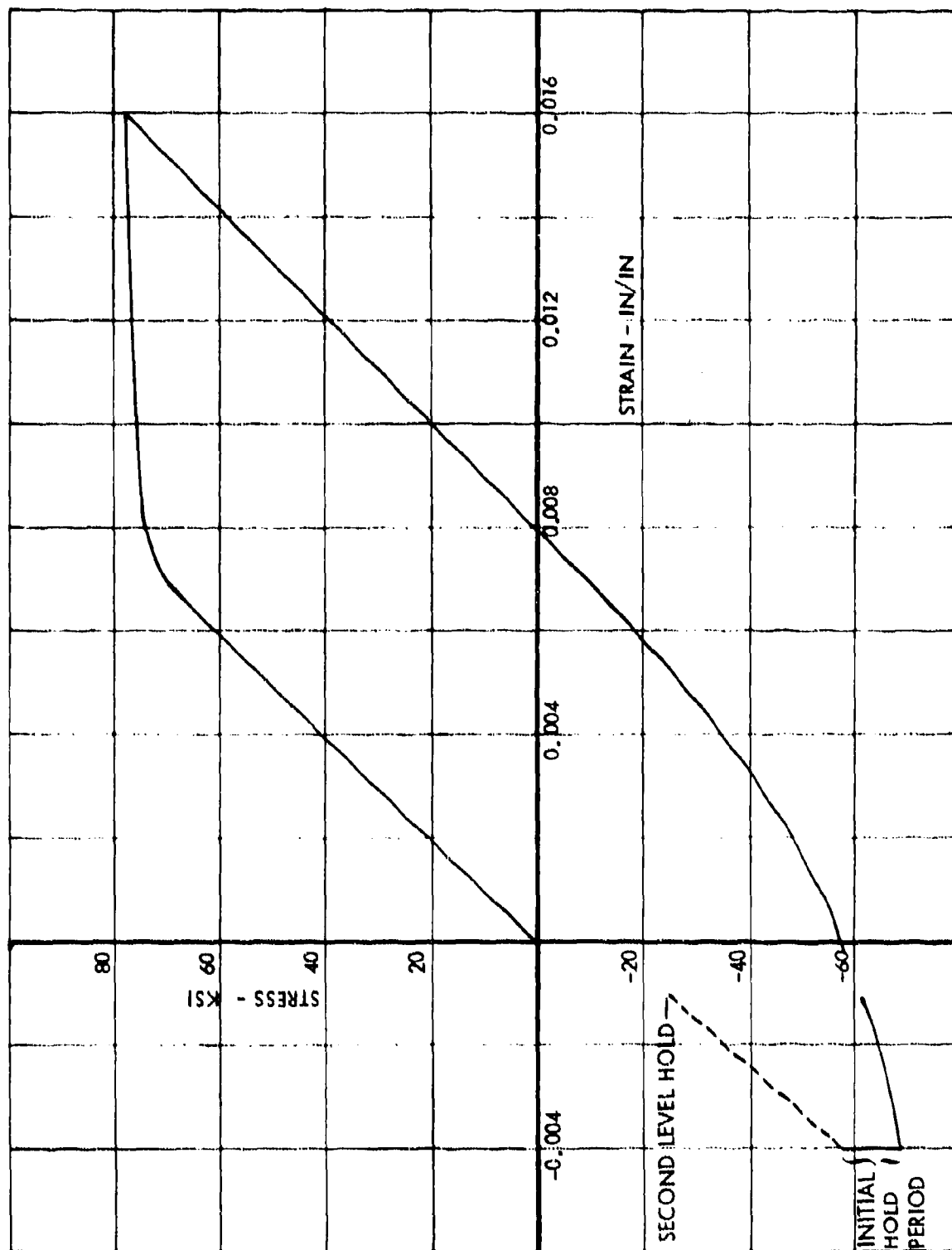


Figure 34. Stress-Strain Curve Coupon Specimen 6A
Strain Control Test

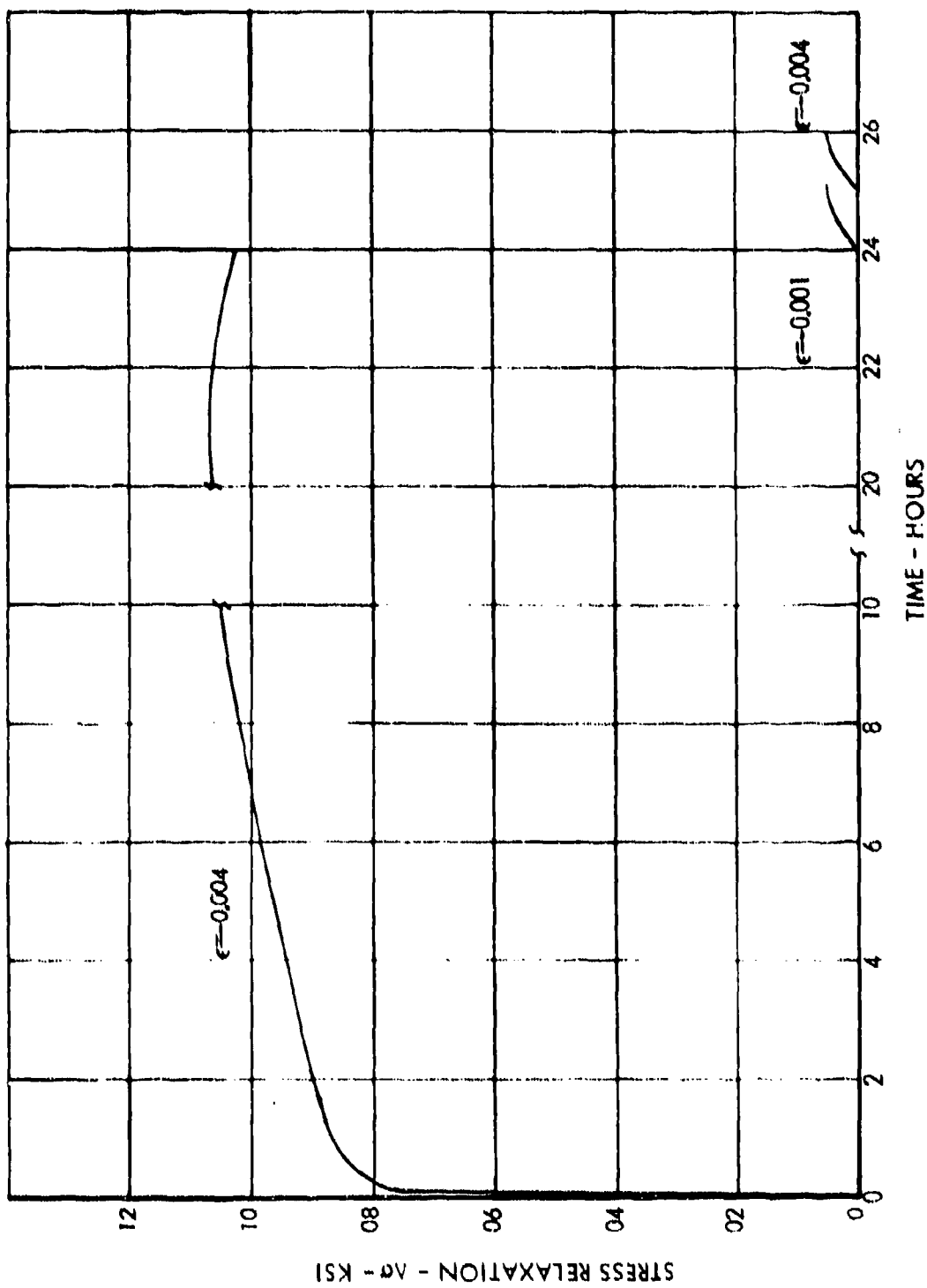


Figure 35. Coupon 6A Stress Relaxation

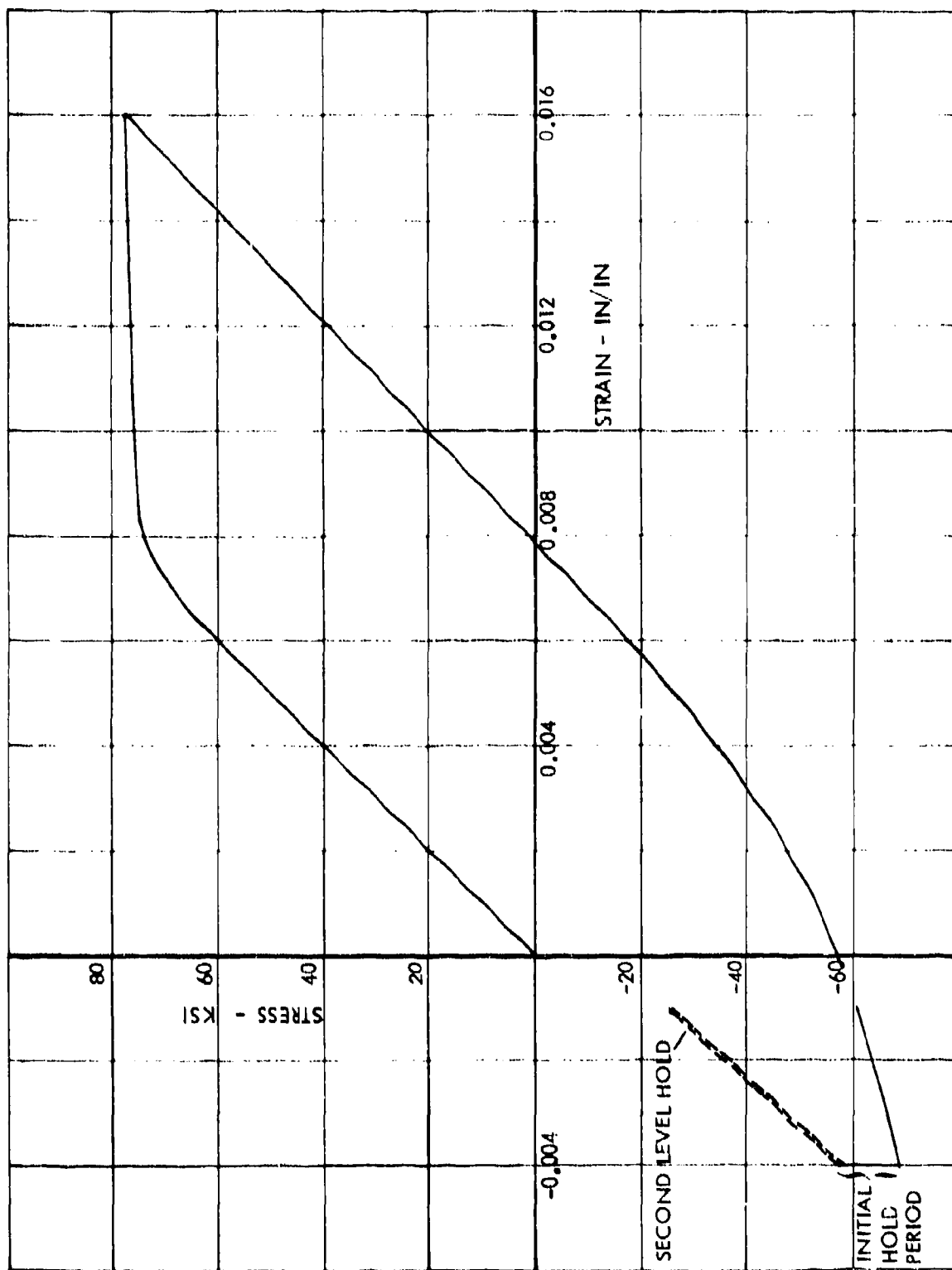


Figure 36. Stress-Strain Curve Coupon Specimen 68
Strain Control Test

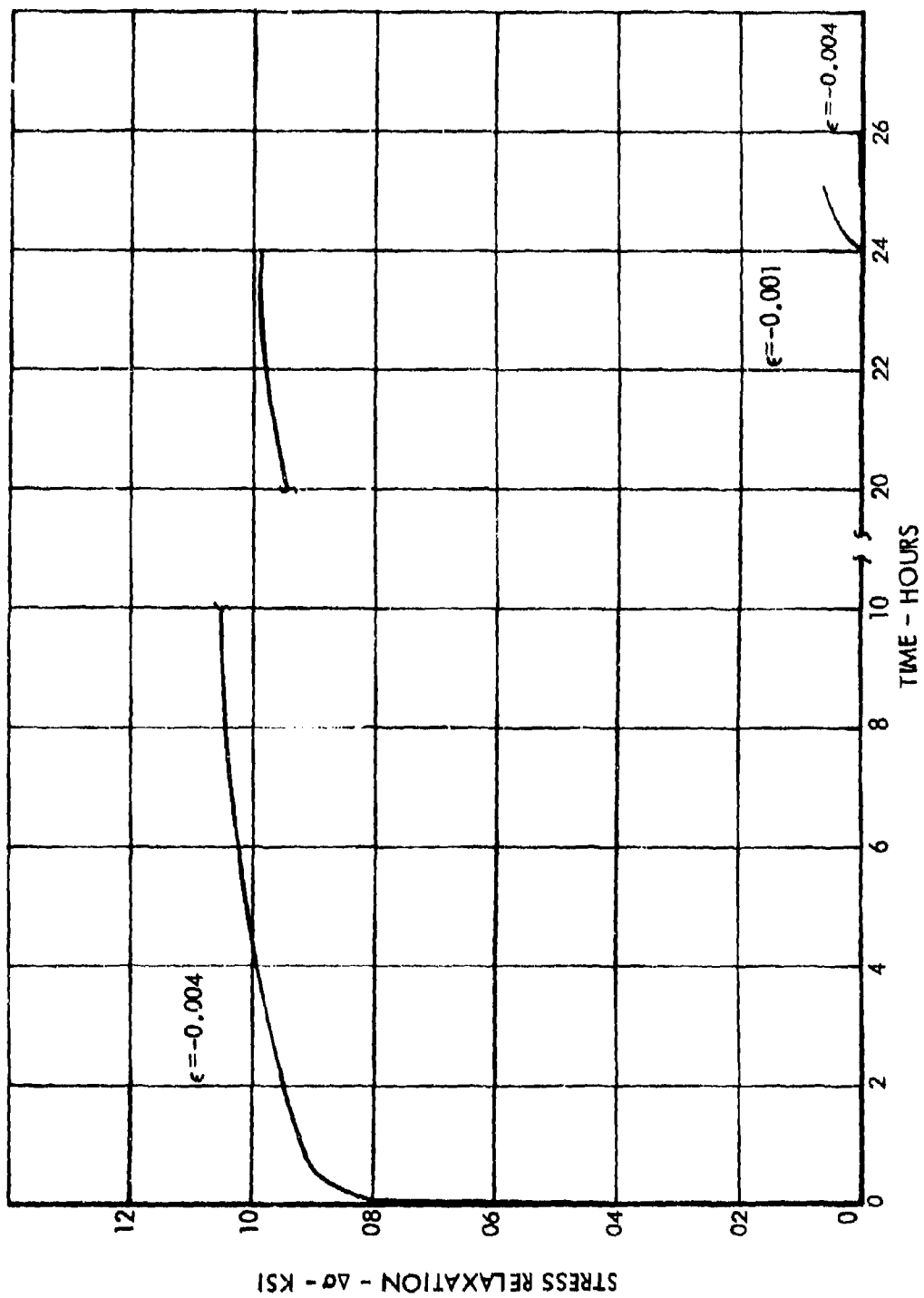


Figure 37. Coupon 6B Stress Relaxation

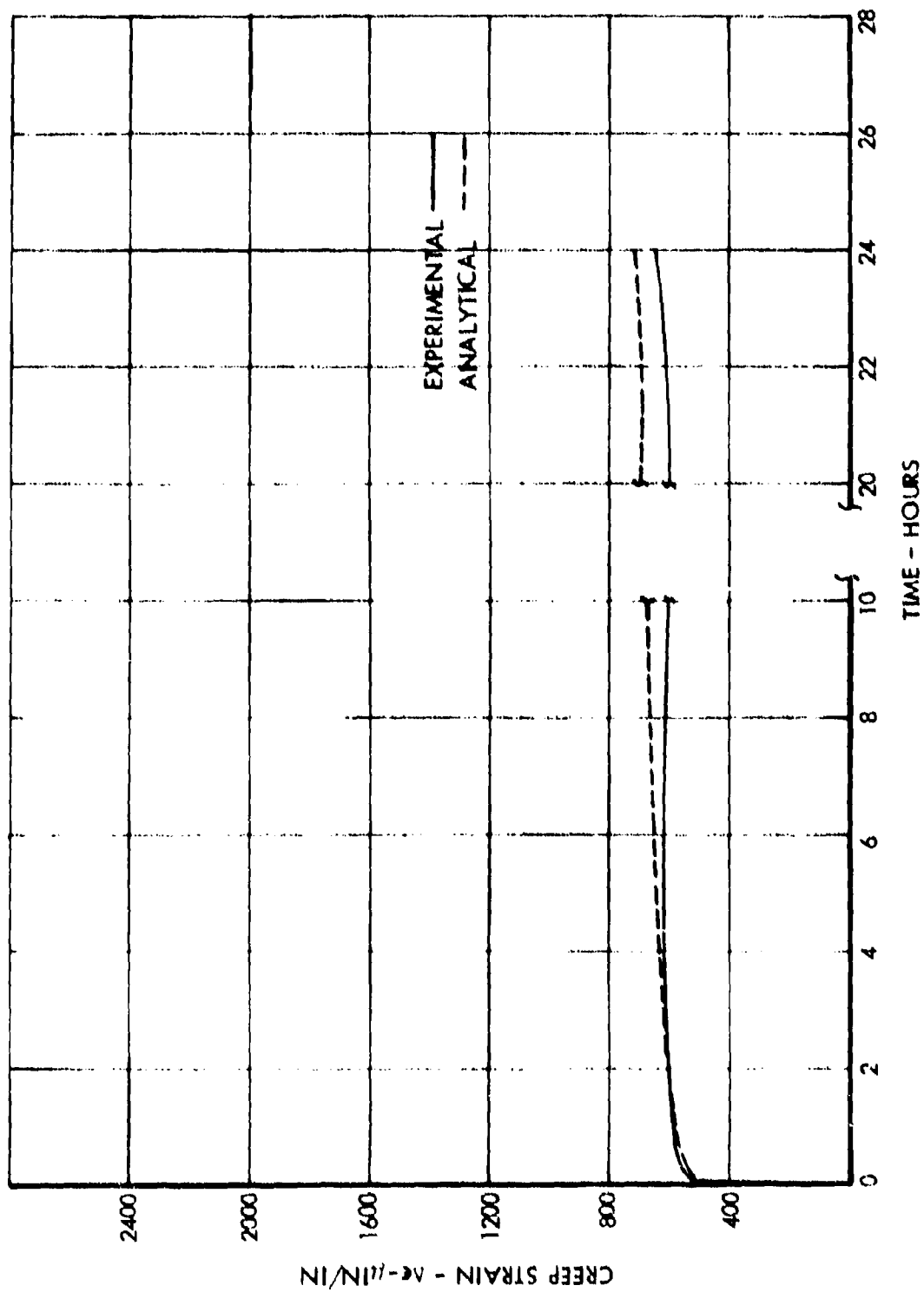


Figure 38. Comparison of Predicted and Measured Creep at -40 Ksi

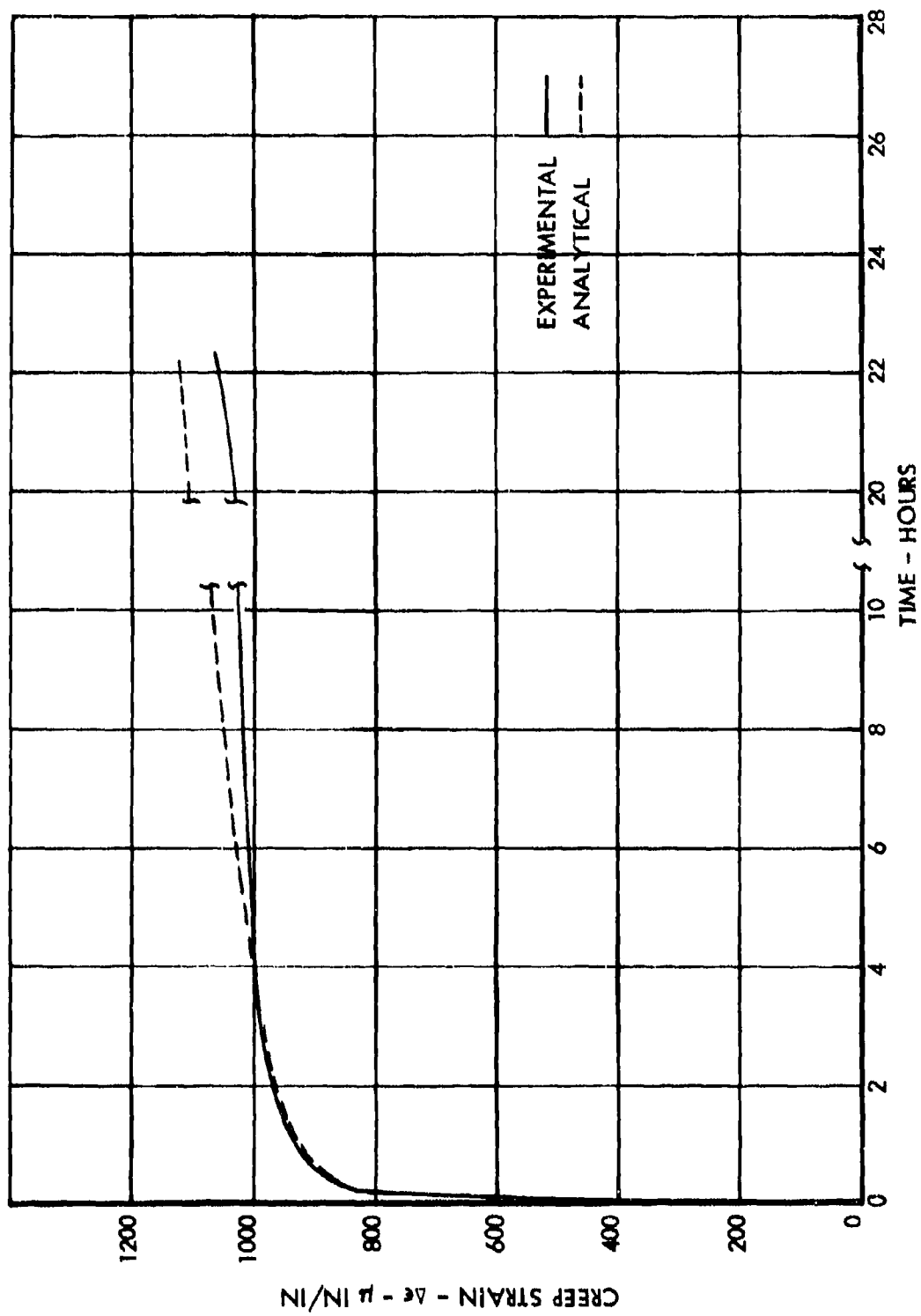


Figure 39. Comparison of Predicted and Measured Creep at -50 Ksi

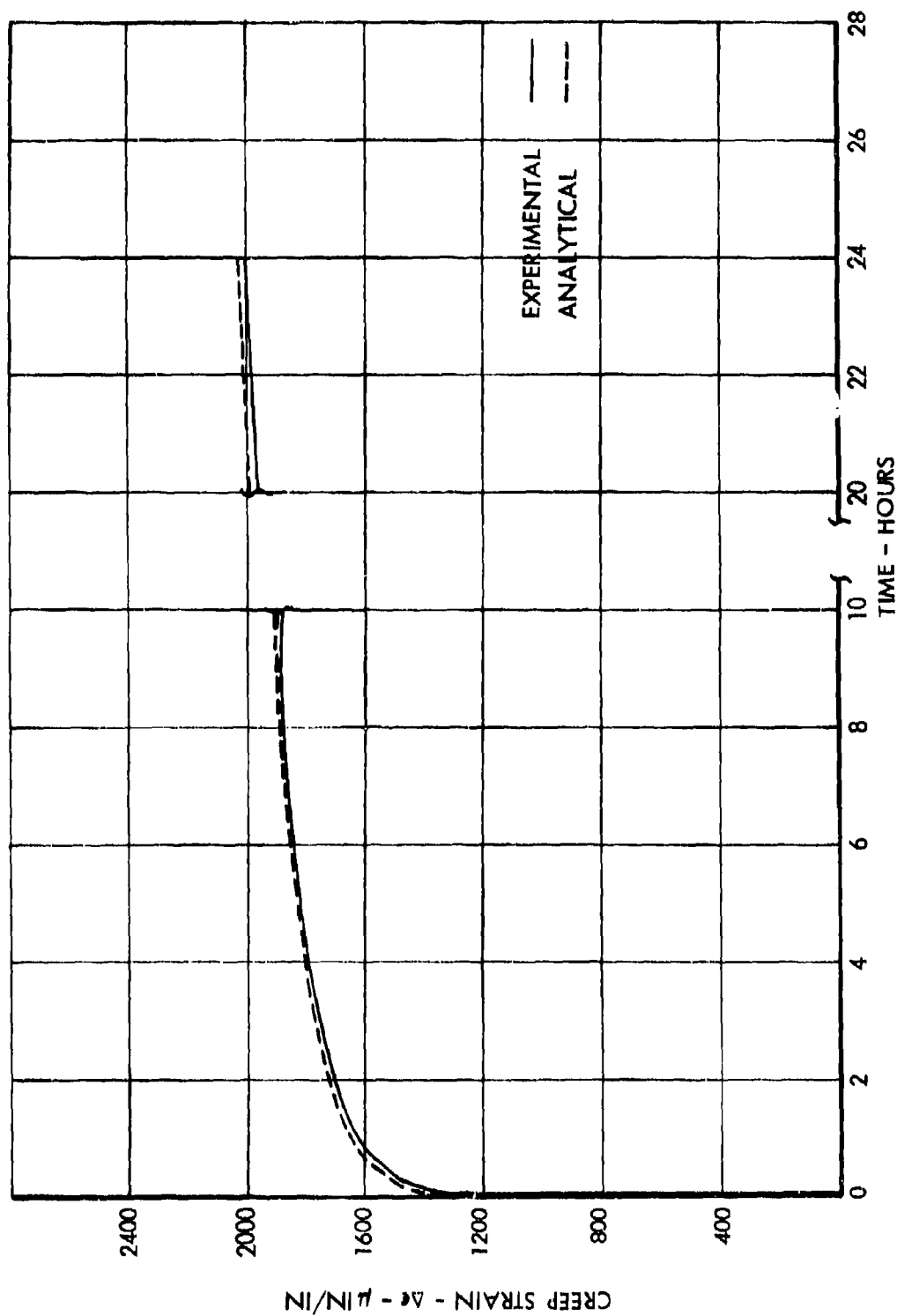


Figure 40. Comparison of Predicted and Measured Creep at -66 Ksi

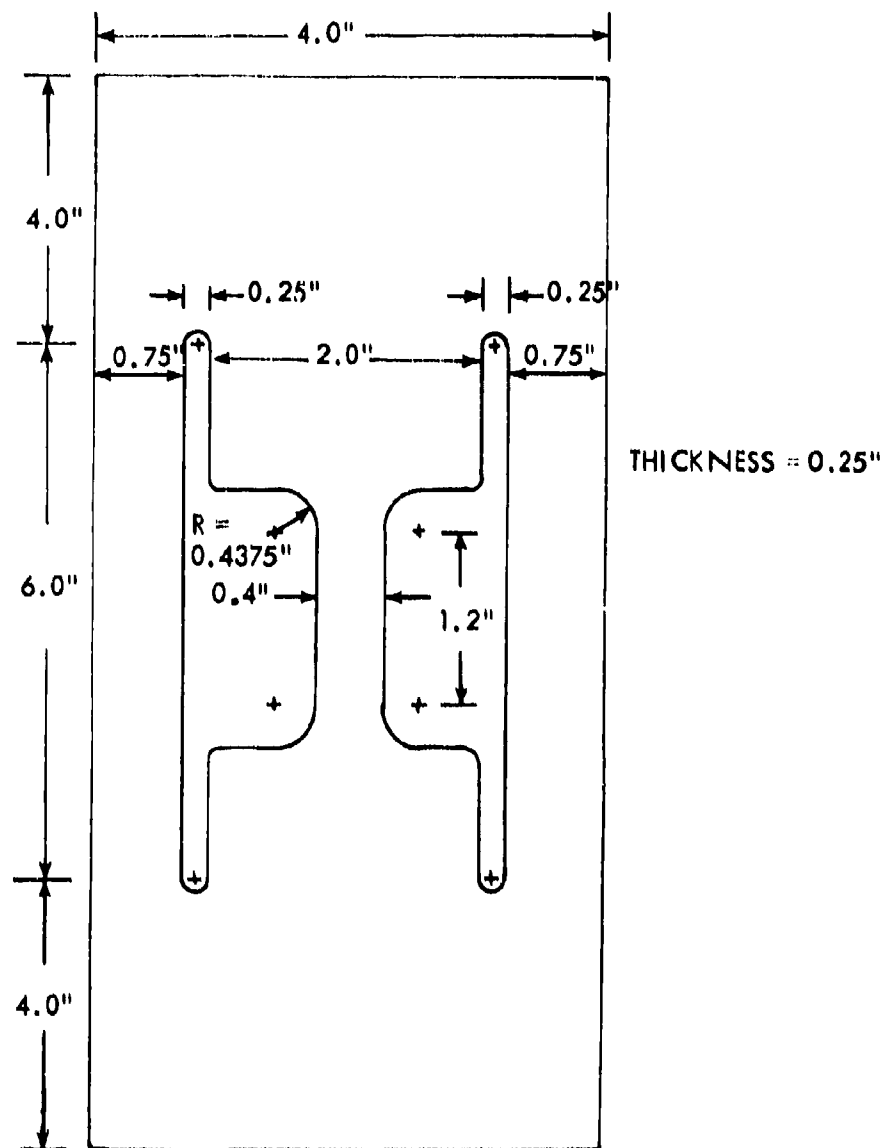


Figure 41. Simplified Stress Concentration Specimen

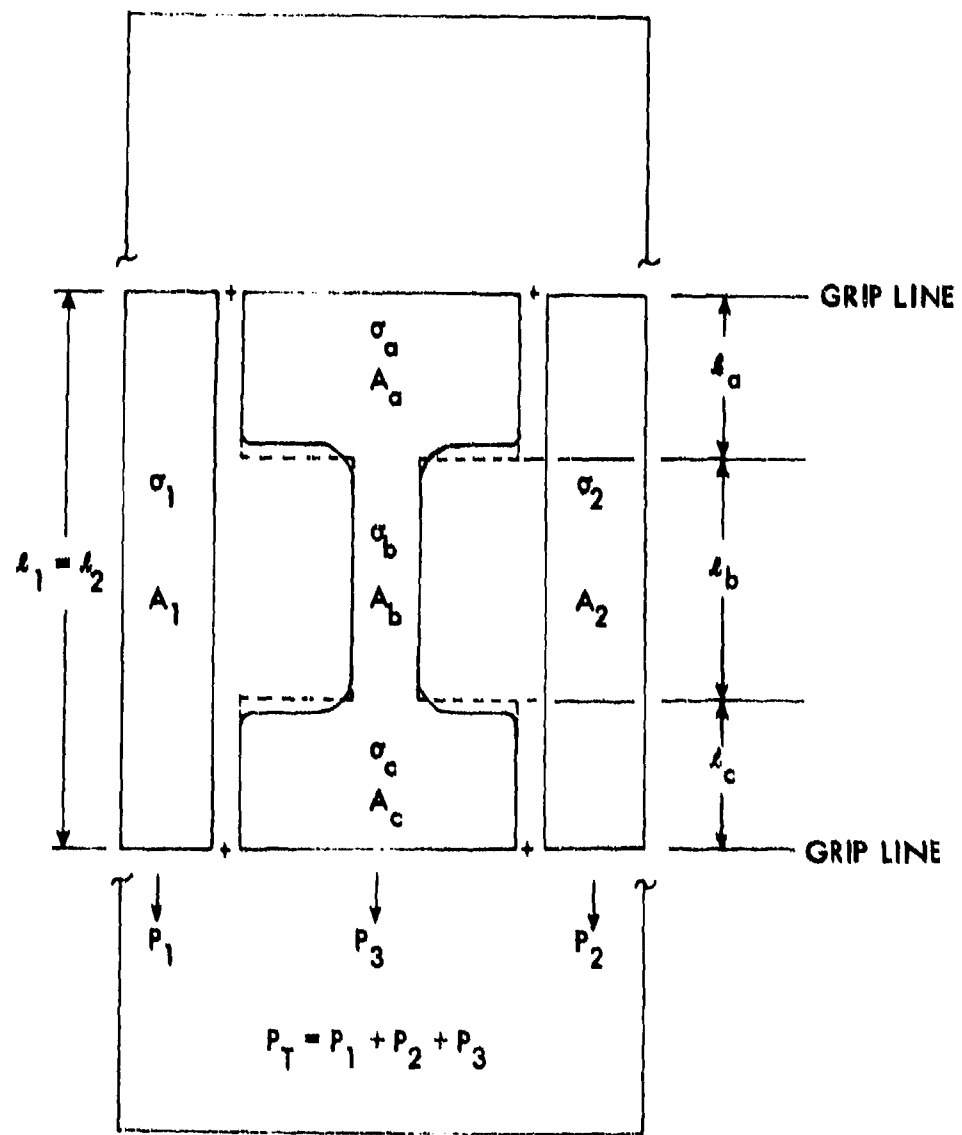
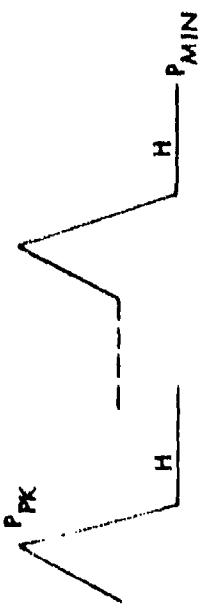
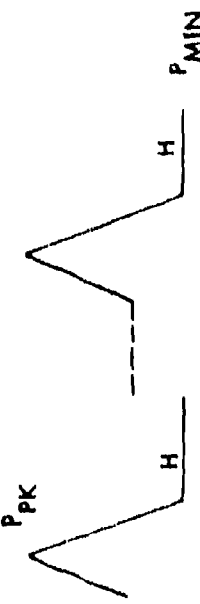
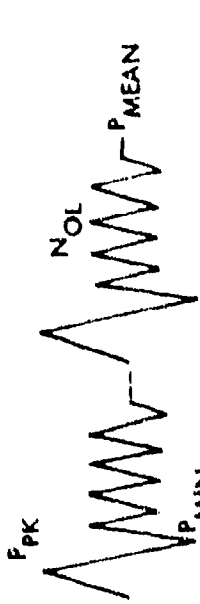
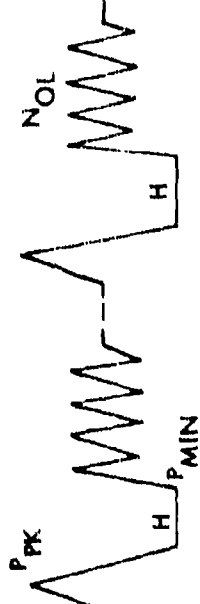


Figure 42. Schematic Representation of Simplified Stress Concentration Specimen

TEST SEQUENCE	LOADING CONDITIONS	COMMENTS
	$P_{PK} = 25 \text{ KIPS}$ $P_{MIN} = -9.5 \text{ KIPS}$ $H = 24 \text{ HRS.}$ $\epsilon_{PK} \approx 0.016 \text{ IN/IN}$ $\epsilon_{MIN} \approx 0.0015 \text{ IN/IN}$	Two Specimens Tested. See Data in Figures 45 through 48. Test for Creep/Stress Relaxation Only.
	$P_{PK} = 25 \text{ KIPS}$ $P_{MIN} = -5.8 \text{ KIPS}$ $H = 24 \text{ Hrs.}$ $\epsilon_{PK} \approx 0.016 \text{ IN/IN}$ $\epsilon_{MIN} \approx 0.0035 \text{ IN/IN}$	One Specimen Tested. See Figures 49 and 50. Creep/Stress Relaxation Only.
	$P_{PK} = 25 \text{ KIPS}, P_{MIN} = -9.8 \text{ KIPS}$ $P_{MEAN} = 8.8 \text{ KIPS}$ $N_{OL} = 1000 \text{ CYCLES}$ $-0.0001 \leq \epsilon_{MIN} \leq +0.0036$ $+0.008 \leq \epsilon_{MEAN} \leq +0.0098$	Fatigue Tests. Two Specimens Tested. See Figures 51 through 53.
	$P_{PK} = 25 \text{ KIPS}, P_{MIN} = -9.8 \text{ KIPS}$ $P_{MEAN} = 8.8 \text{ KIPS}$ $N_{OL} = 1000 \text{ CYCLES}$ $0.0013 \leq \epsilon_{MIN} \leq 0.0026$ $0.0078 \leq \epsilon_{MEAN} \leq 0.0092$ $H = 1.0 \text{ HR.}$	Fatigue Test. One Specimen Only. See Figures 54 and 55.

NOTE: 1) P is defined as total load (P_T)

2) ϵ is center bar strain

3) H is sustained load hold time.

4) N_{OL} is number of cycles between overloads.

Figure 43. Test Sequence Definition For Simplified Stress Concentration Specimens

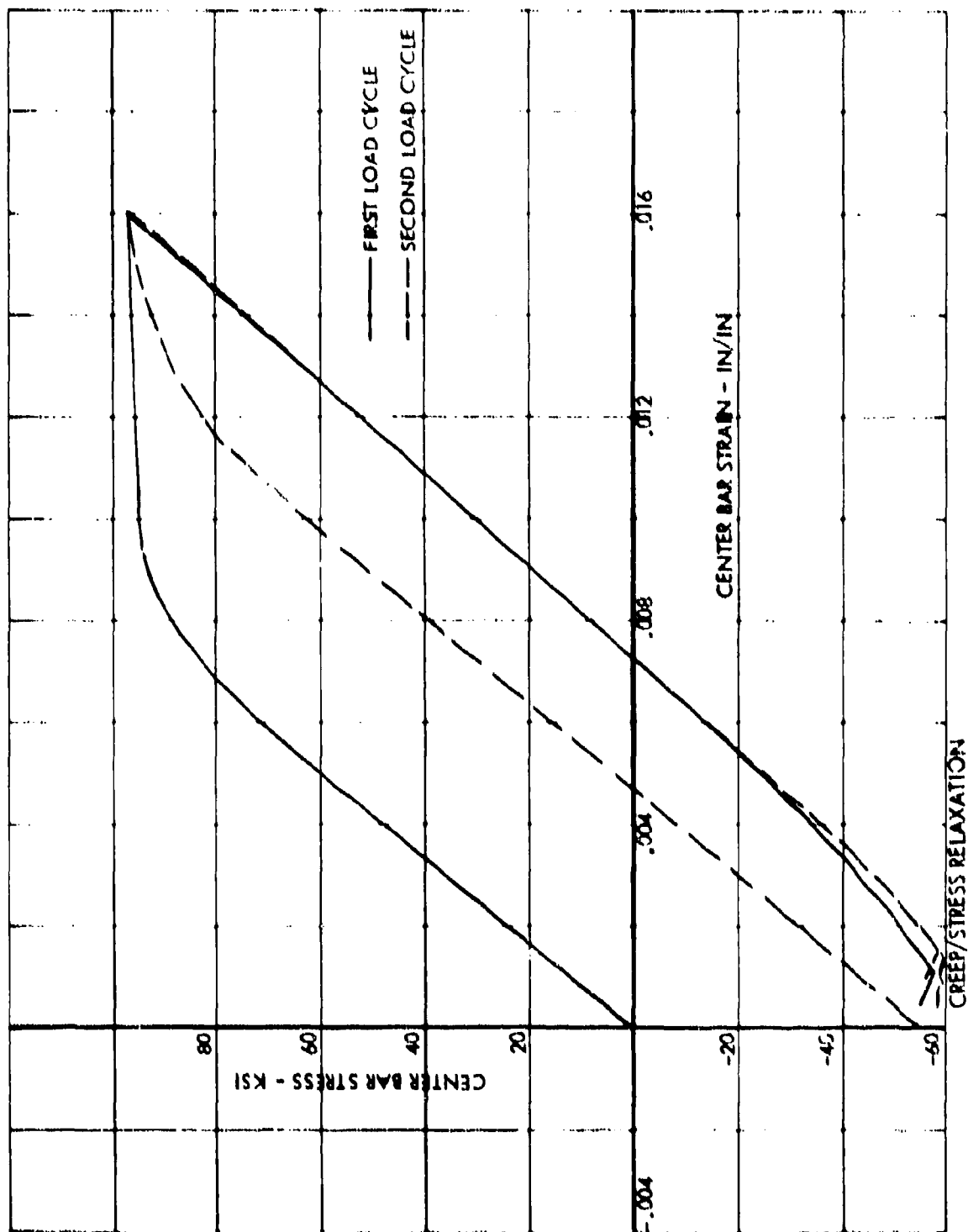


Figure 45. Simplified Stress Concentration Specimen Sequence 1
Center Bar Stress-Strain Data

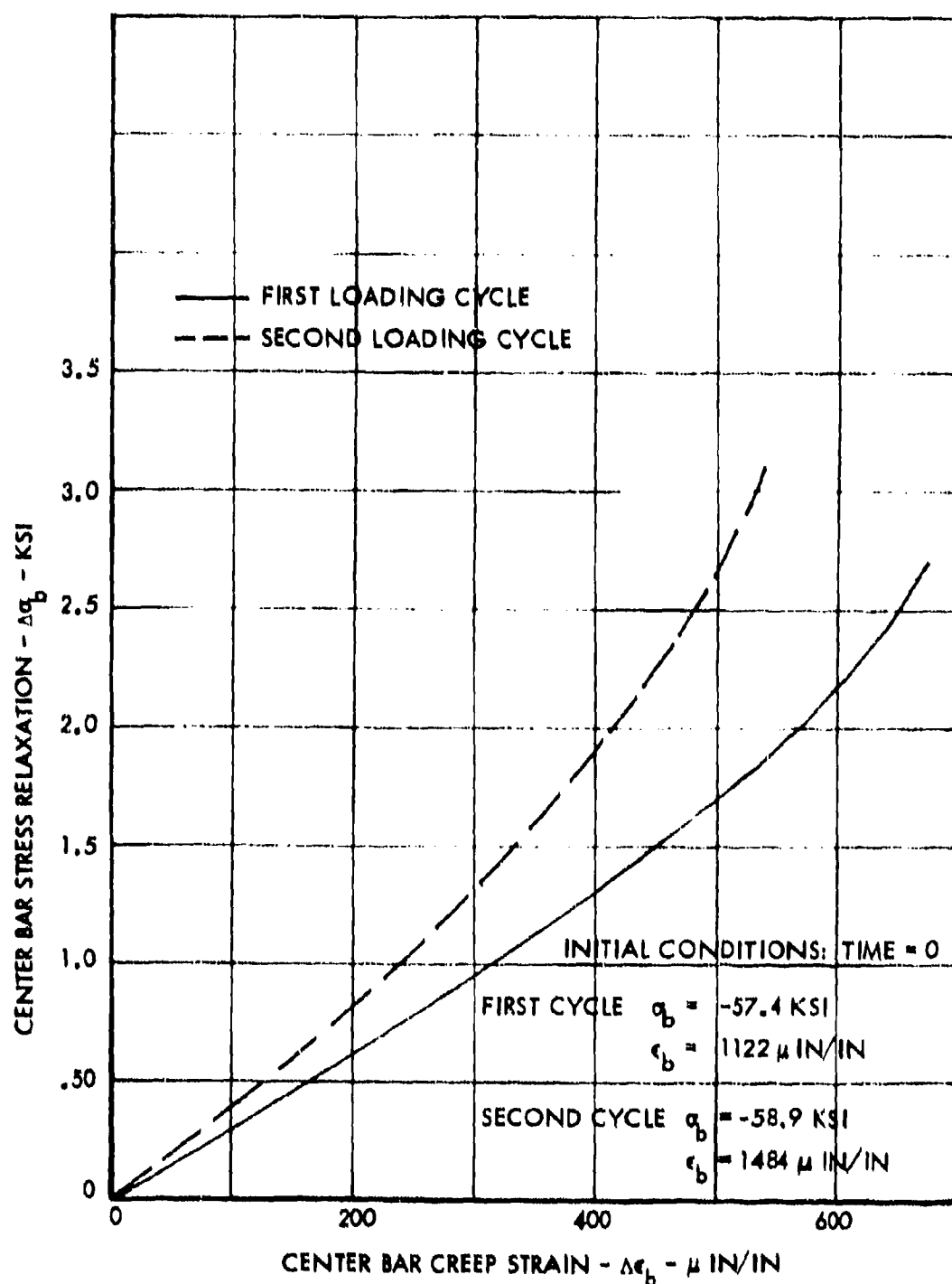


Figure 46. Creep/Stress Relaxation Measurements From Simplified Stress Concentration Specimen - Sequence 1

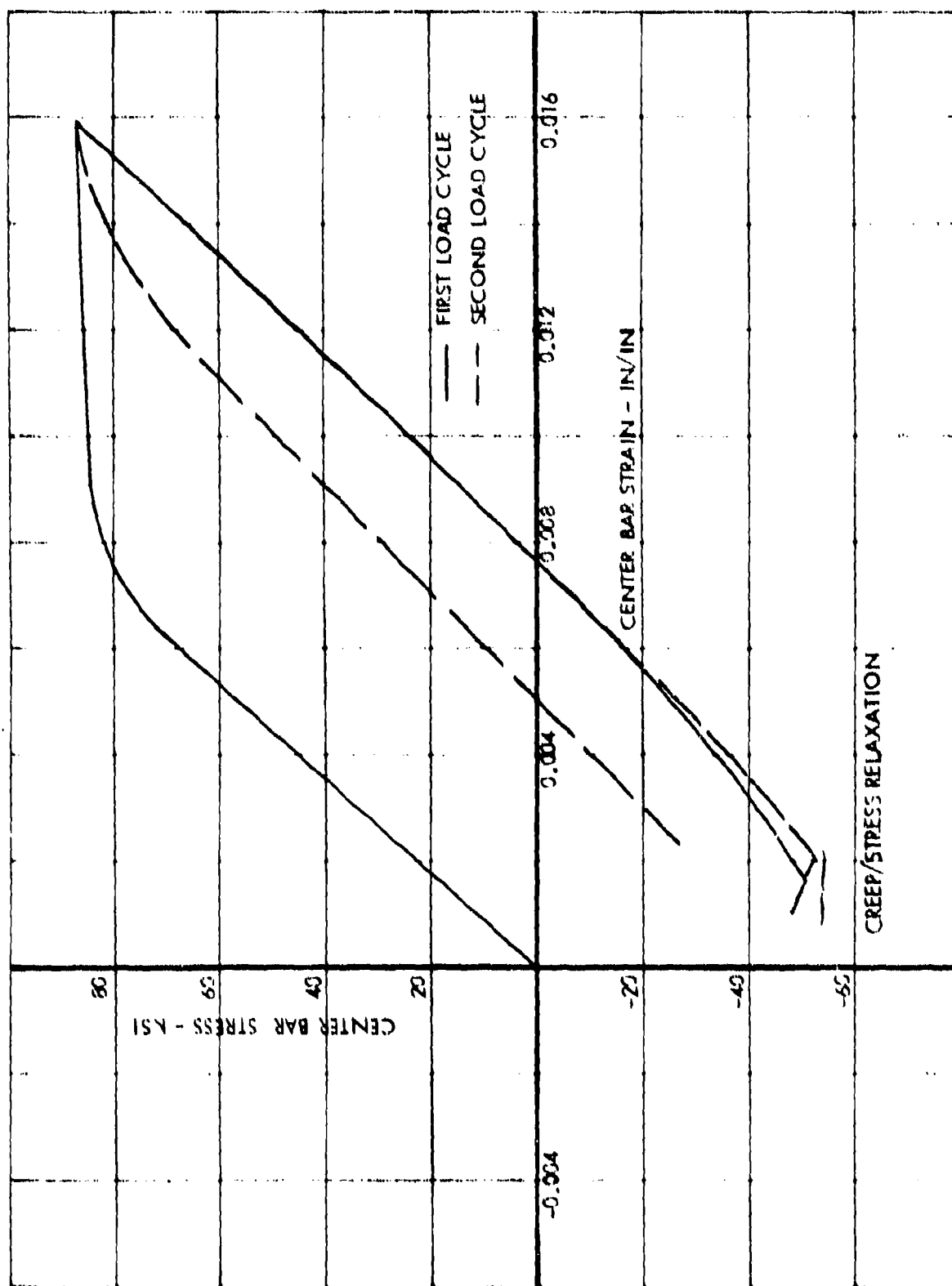


Figure 47. Simplified Stress Concentration Specimen - Center Bar Stress-Strain Data
Sequence No. 1 Repeat

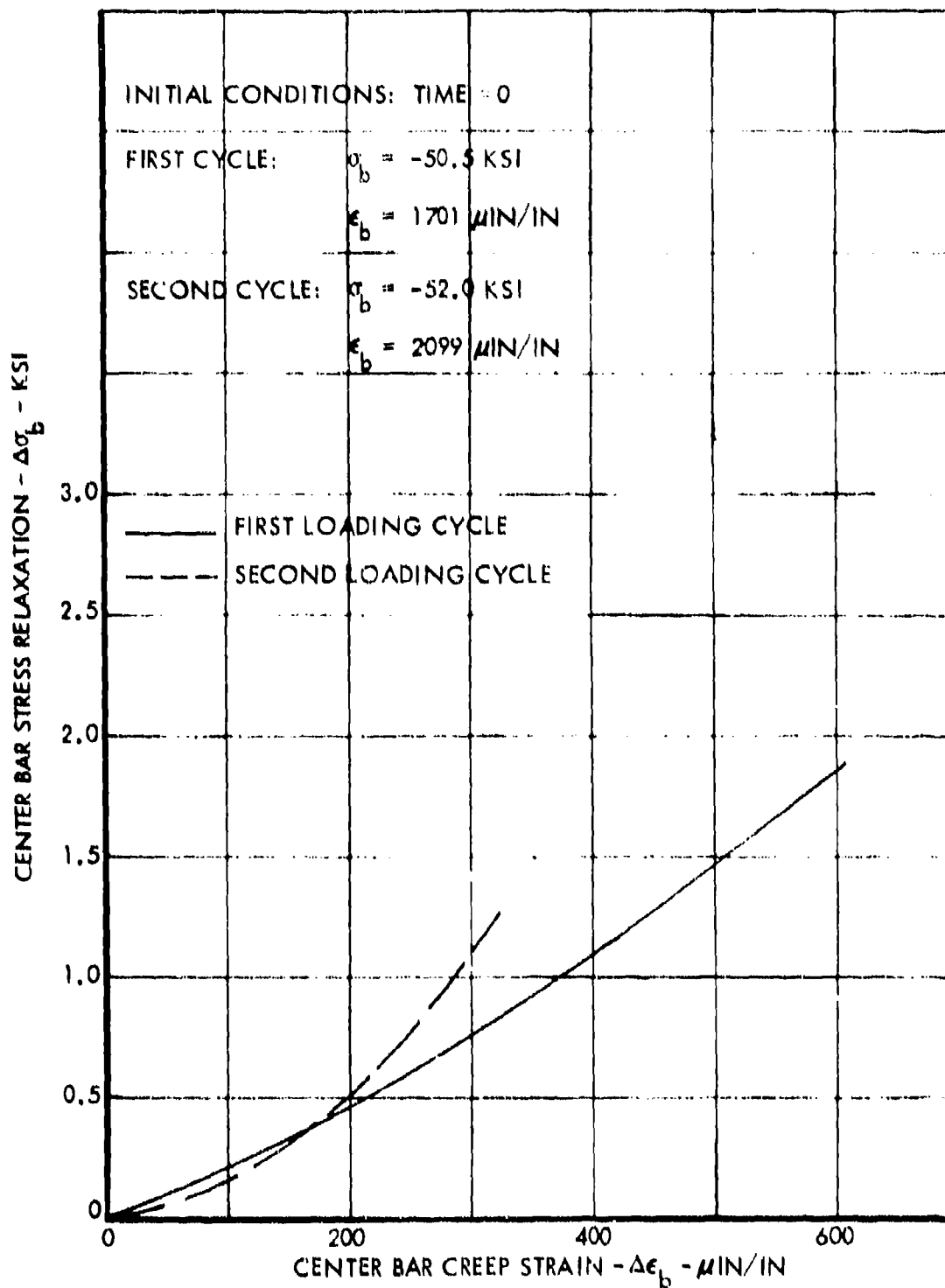


Figure 4B. Creep/Stress Relaxation Measurements From Simplified Stress Concentration Specimen - Repeat Sequence 1.

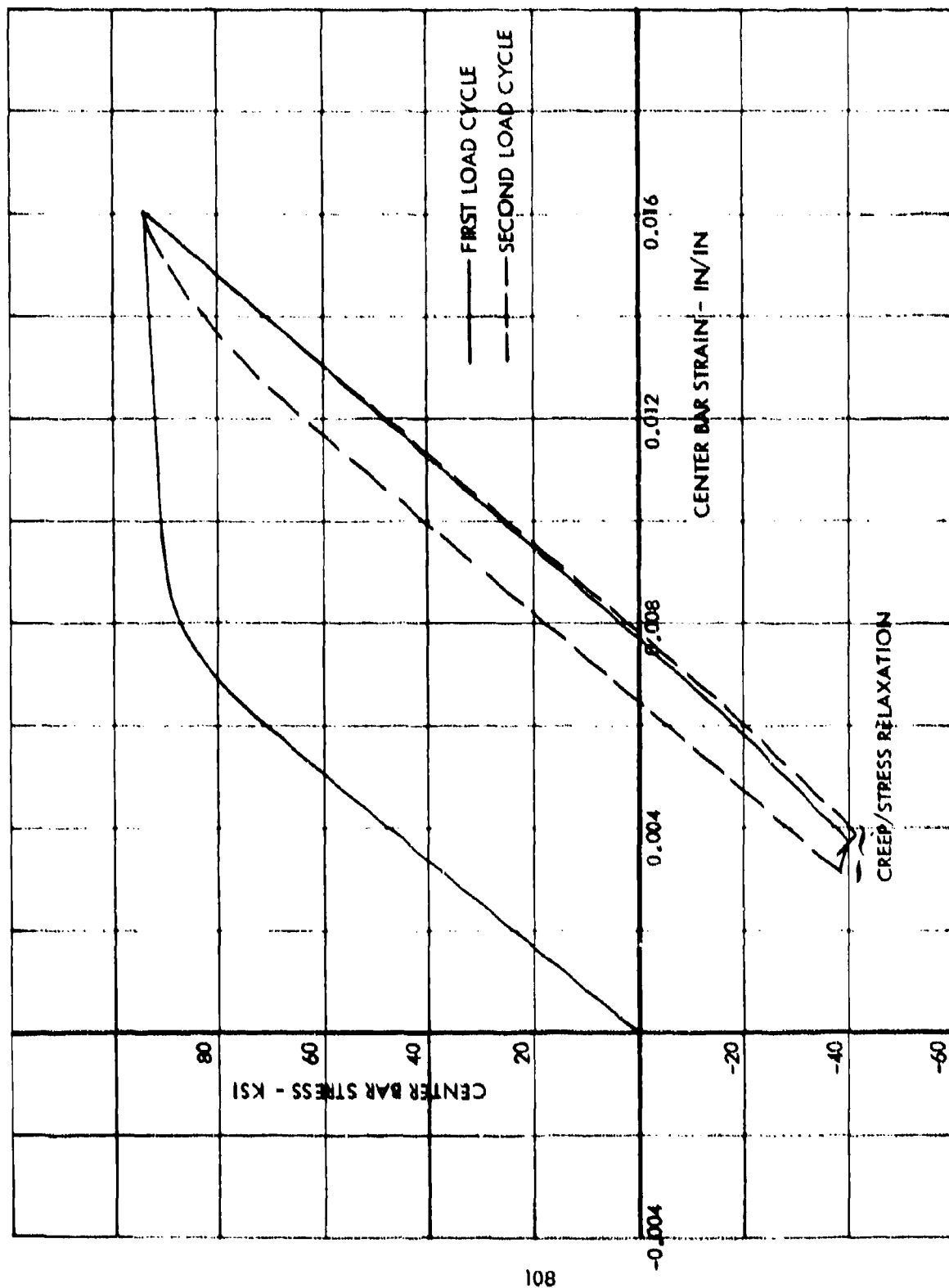


Figure 49. Simplified Stress Concentration Specimen Sequence 2
Center Bar Stress-Strain Data

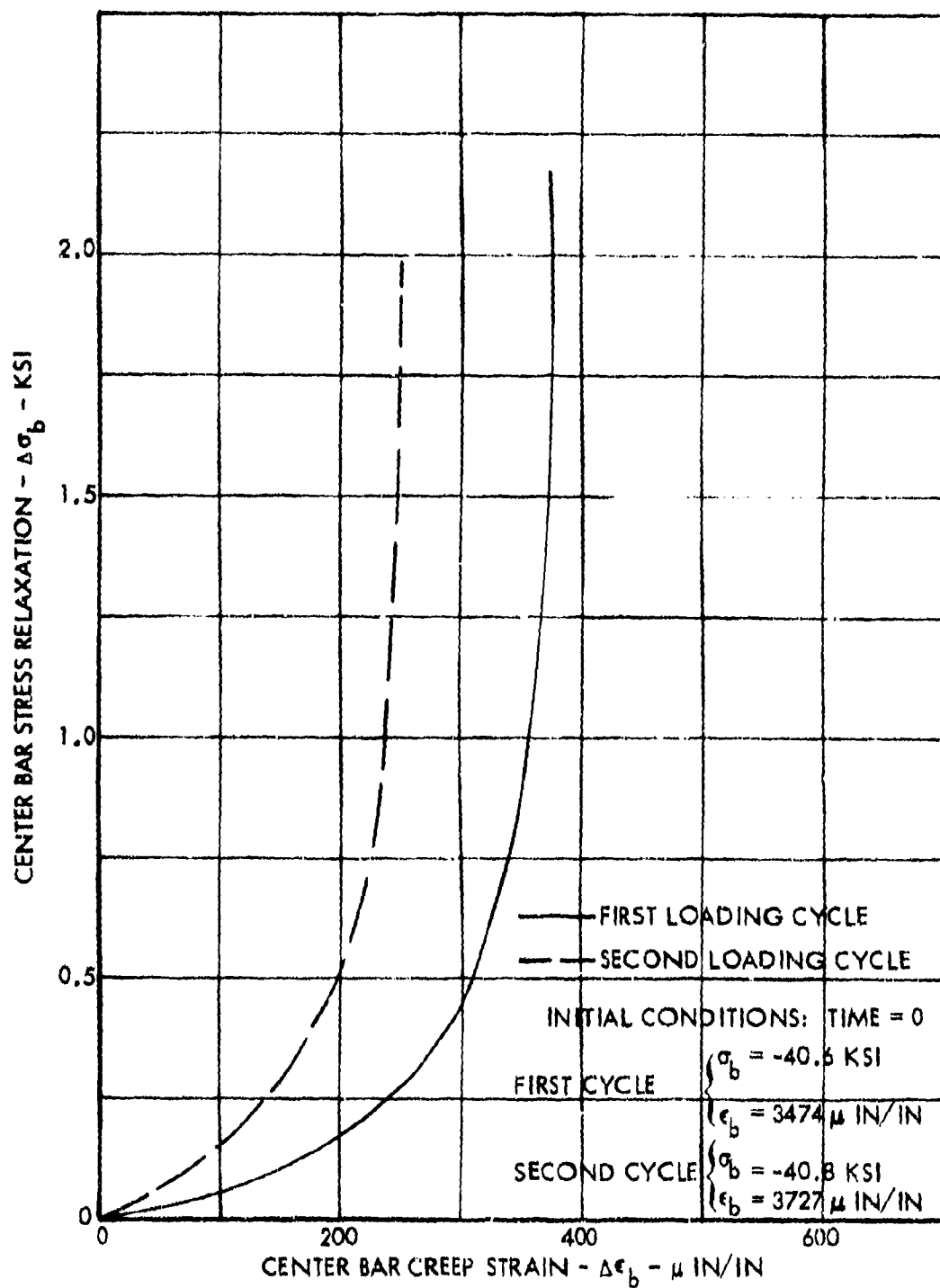


Figure 50. Creep/Stress Relaxation Measurements From Simplified Stress Concentration Specimen - Sequence 2

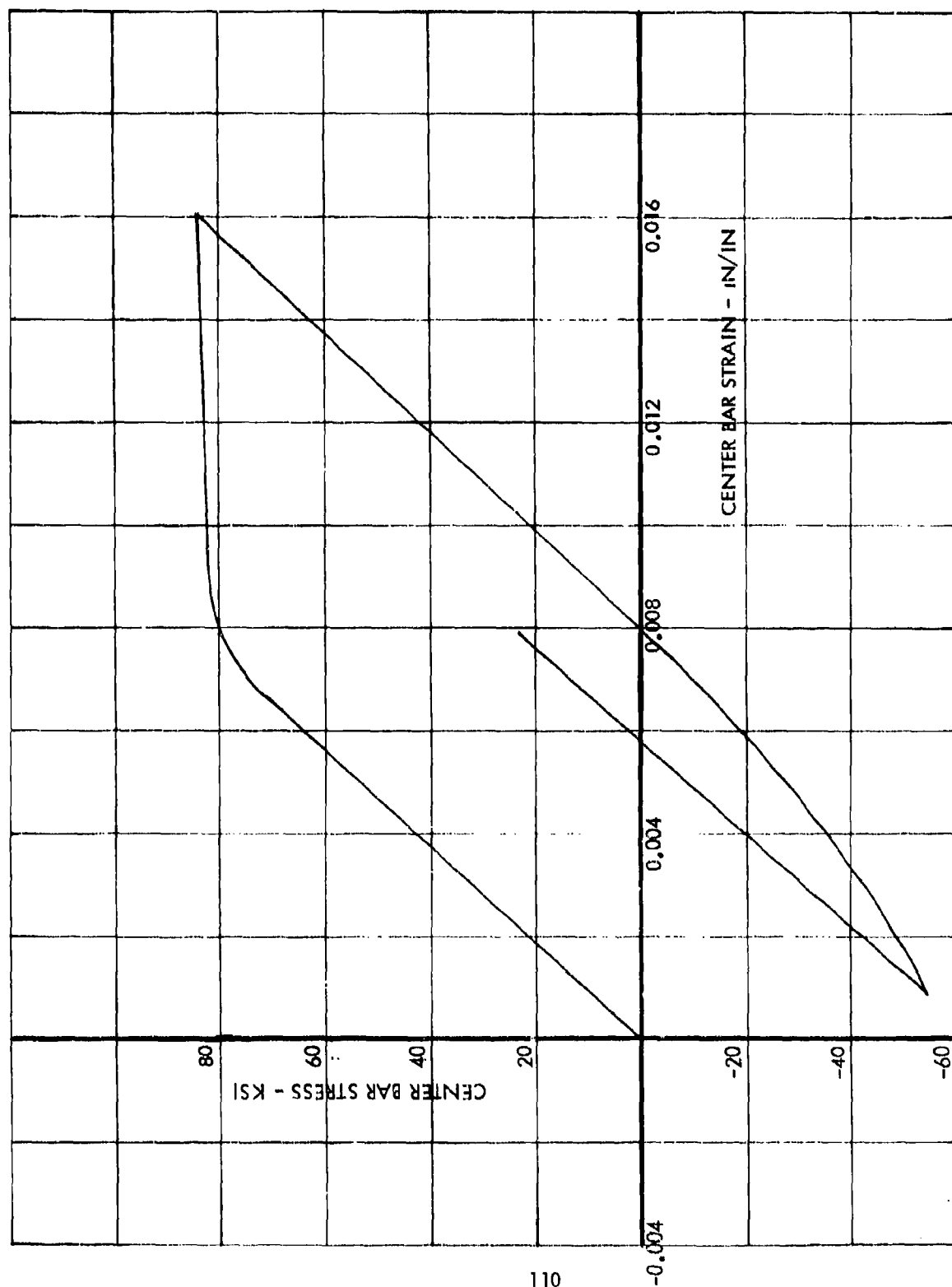


Figure 51. Simplified Stress Concentration Specimen, Sequence 3-1
Center Bar Stress-Strain Data

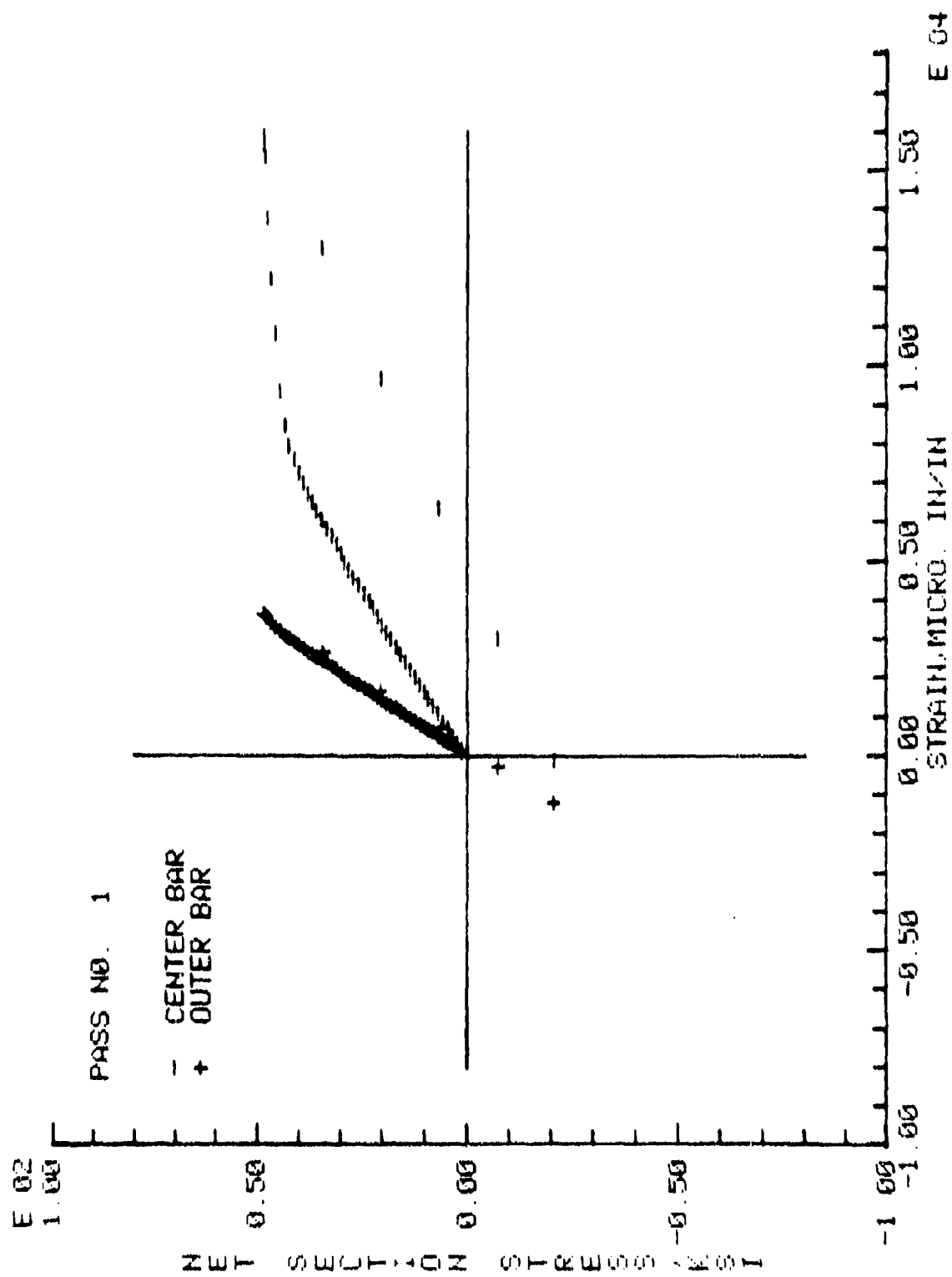


Figure 52. Typical Stress-Strain Data for Simplified Stress Concentration Specimen - Sequence 3, Specimen 2

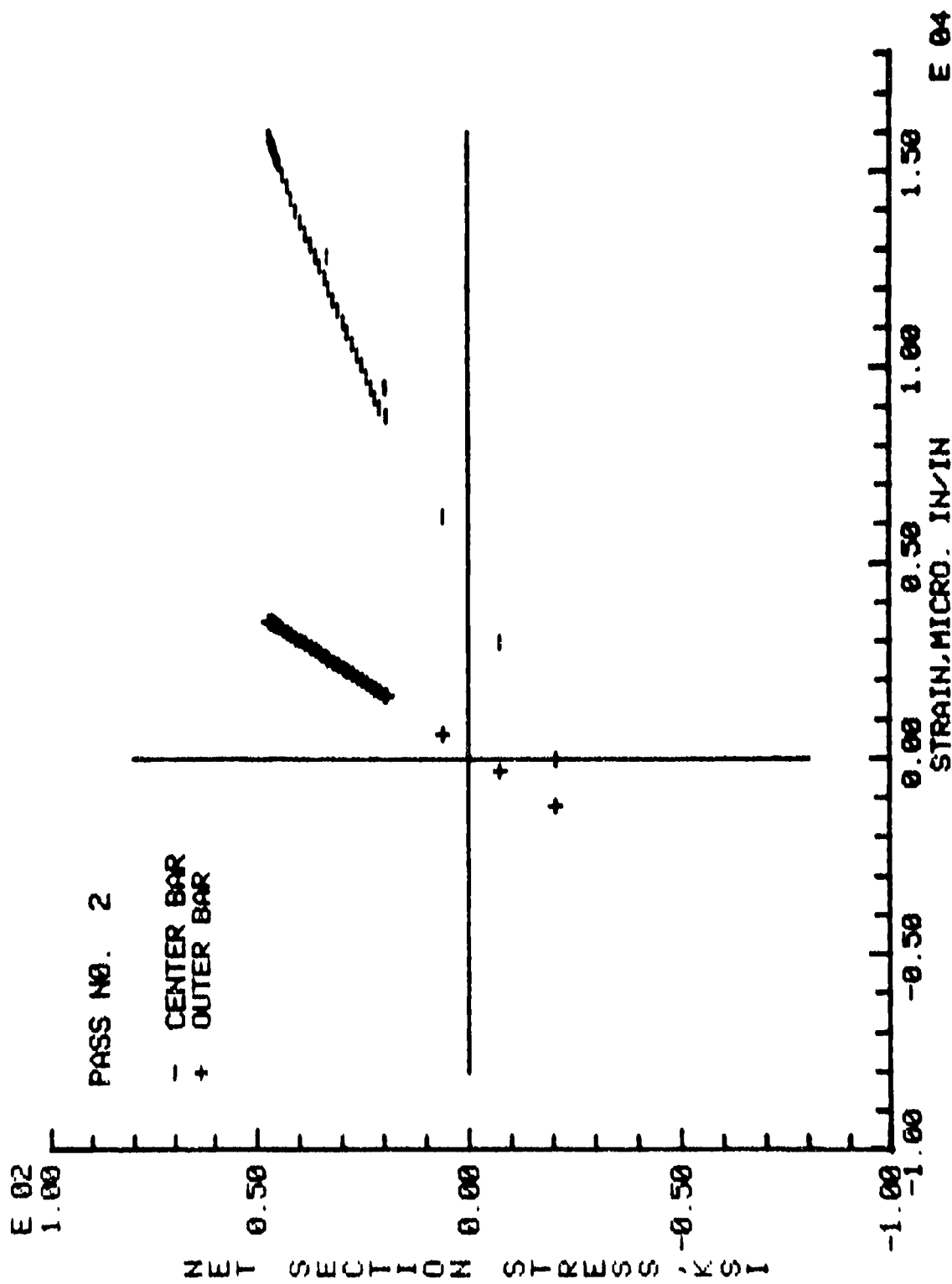


Figure 52. (Continued)

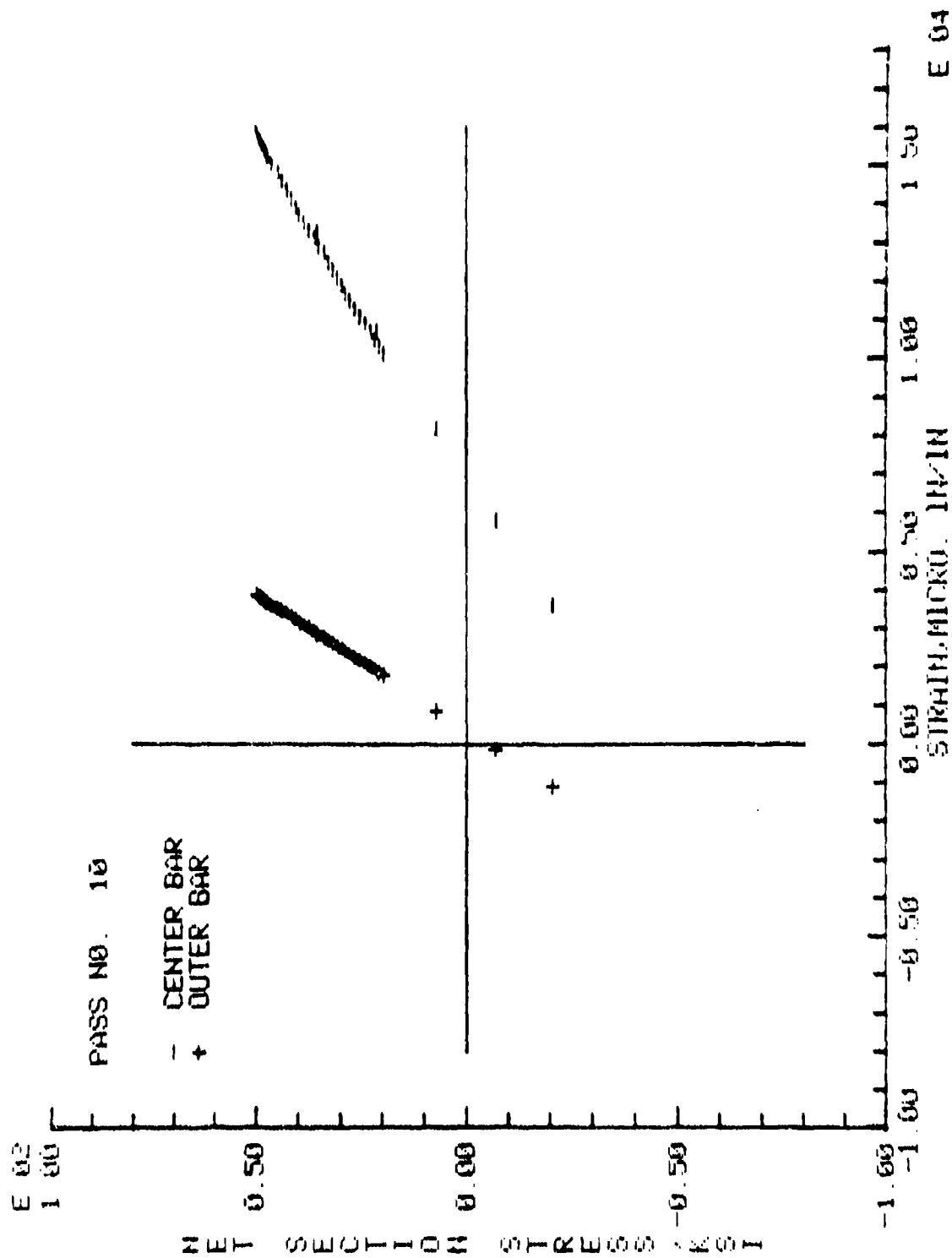


Figure 52. (Concluded)

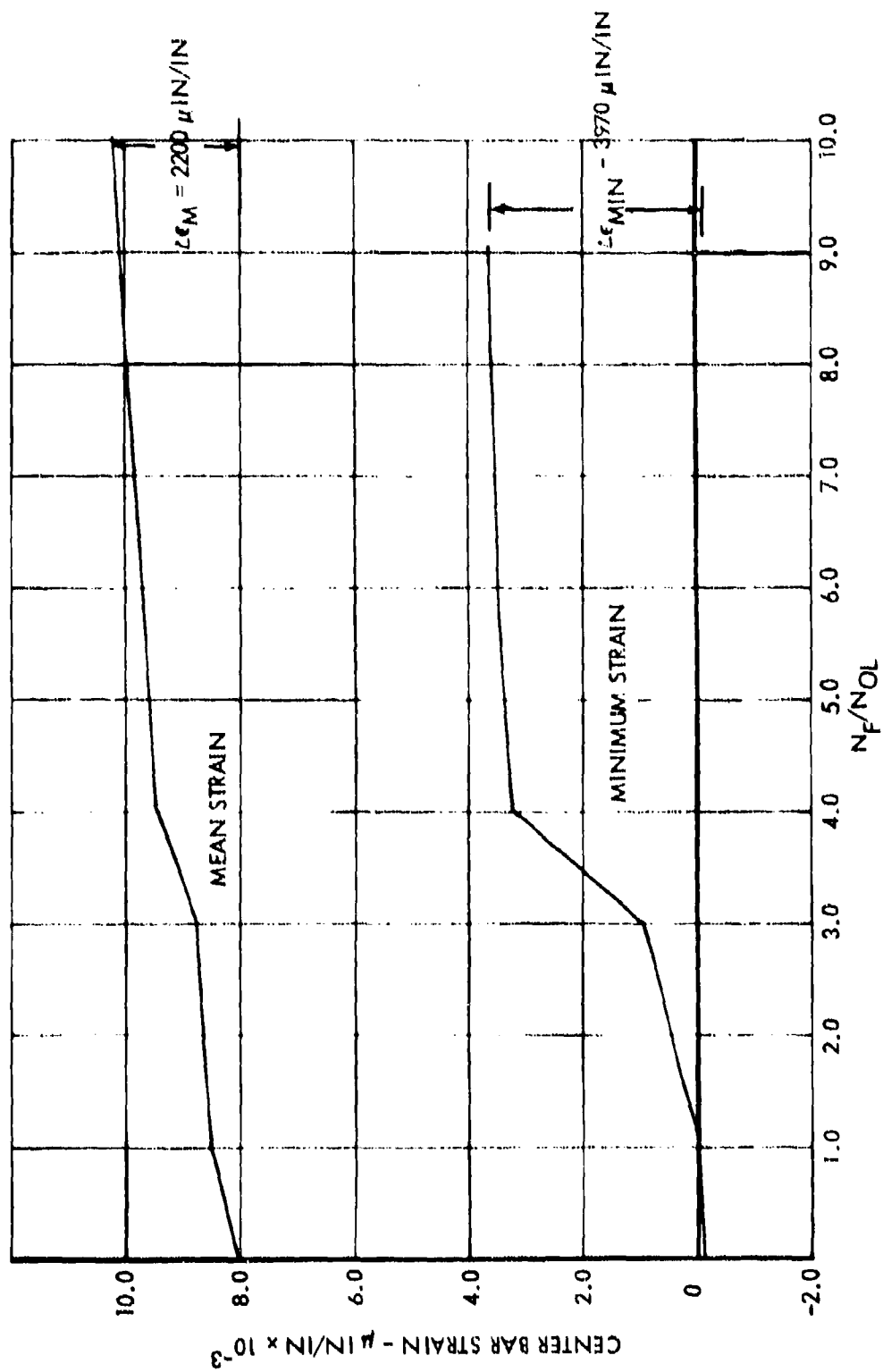


Figure 53. Strain Time History - Simplified Stress Concentration Specimen, Sequence No. 3 Test

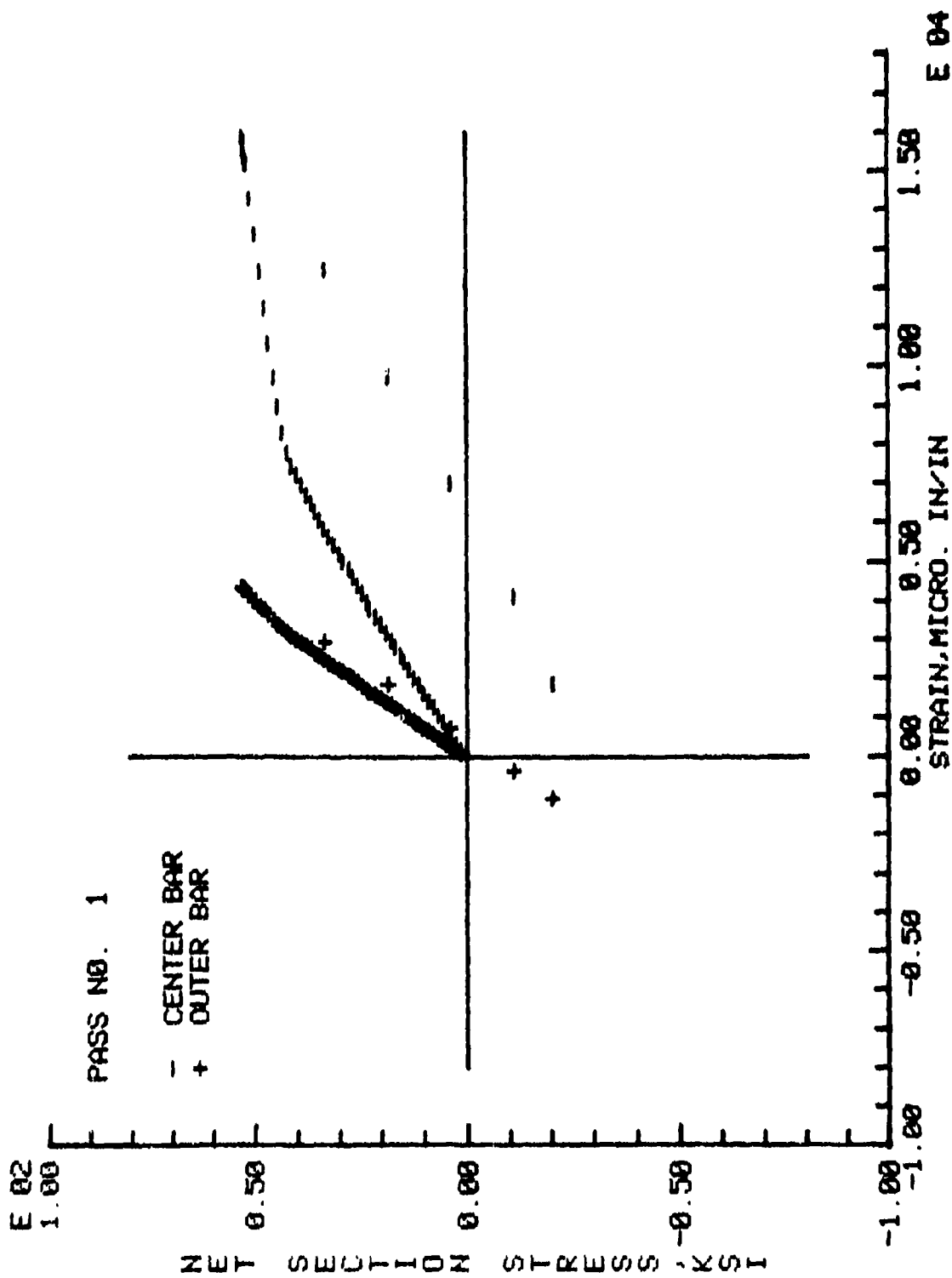


Figure 54. Stress-Strain Curve for Simplified Stress Concentration Specimen - Sequence 4

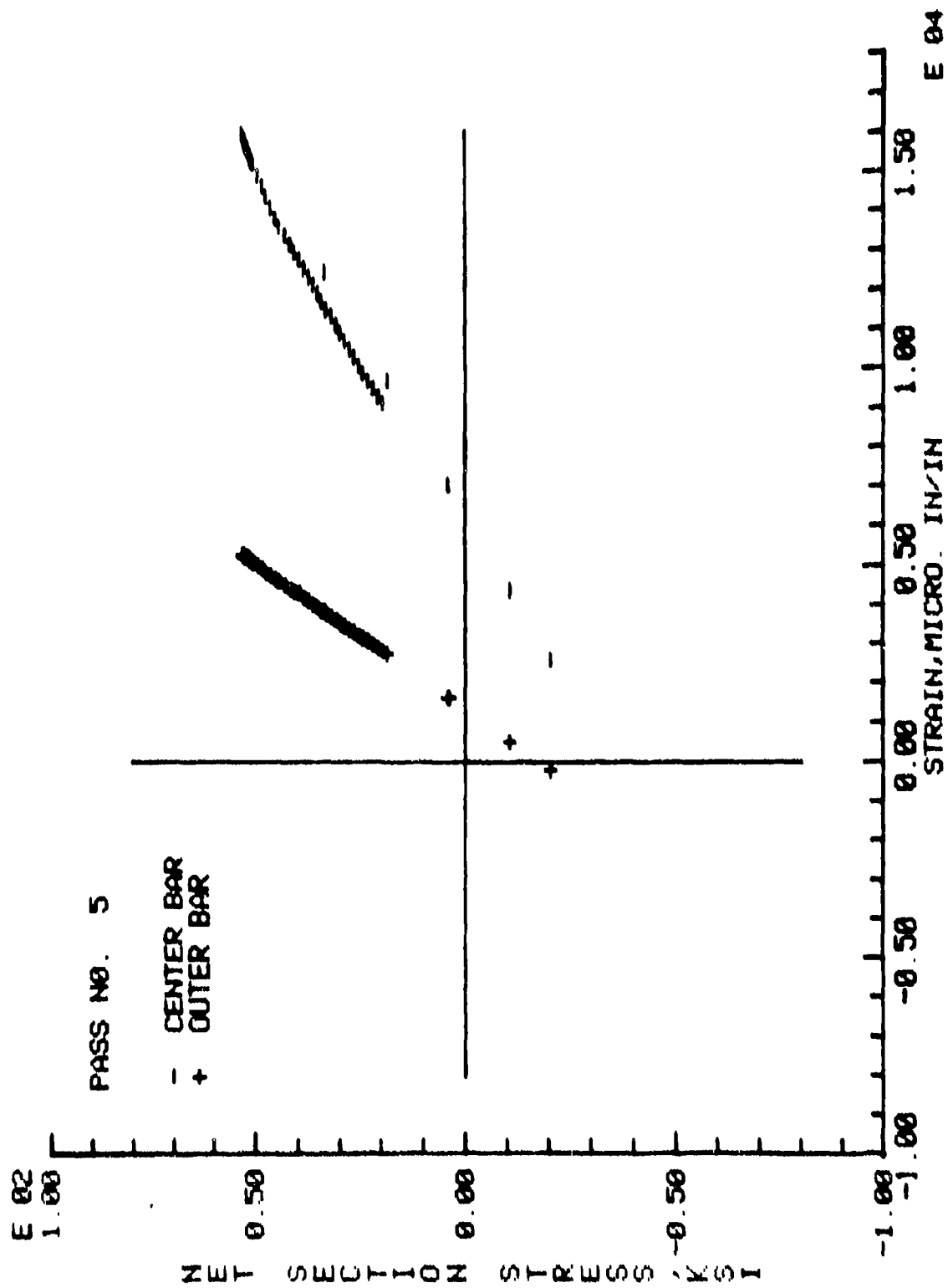


Figure 54. (Continued)

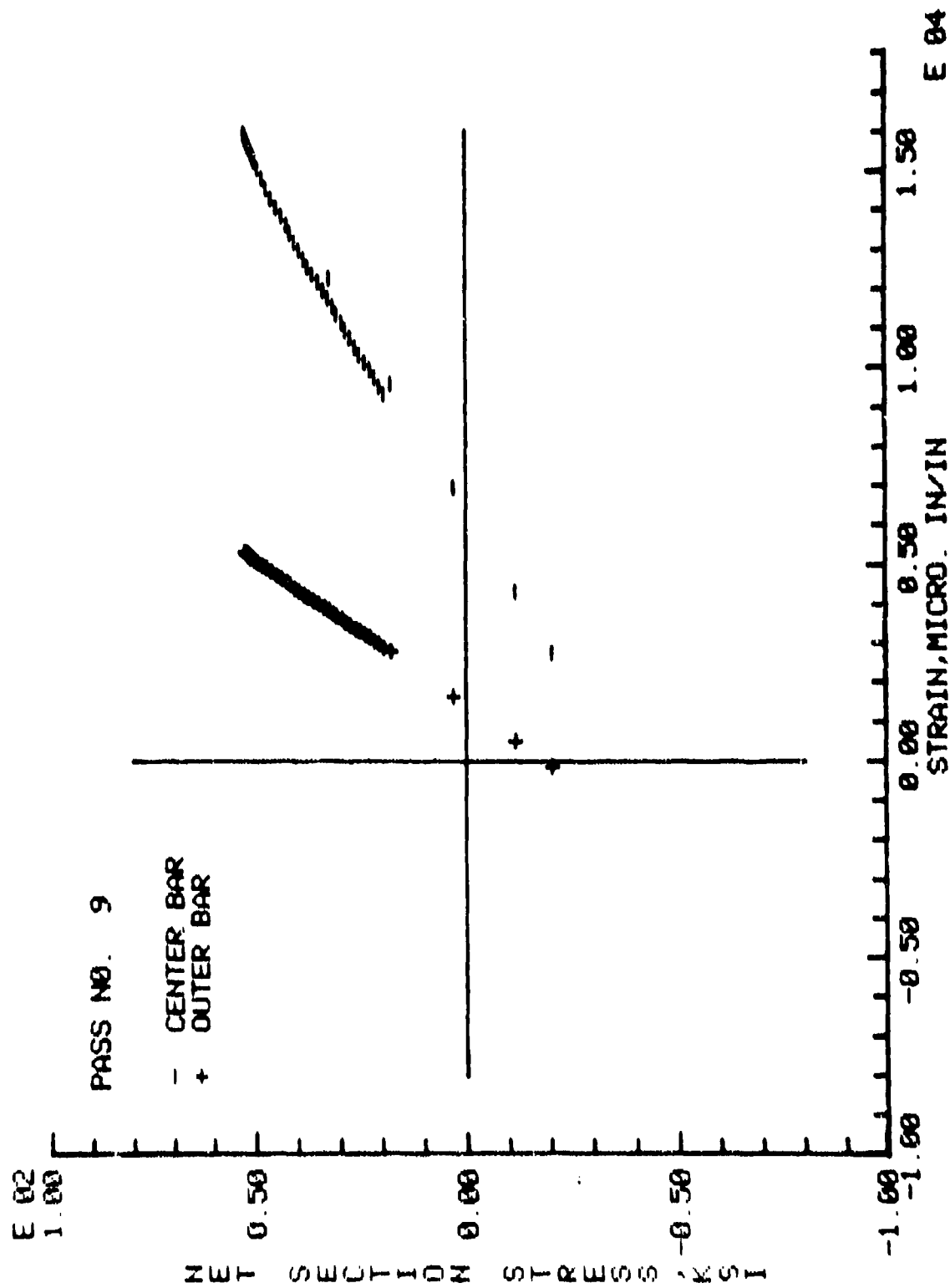


Figure 54. (Concluded)

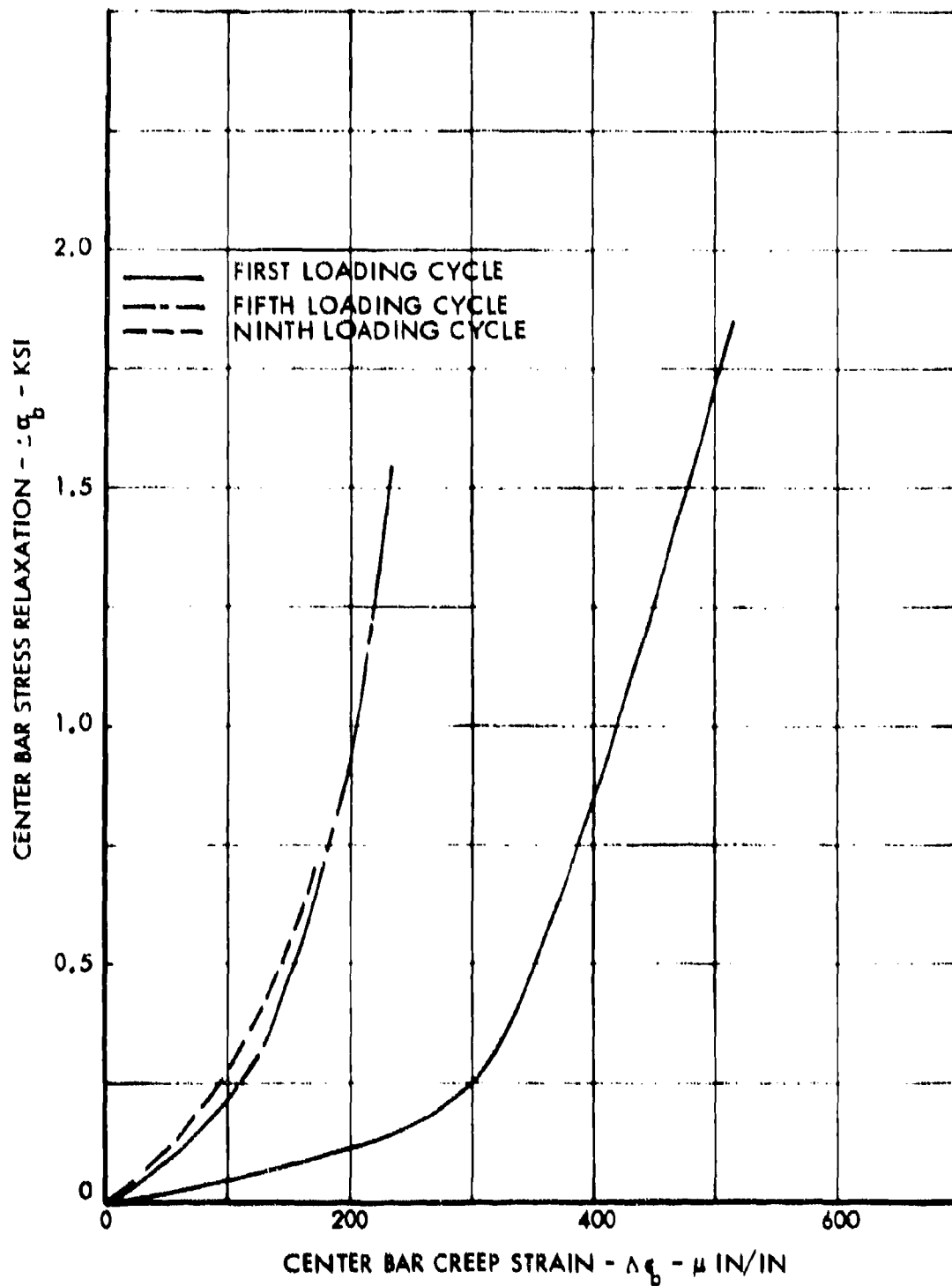


Figure 55. Creep/Stress Relaxation Measurements From Simplified Stress Concentration Specimen - Sequence No. 4

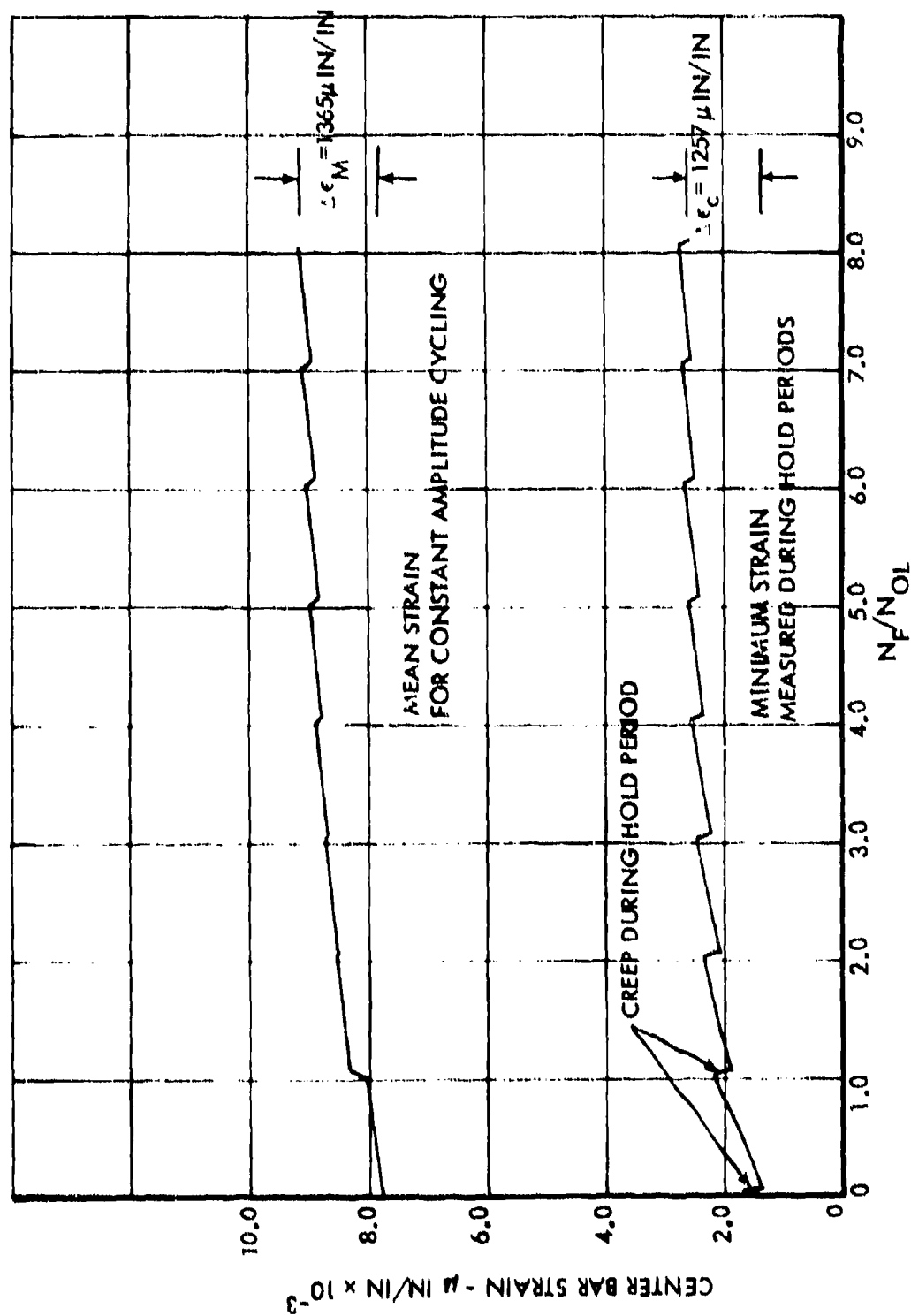


Figure 56. Time History of Cyclic Strain Change
Sequence No. 4

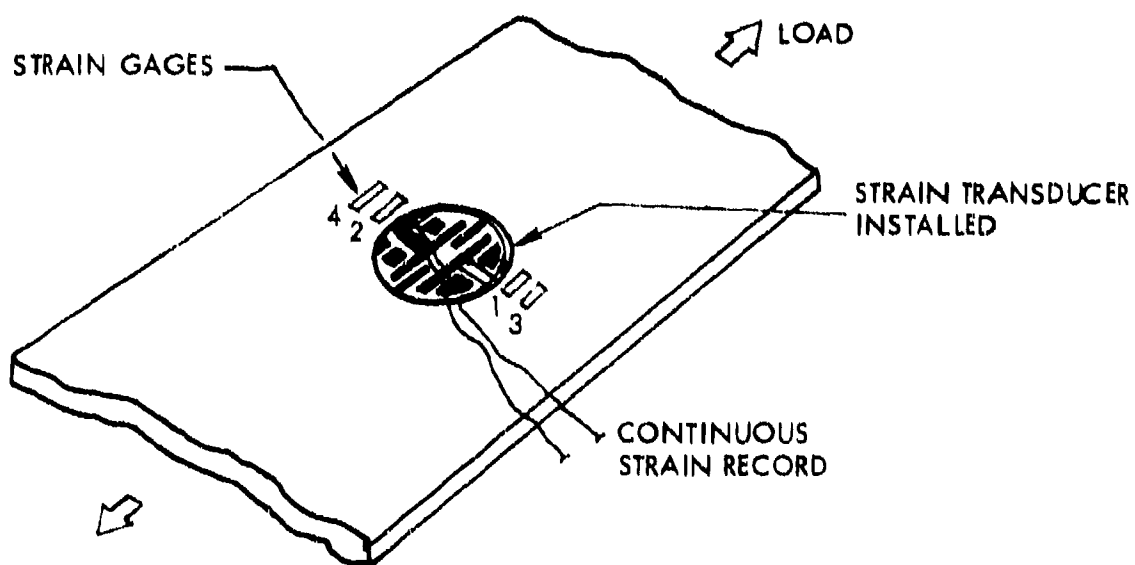
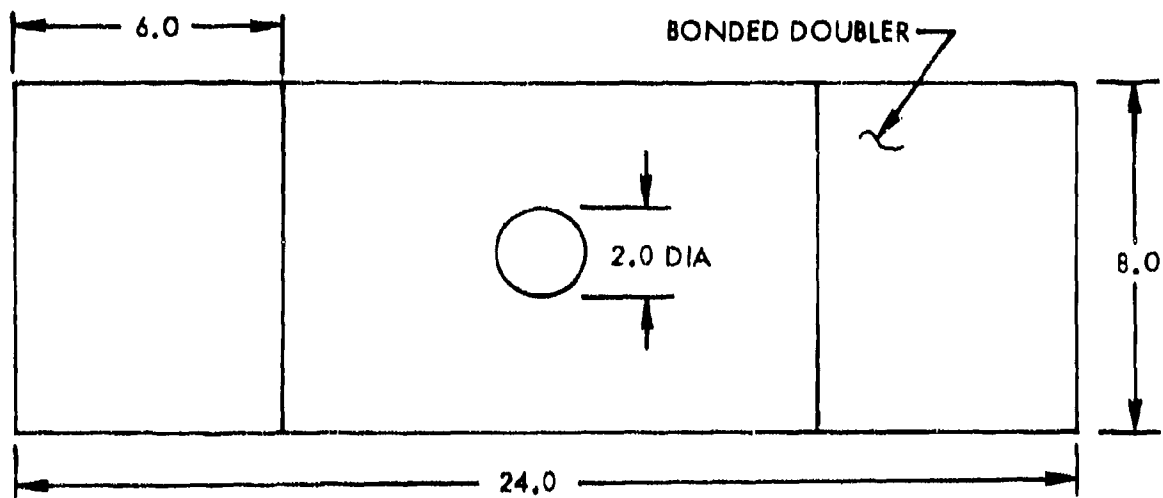


Figure 57. Super-Scale Test Specimen

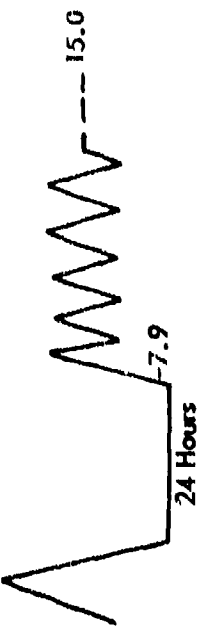
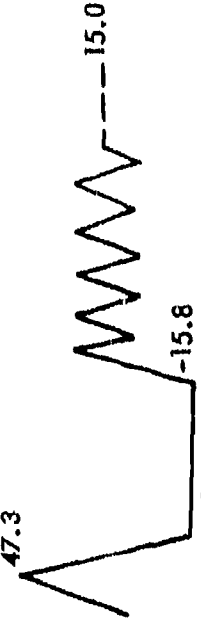
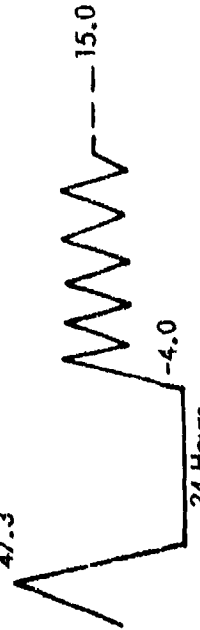
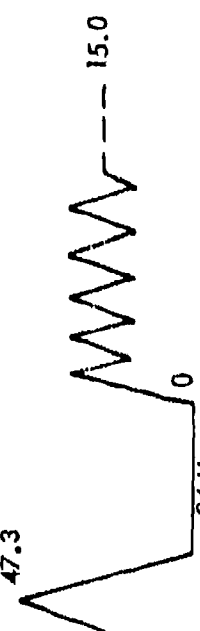
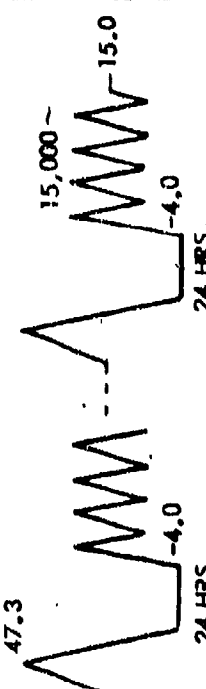
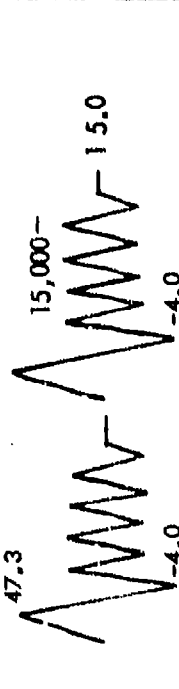

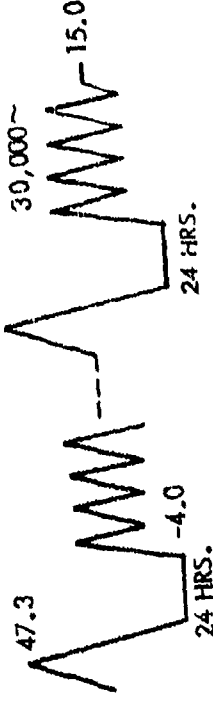
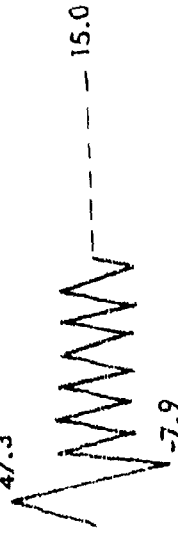
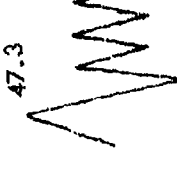
SEQUENCE DEFINITION	TEST CONDITIONS	TEST LIFE
<p>1. </p>	<p>TY - 24 Hours @ -7.9 - CA, $F_M = 15$</p>	<p>80,018 Cycles</p>
<p>2. </p>	<p>TY - 24 Hours @ -15.8 - CA, $F_M = 15$</p>	<p>49,867 Cycles</p>
<p>3. </p>	<p>TY - 24 Hours @ -4.0 - CA, $F_M = 15$</p>	<p>44,277 Cycles</p>
<p>4. </p>	<p>TY - 24 Hours @ Zero - CA, $F_M = 15$</p>	<p>135,801 Cycles</p>

Figure 58. Super-Scale Specimen Test Summary

SEQUENCE DEFINITION	TEST DEFINITIONS	TEST LIFE
5. 	TY, 24 Hours @ -4.0 KSI CA, $F_M = 15$, $F_V = \pm 10$ $N_{OL} = 15,000$	1,375,000 Cycles
6. 	TY, -4.0 KSI, CA. $F_M = 15$, $F_V = \pm 10$ $N_{OL} = 15,000$	1,269,770 Cycles
7. 	TY, 24 Hours @ -7.9 KSI, CA, $F_M = 15$, $F_V = \pm 10$ $N_{OL} = 30,000$	① 214,546 Cycles ② 88,000 Cycles

- ① Three 24-hour hold periods then cycled to failure.
- ② Five 24-hour hold periods then cycled to failure.

Figure 58. (Continued)

SEQUENCE DEFINITION	TEST DEFINITION	TEST LIFE
<p>8.</p> 	<p>TY, 24 Hours @ -4.0 KSI CA, $F_M = 15$, $F_V = \pm 10$, $N_{OL} = 30,000$ Cycles</p>	<p>581,557 Cycles</p>
<p>9.</p> 	<p>TY, -7.9 KSI CA, $F_M = 15$, $F_V = \pm 10$ KSI</p>	<p>286,838 Cycles</p>
<p>10.</p> 	<p>TY, -4.0 KSI CA, $F_M = 15$, $F_V = \pm 10$ KSI</p>	<p>131,956 Cycles</p>

TY = ± 47.3 KSI Overload
CA = Constant Amplitude, $F_M = 15$ KSI, $F_V = \pm 10$ KSI
 N_{OL} = Cycles Between Overloads

Figure 58. (Concluded)

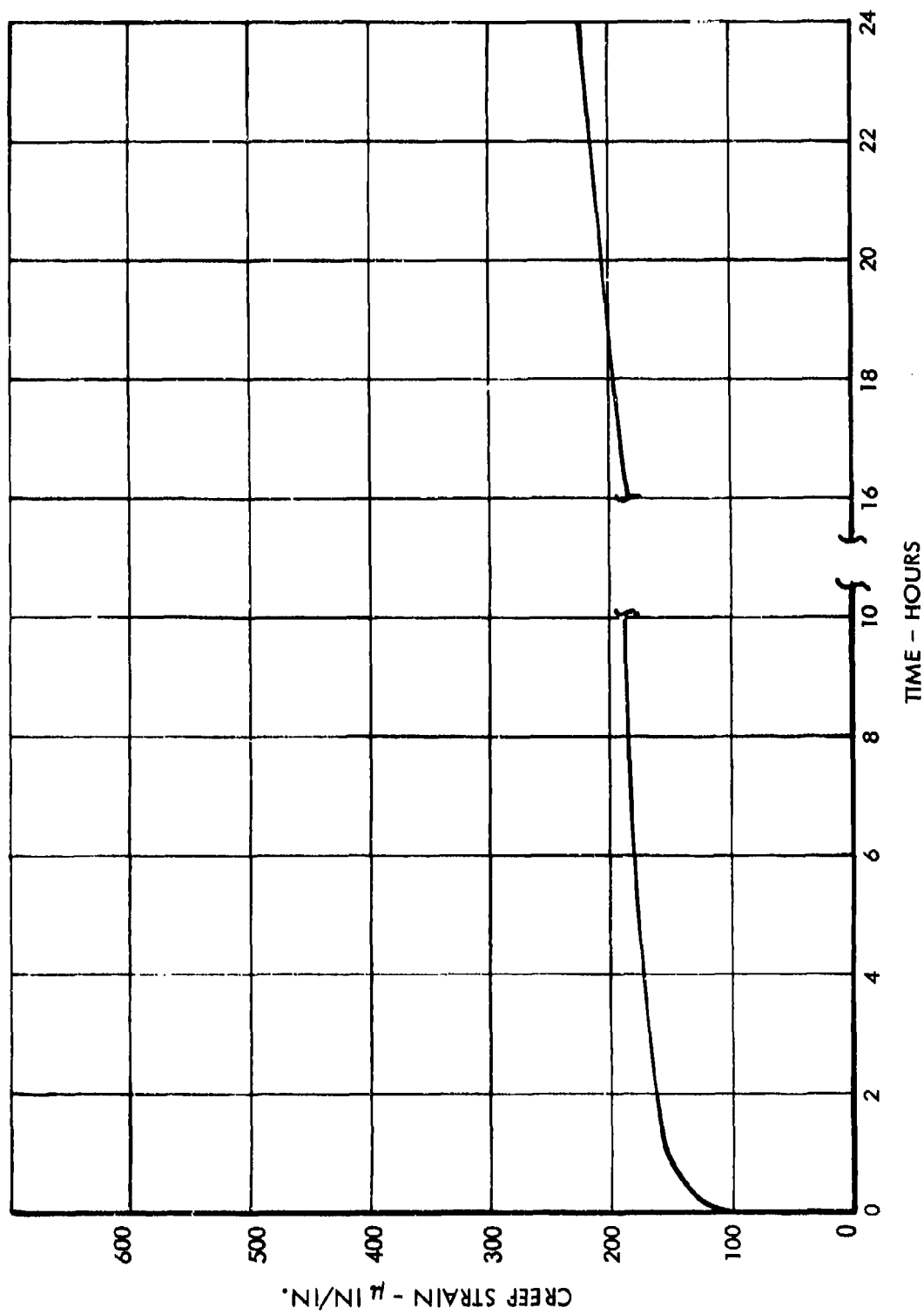


Figure 59. Creep Strain - Super-Scale Test Sequence 5

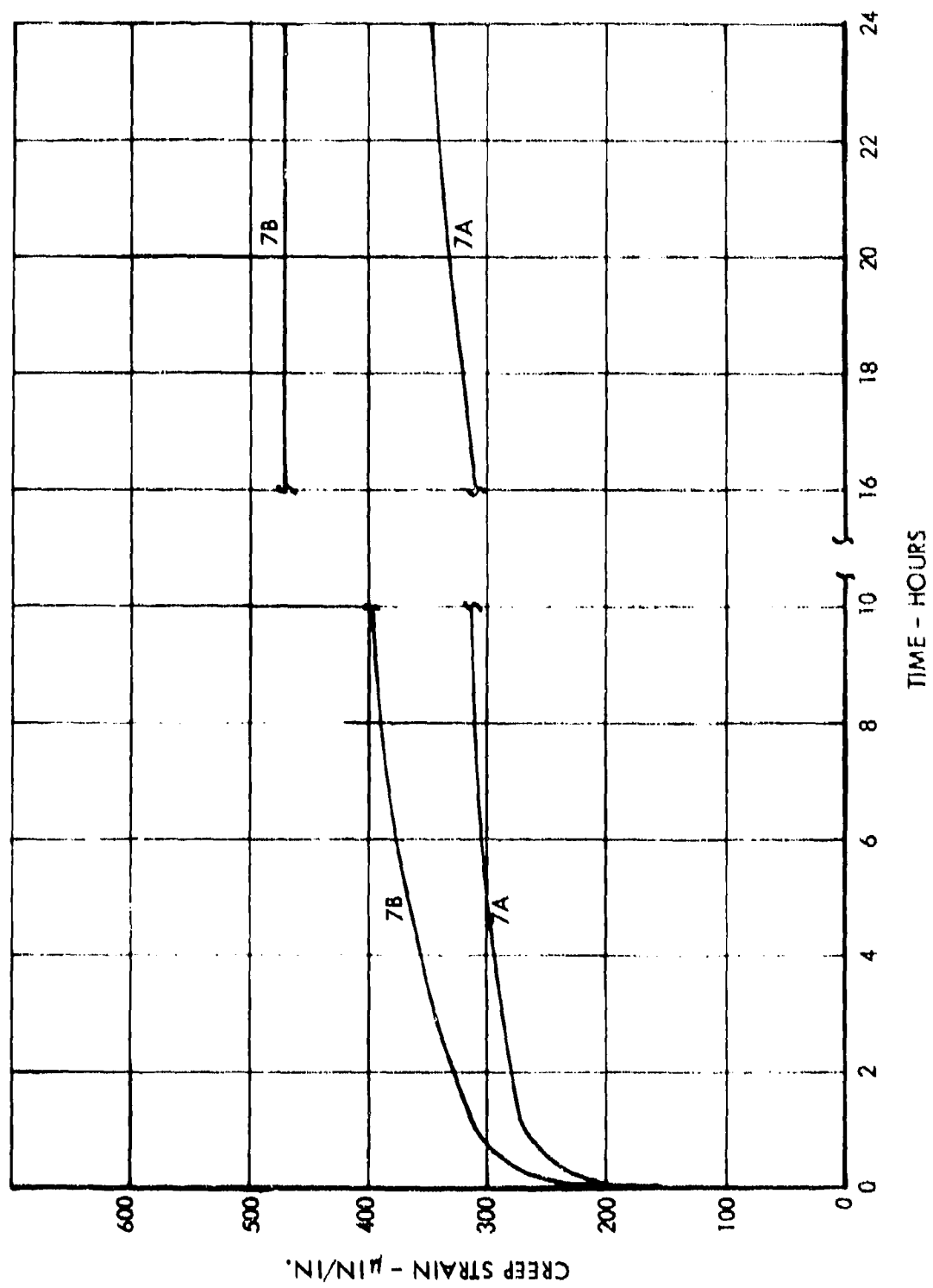


Figure 60. Creep Strain - Super-Scale Test Sequences 7A and 7B

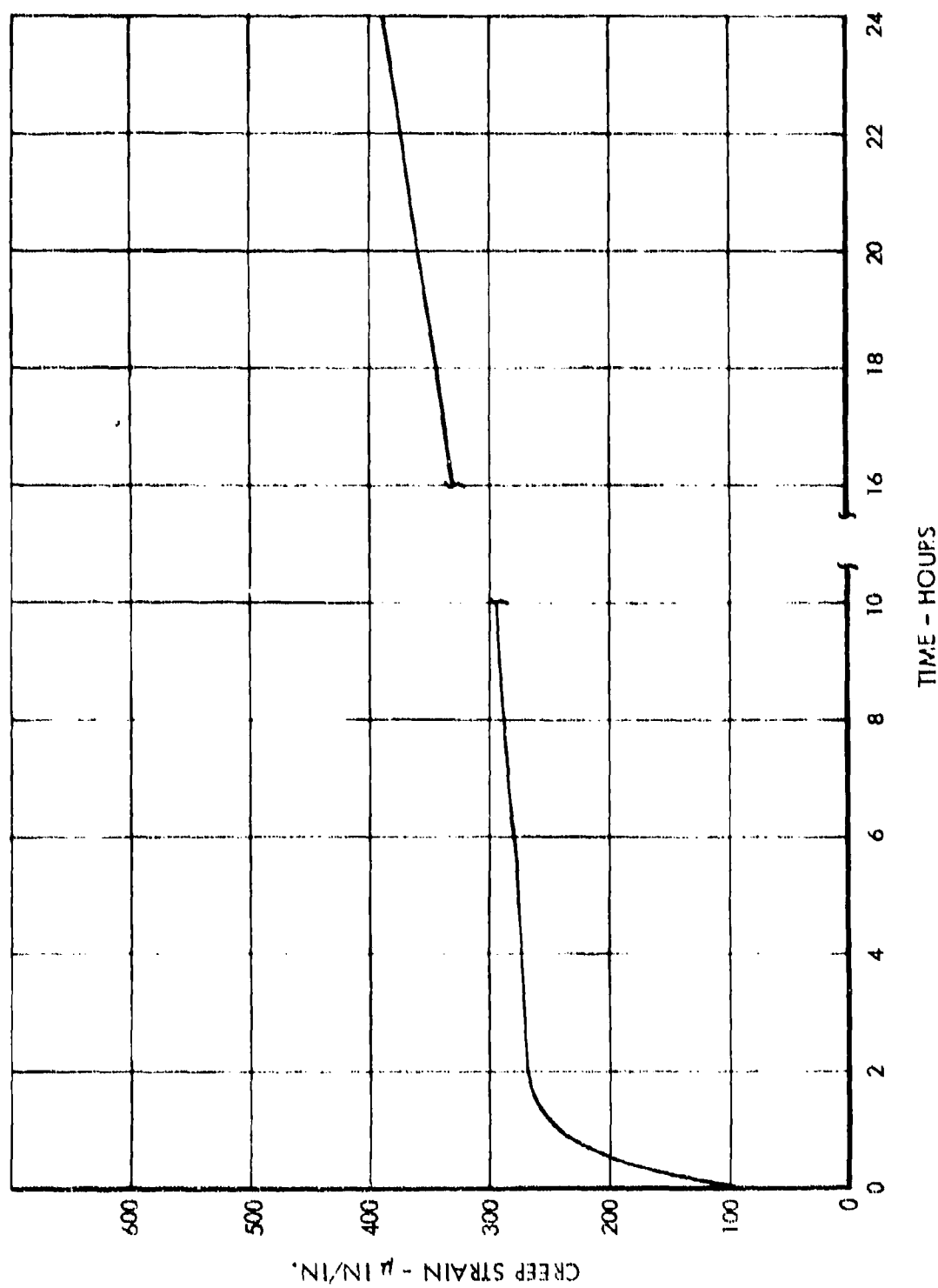


Figure 61. Creep Strain - Super-Scale Test Sequence 8

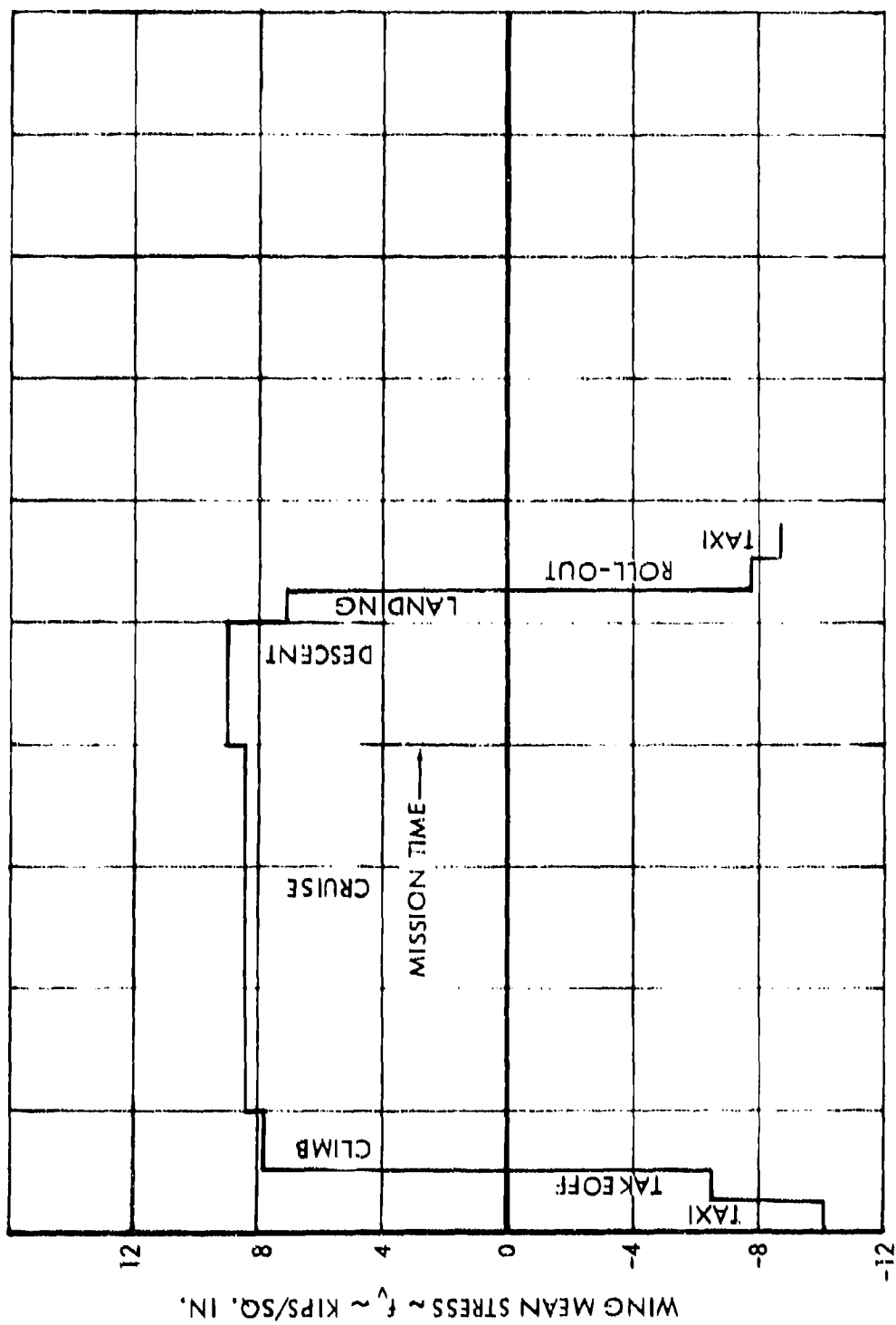


Figure 62. Typical Wing Lower Surface Mean Stresses - Transport Logistics Mission

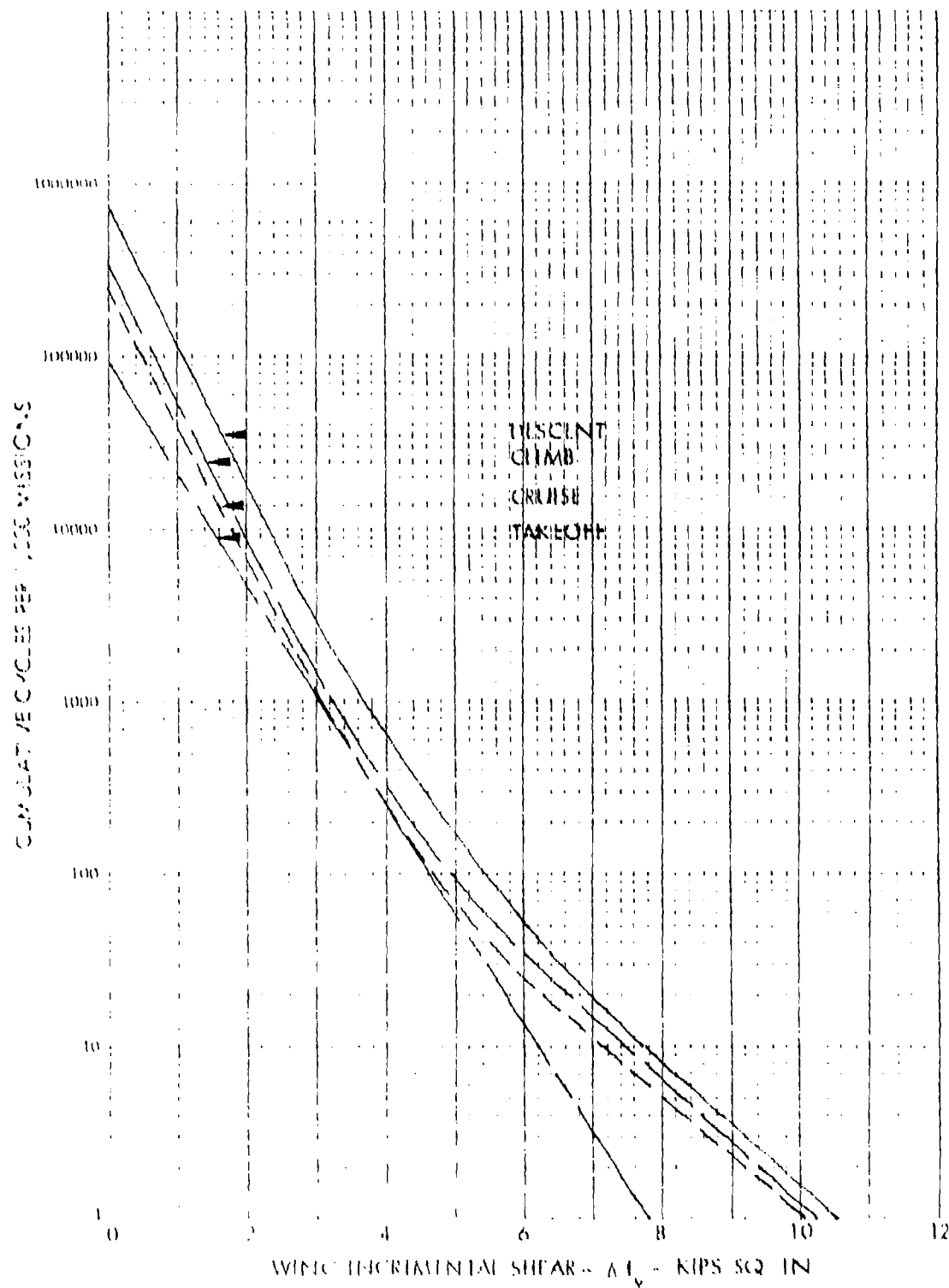


Figure 63. Wing Lower Surface Flight Spectra - Transport Logistics Mission

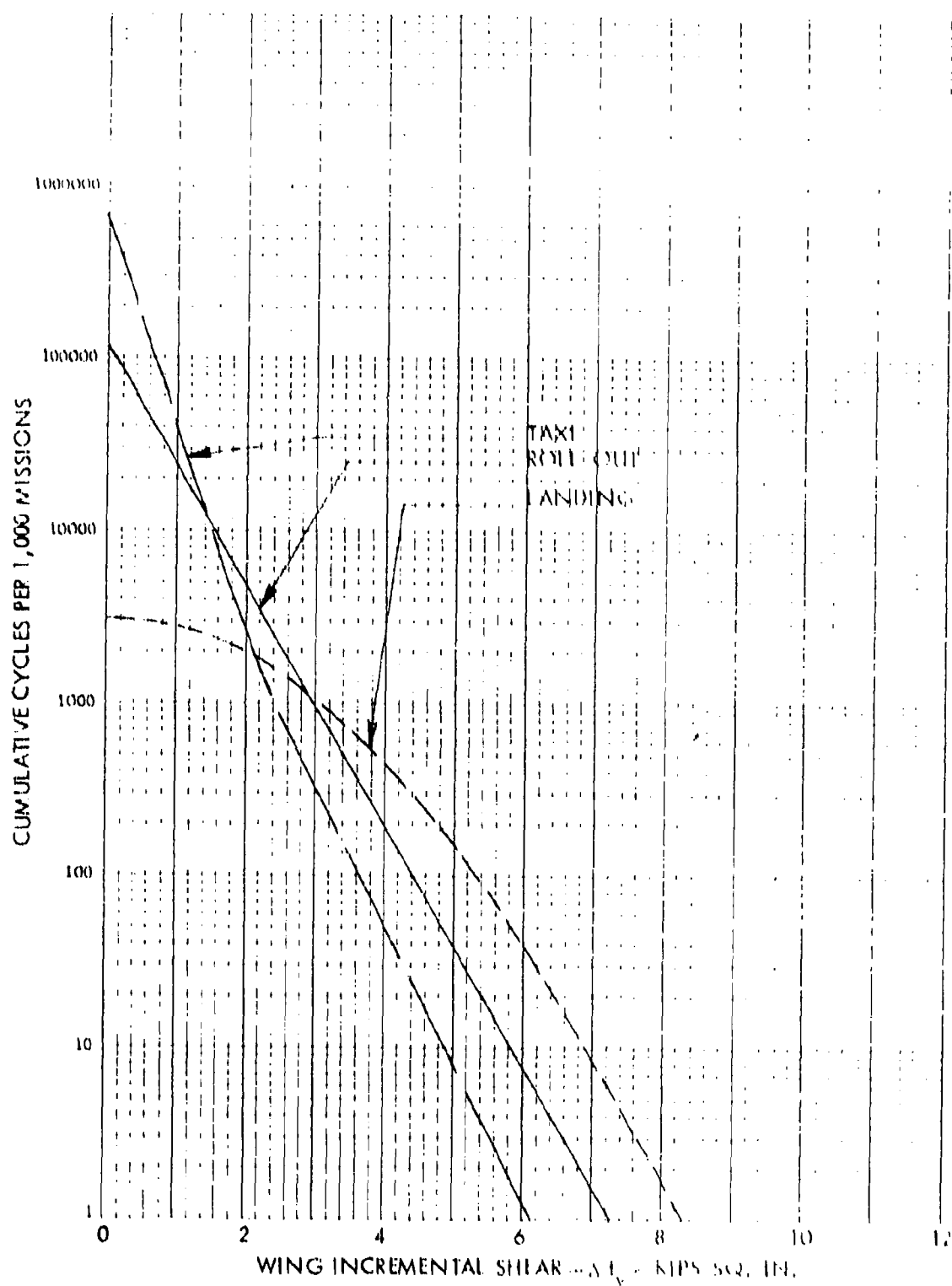


Figure 64. Wing Lower Surface Taxi Spectra - Transport Logistics Mission

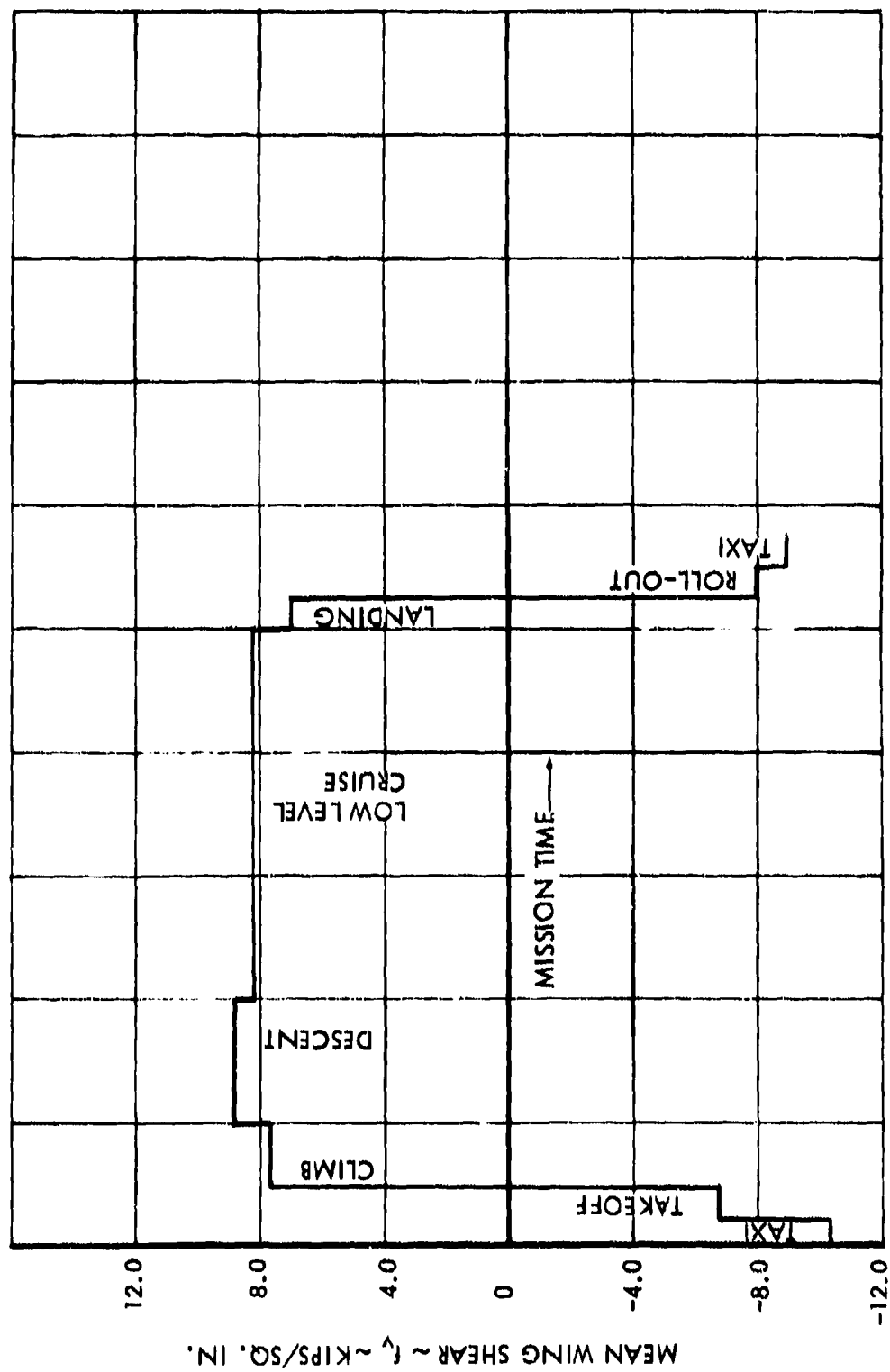


Figure 65. Typical Wing Lower Surface Mean Stresses - Transport Low Level Mission

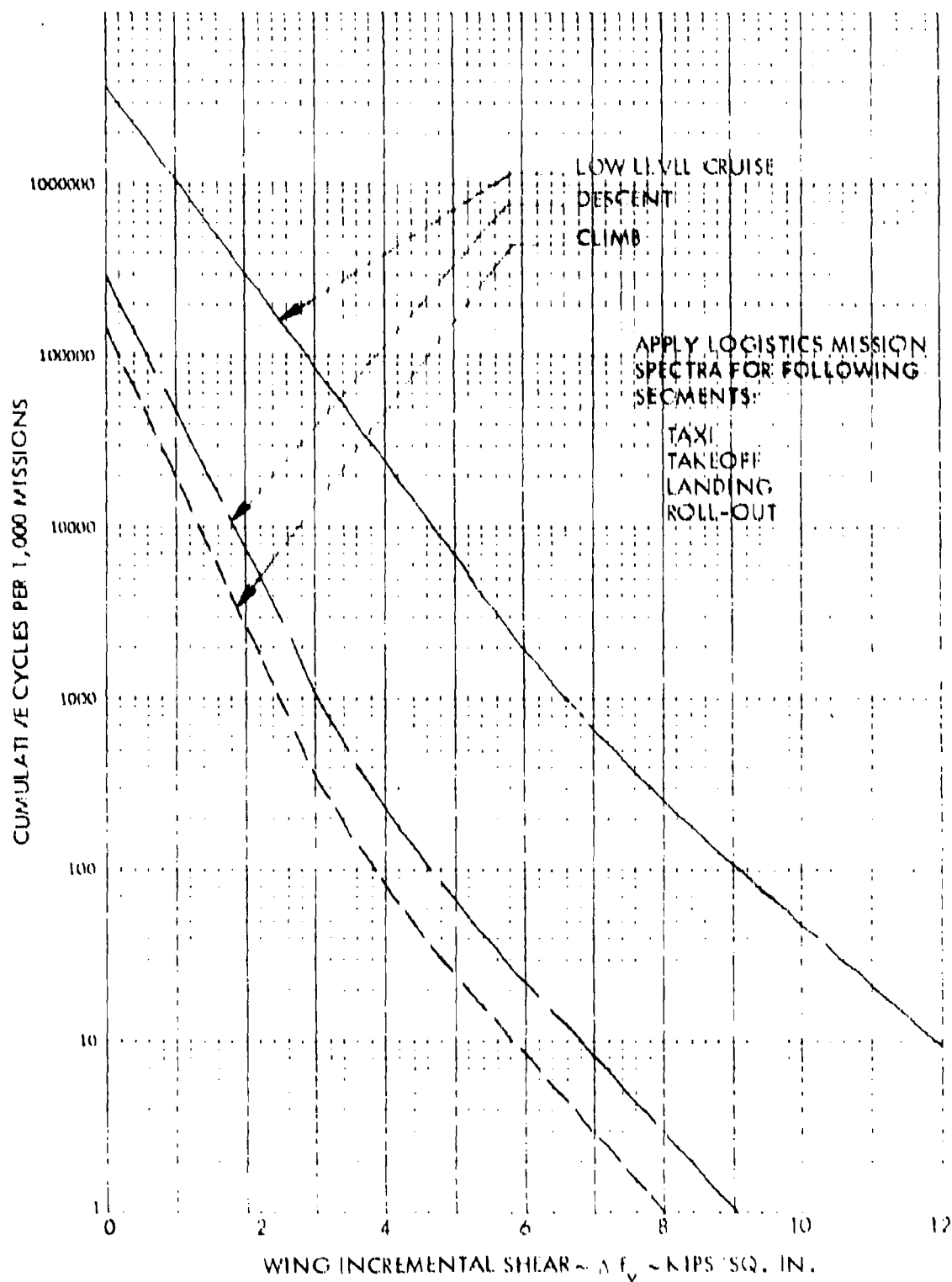


Figure 66. Wing Lower Surface Flight Spectra - Transport Low Level Mission

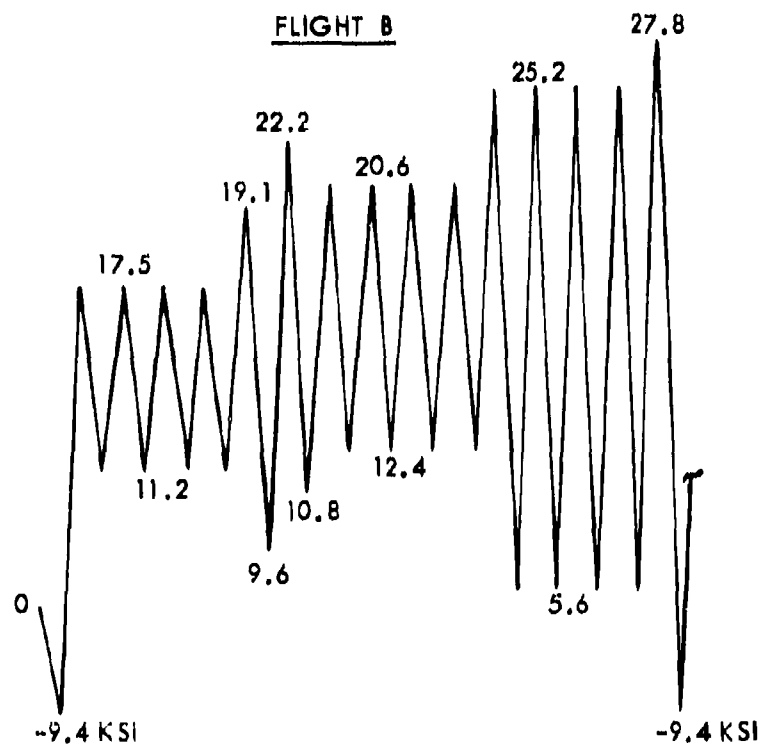
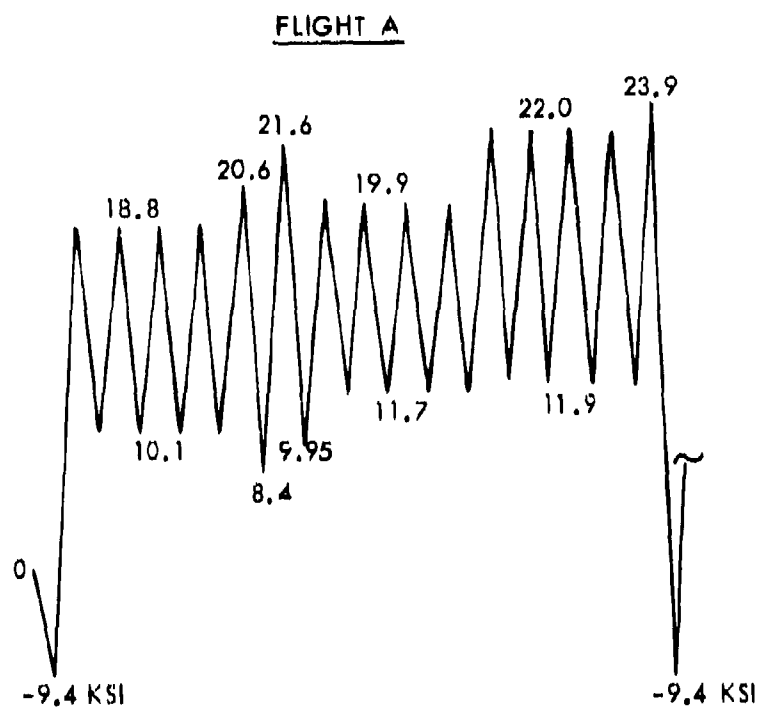


Figure 67. Flight Definition For Spectrum Fatigue Tests

FLIGHT C

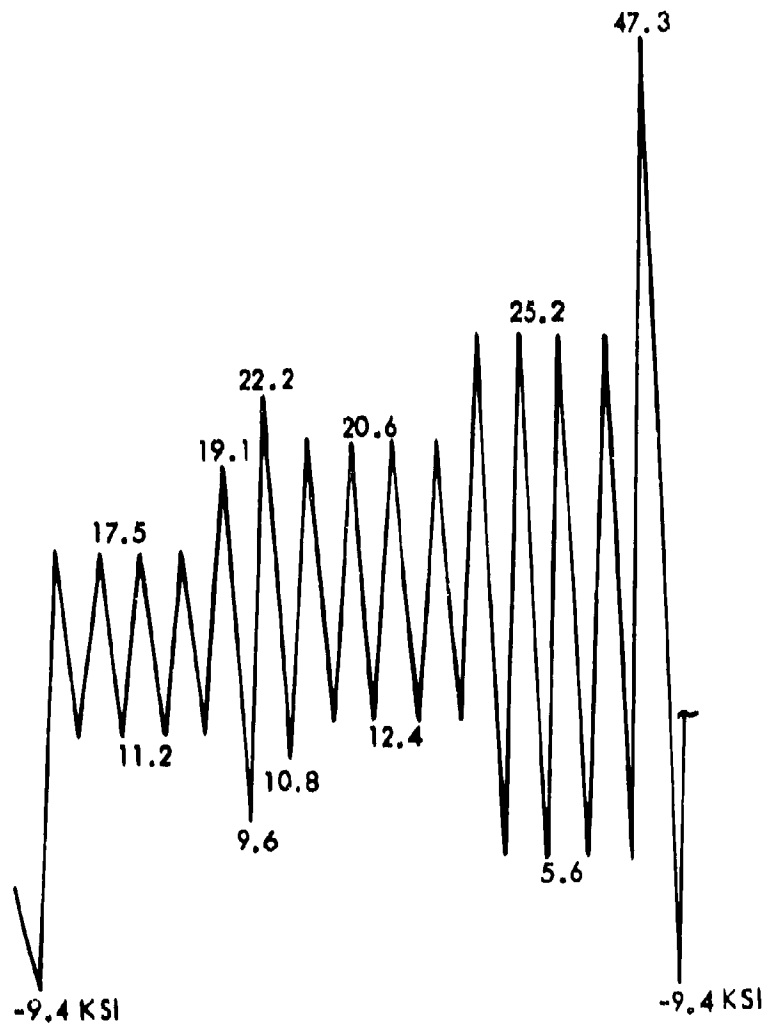


Figure 67. (Continued) Flight Definition For Spectrum Fatigue Tests

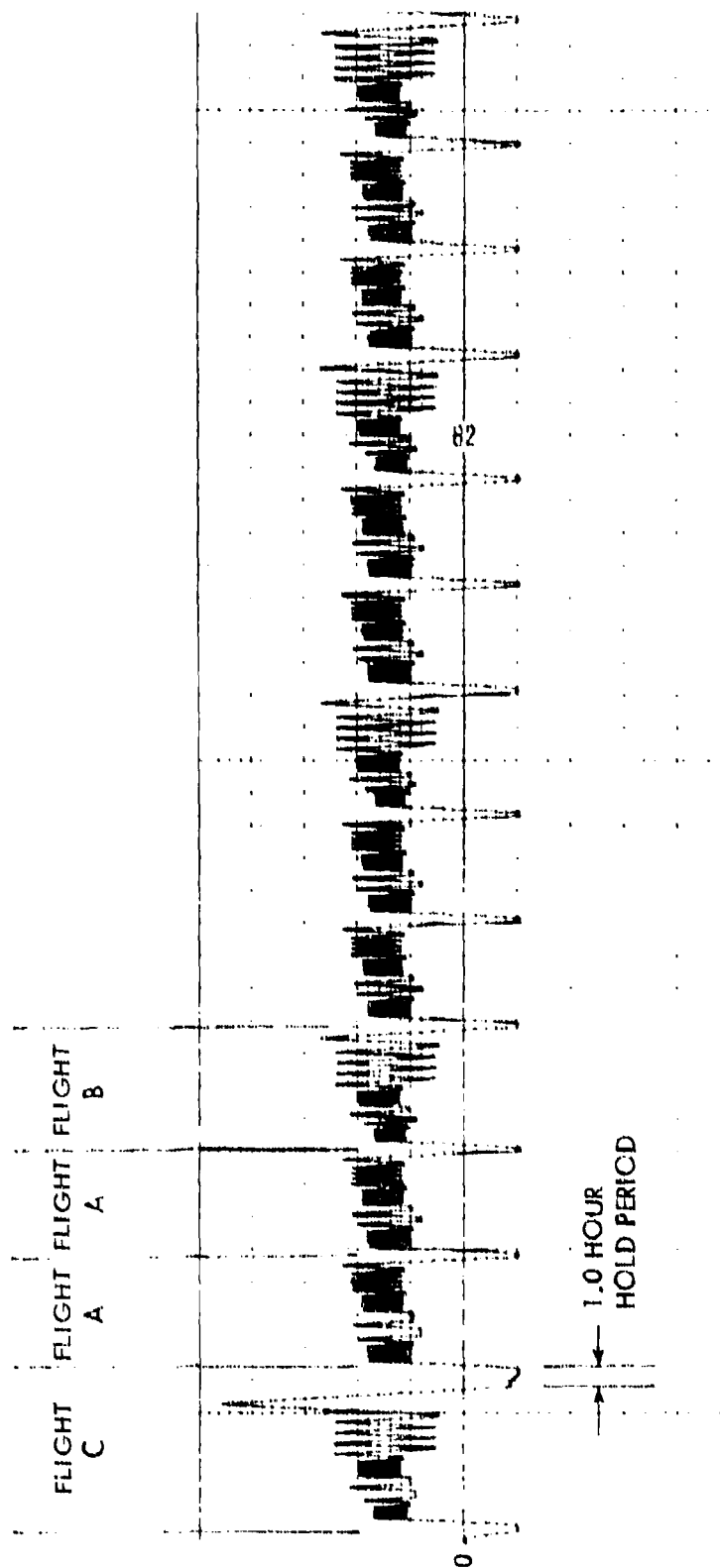
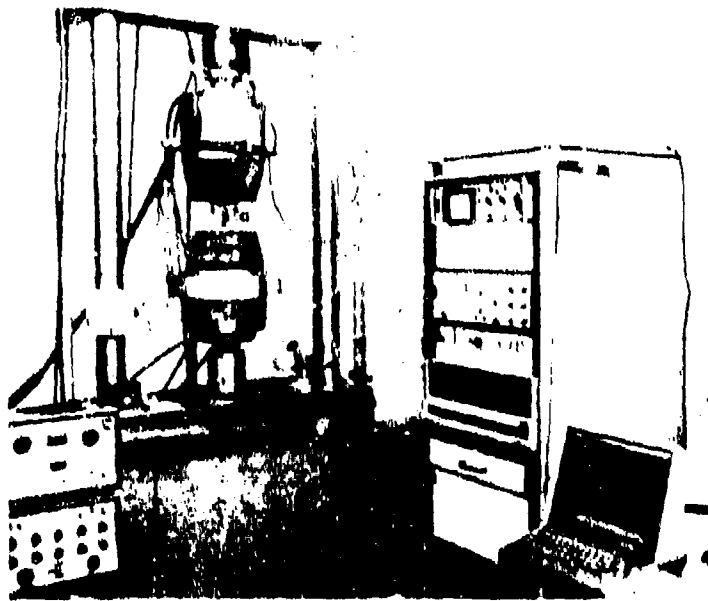
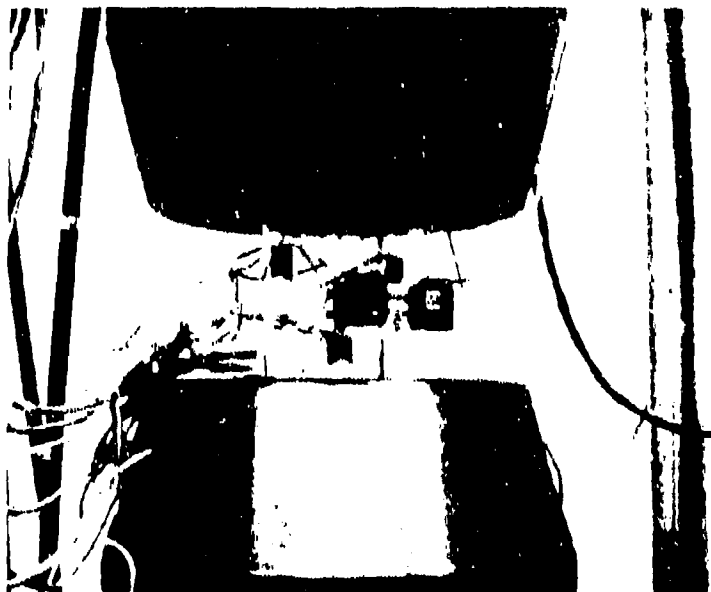


Figure 68. Sample of Strip Chart Record of Spectrum 2 Fatigue Loading



a. Computer Controlled Test Equipment



b. Extensometer and Strain Gages Located on Simplified Stress Concentration Specimen

Figure 69. Test Equipment and Test Arrangement for Simplified Stress Concentration Specimen

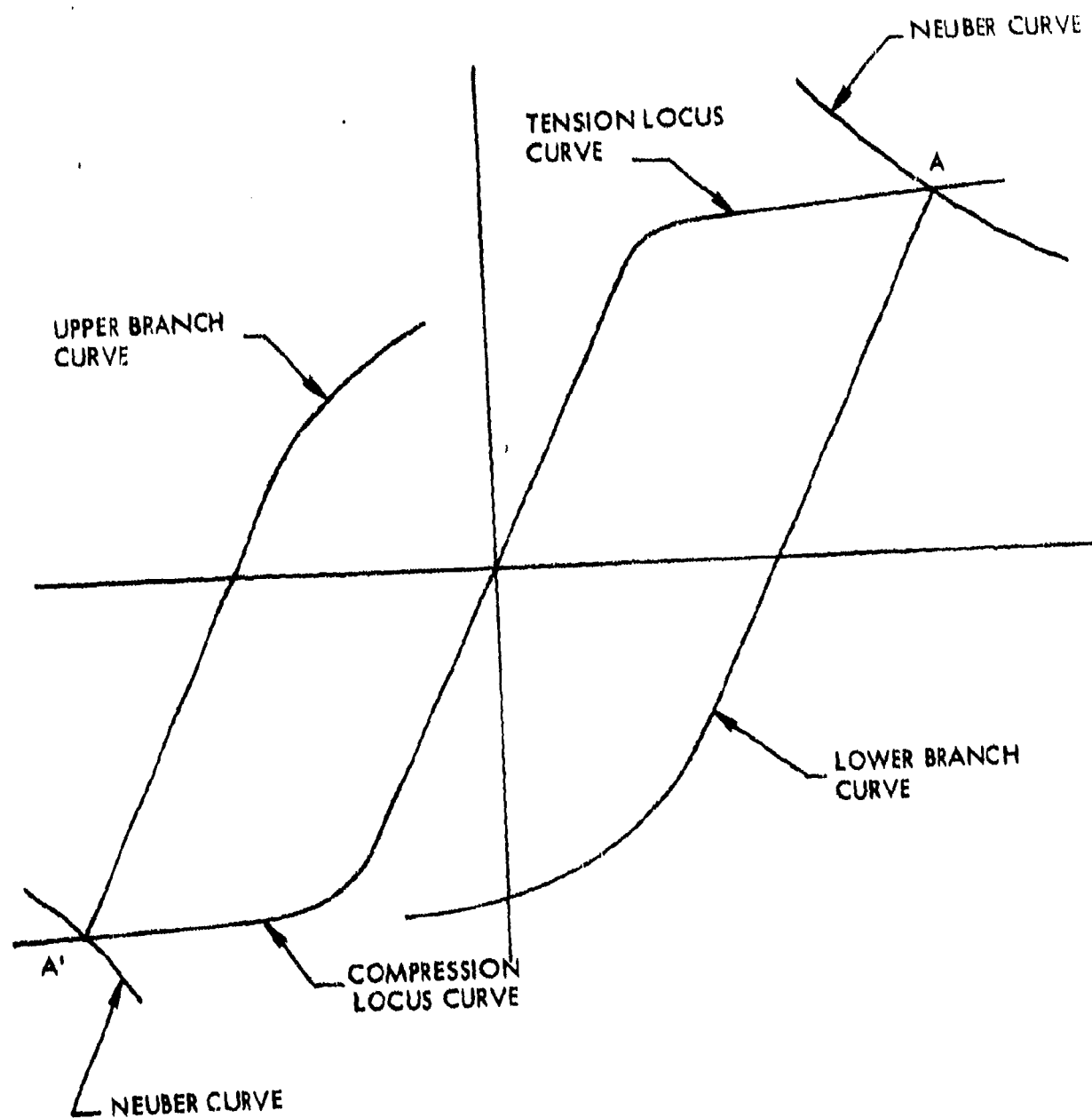


Figure 70. Material Response Algorithm for Hysteresis Analysis Formulation

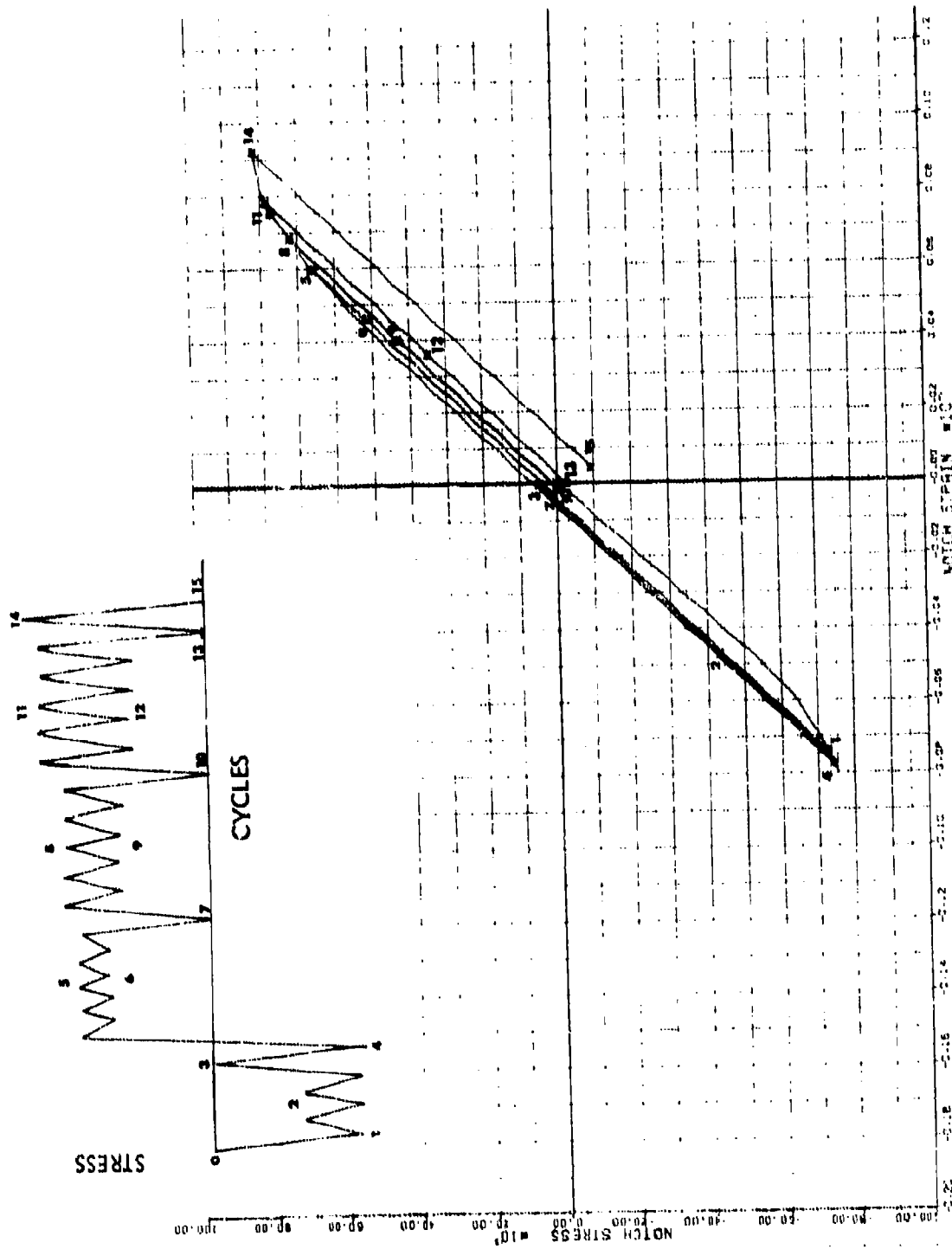


Figure 71. Notch Stress-Notch Strain from Hysteresis Analysis

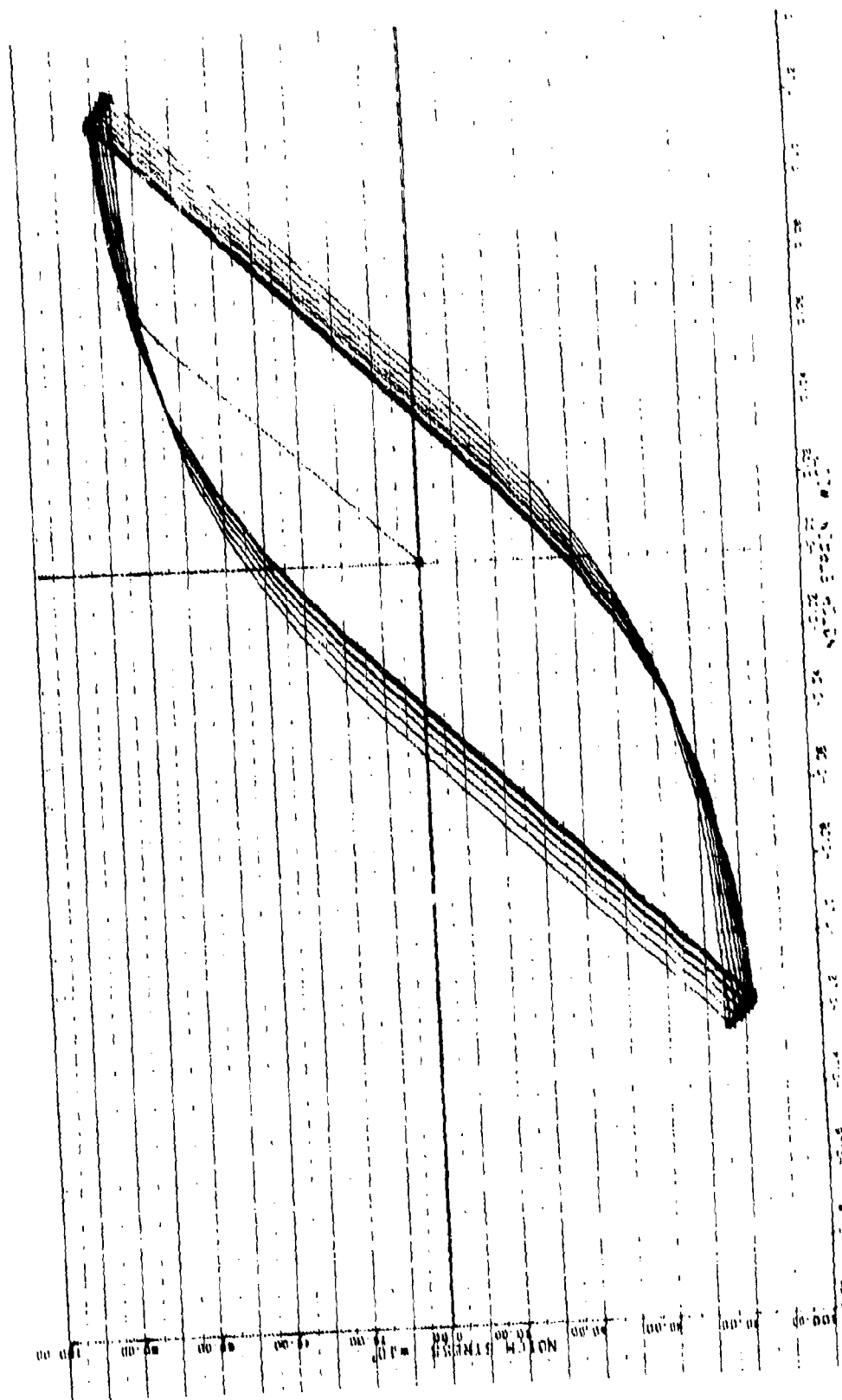


Figure 72. Example of Cyclic Hardening from Hysteresis Analysis

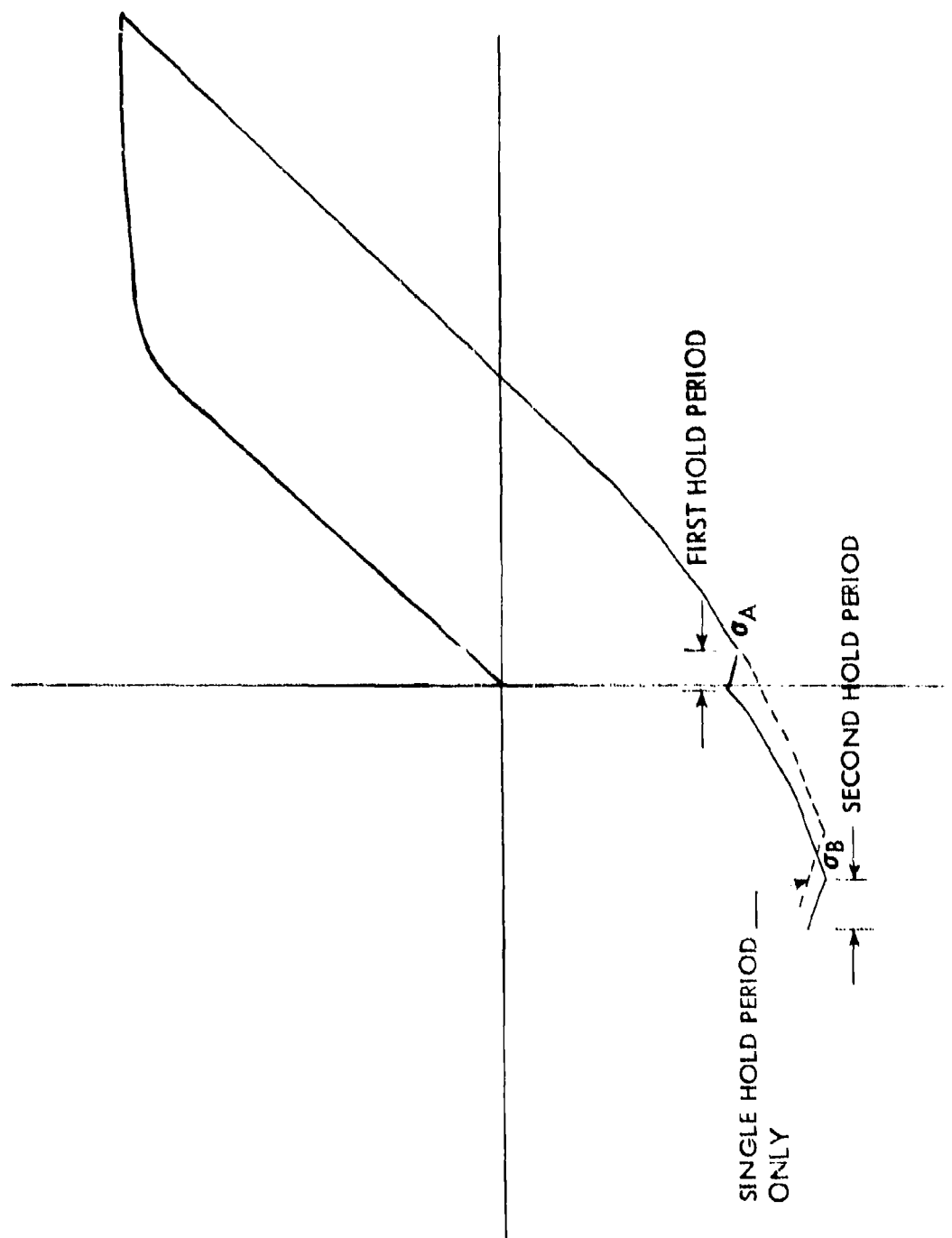


Figure 73. Illustration of Single and Multiple Level Creep During Sustained Load Hold Periods

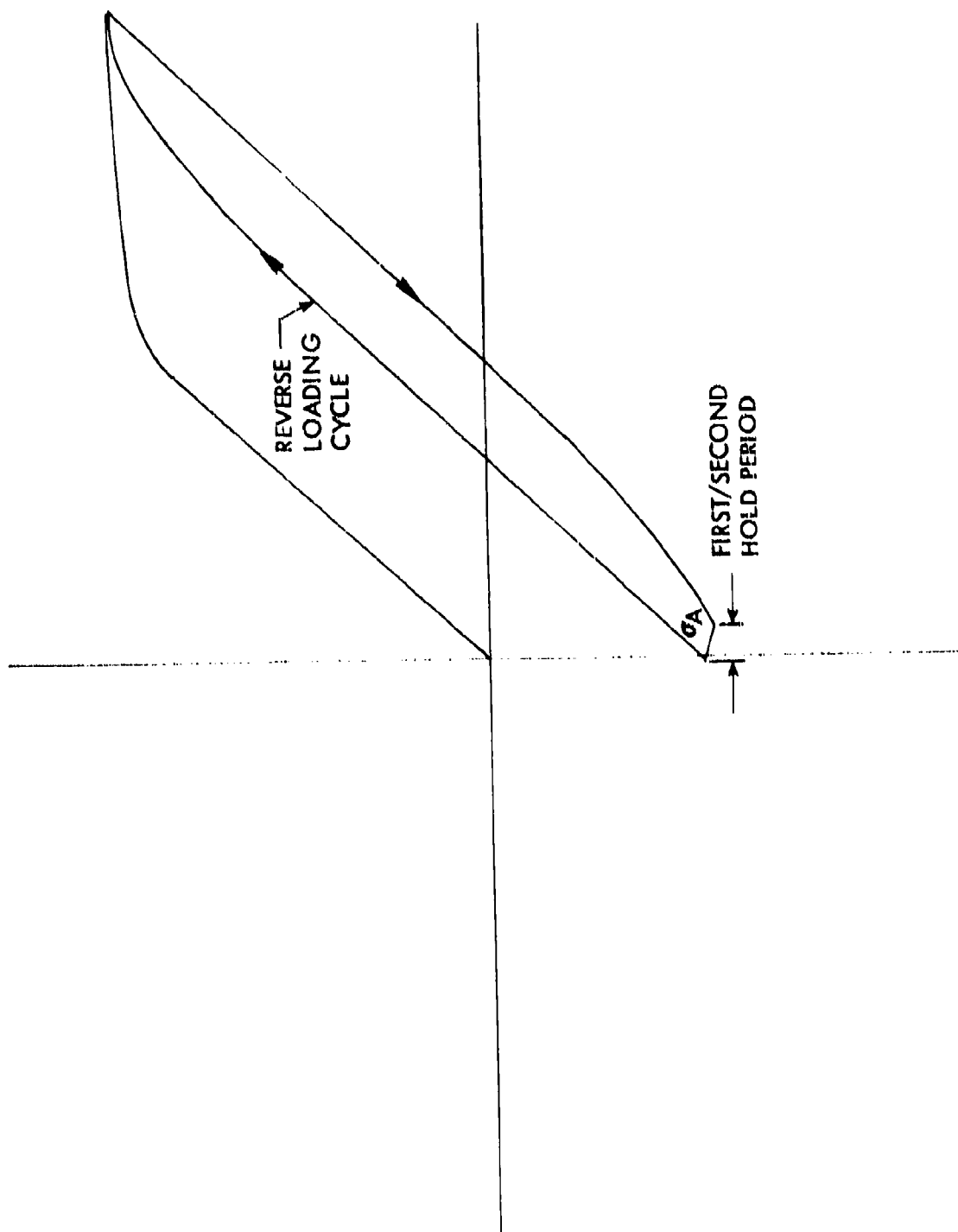


Figure 74. Illustration of Creep During Sustained Load Hold Periods with Completely Reversed Loading

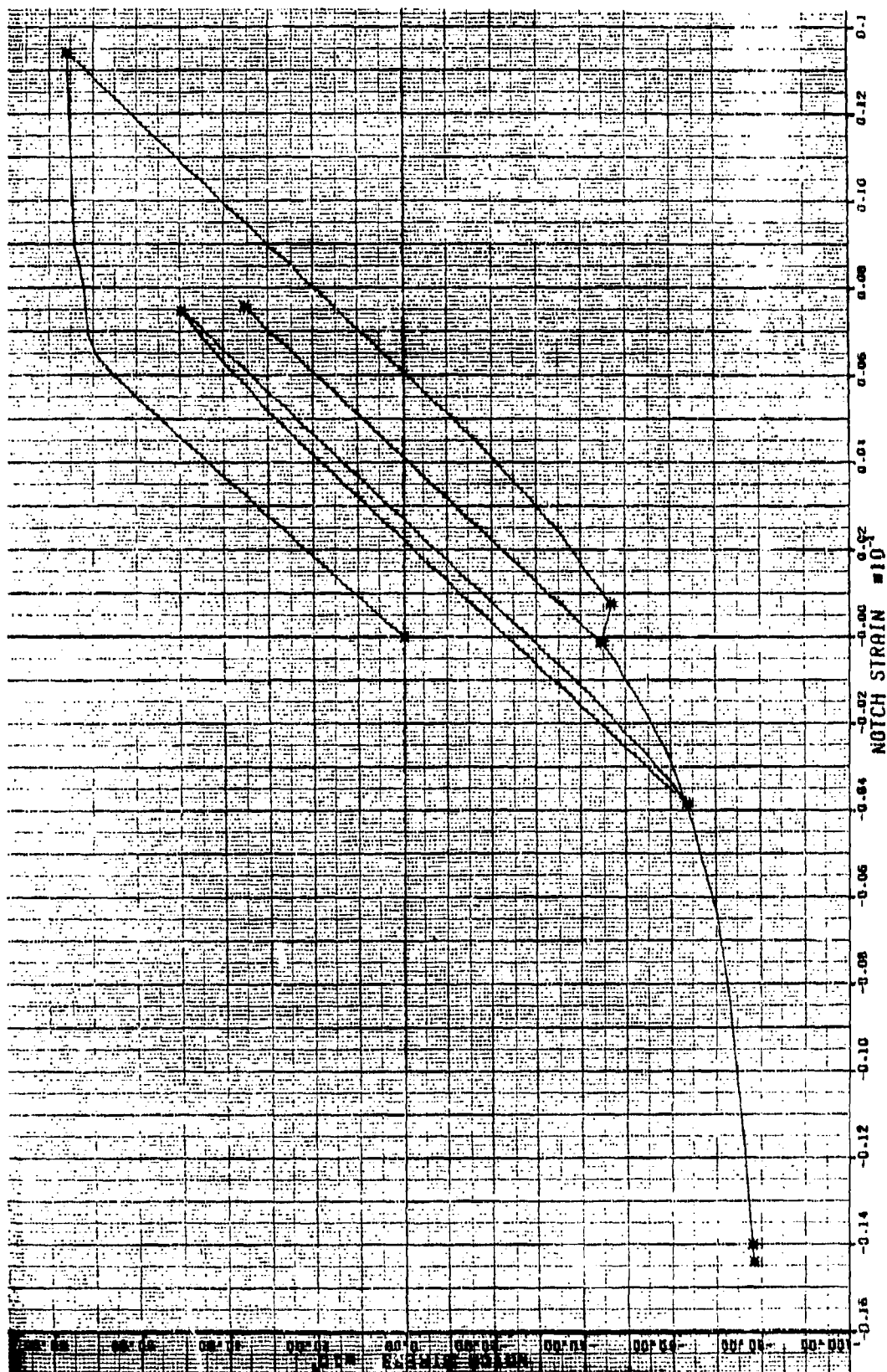


Figure 75. Hysteresis Analysis for Loading Sequence with a Single Sustained Load Hold Period

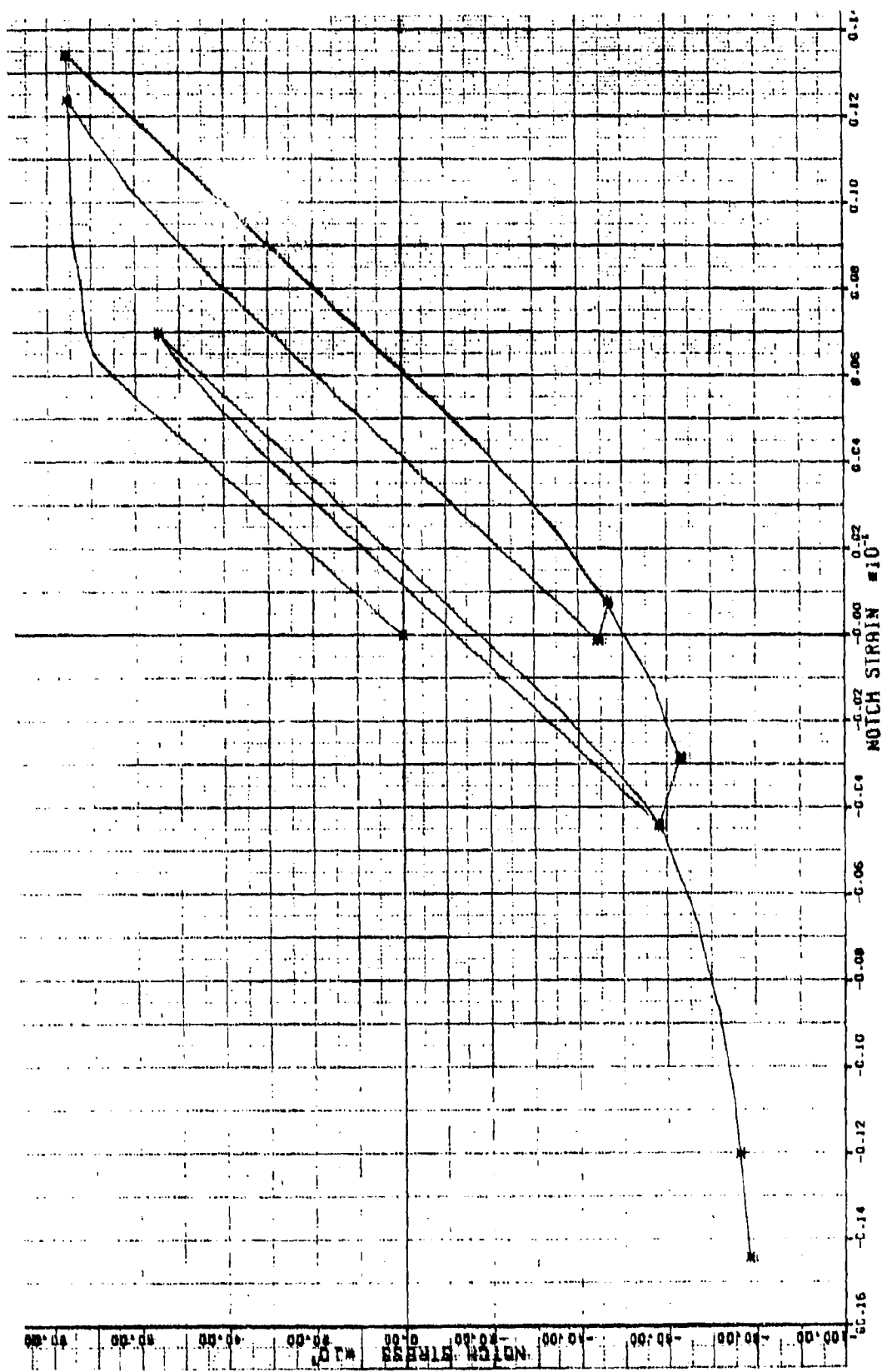


Figure 76. Hysteresis Analysis for Loading Sequence with Two Sustained Load Hold Periods and Completely Reversed Loading

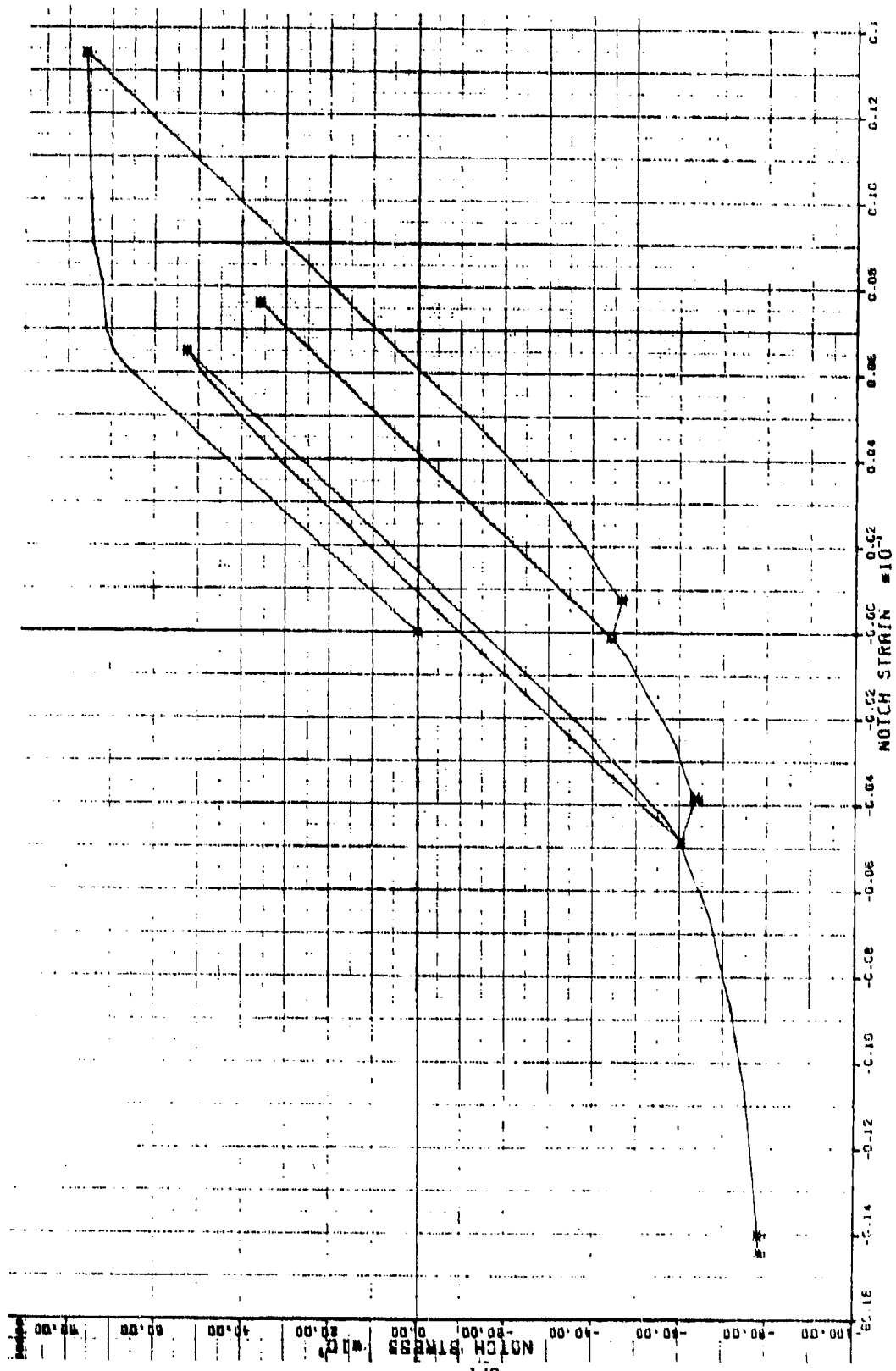
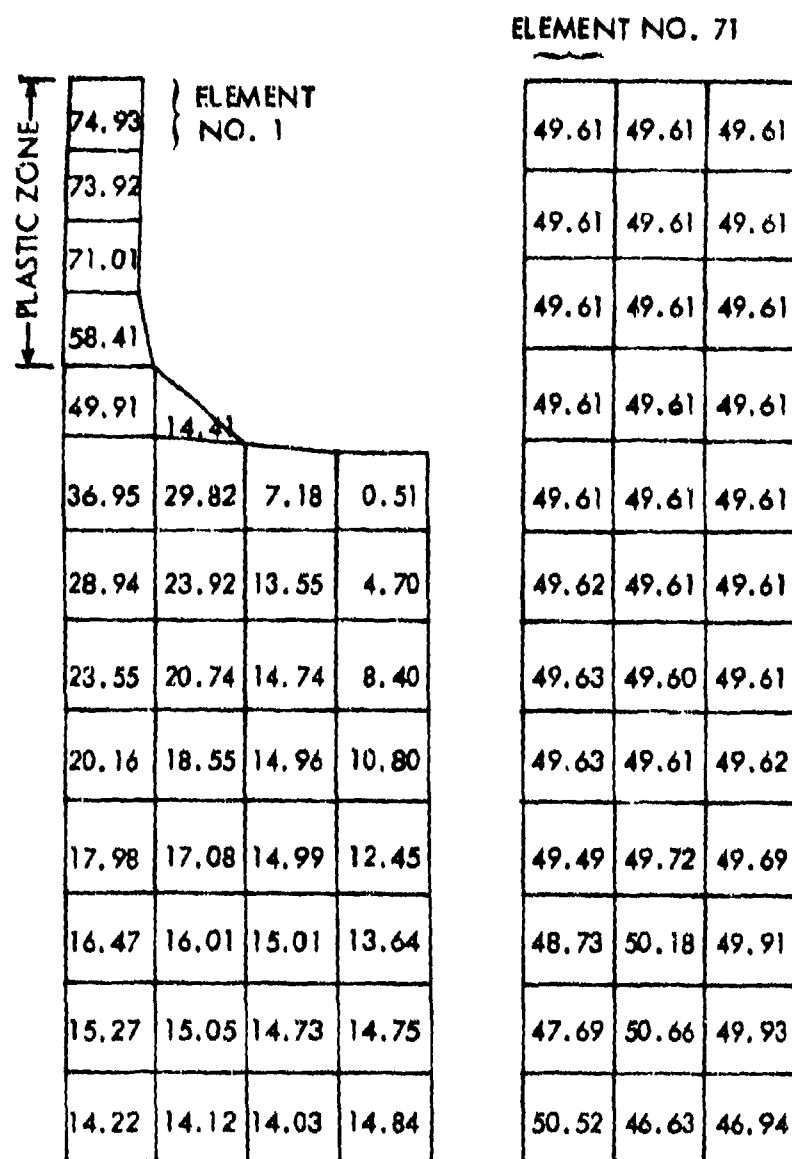
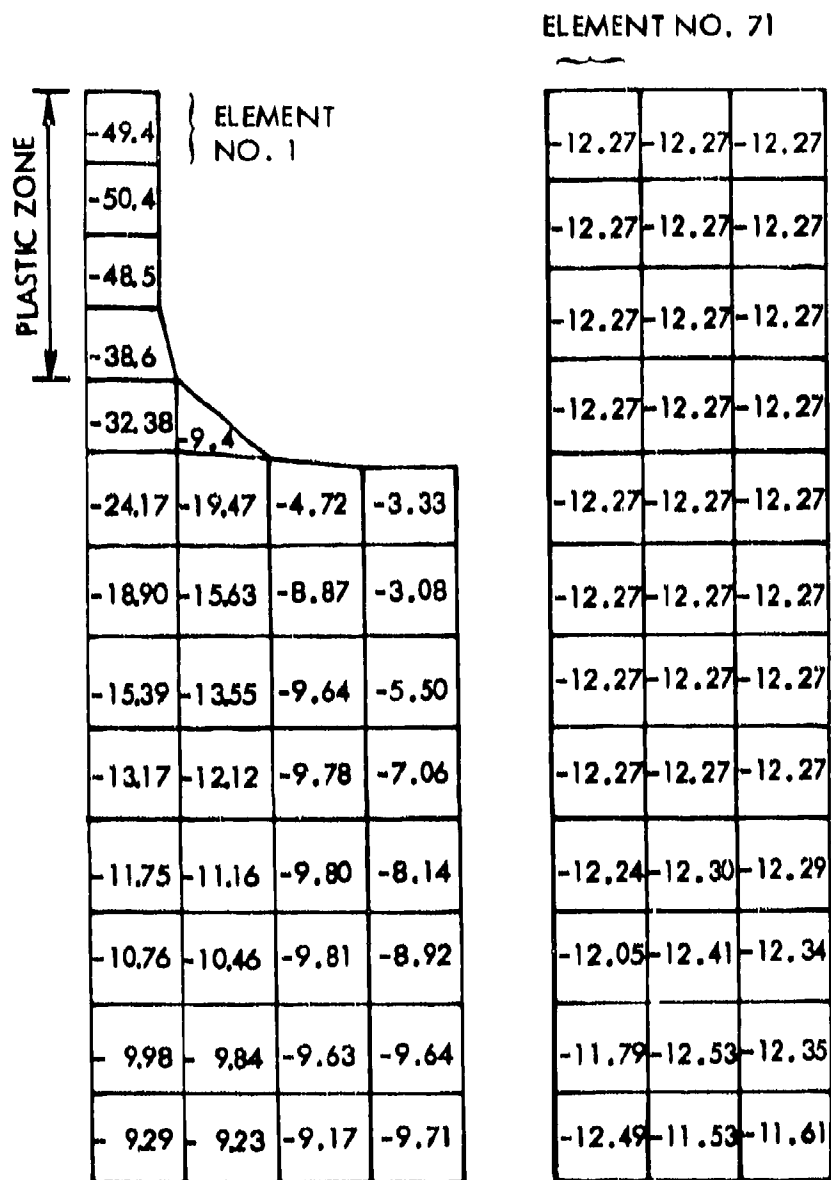


Figure 77. Hysteresis Analysis for Loading Sequence with Two Sustained Load Hold Periods



- (1) All Stresses in ksi
- (2) Applied Load = 13,060 lbs.

Figure 78. Finite Element Analysis Stress Distribution
For Maximum Applied Tensile Load For
Simplified Stress Concentration Specimen



- (1) All Stresses in ksi
- (2) Net Section Compression Stress = -20 ksi

Figure 79. Finite Element Analysis Stress Distribution For Sustained Compression Loading On Simplified Stress Concentration Specimen

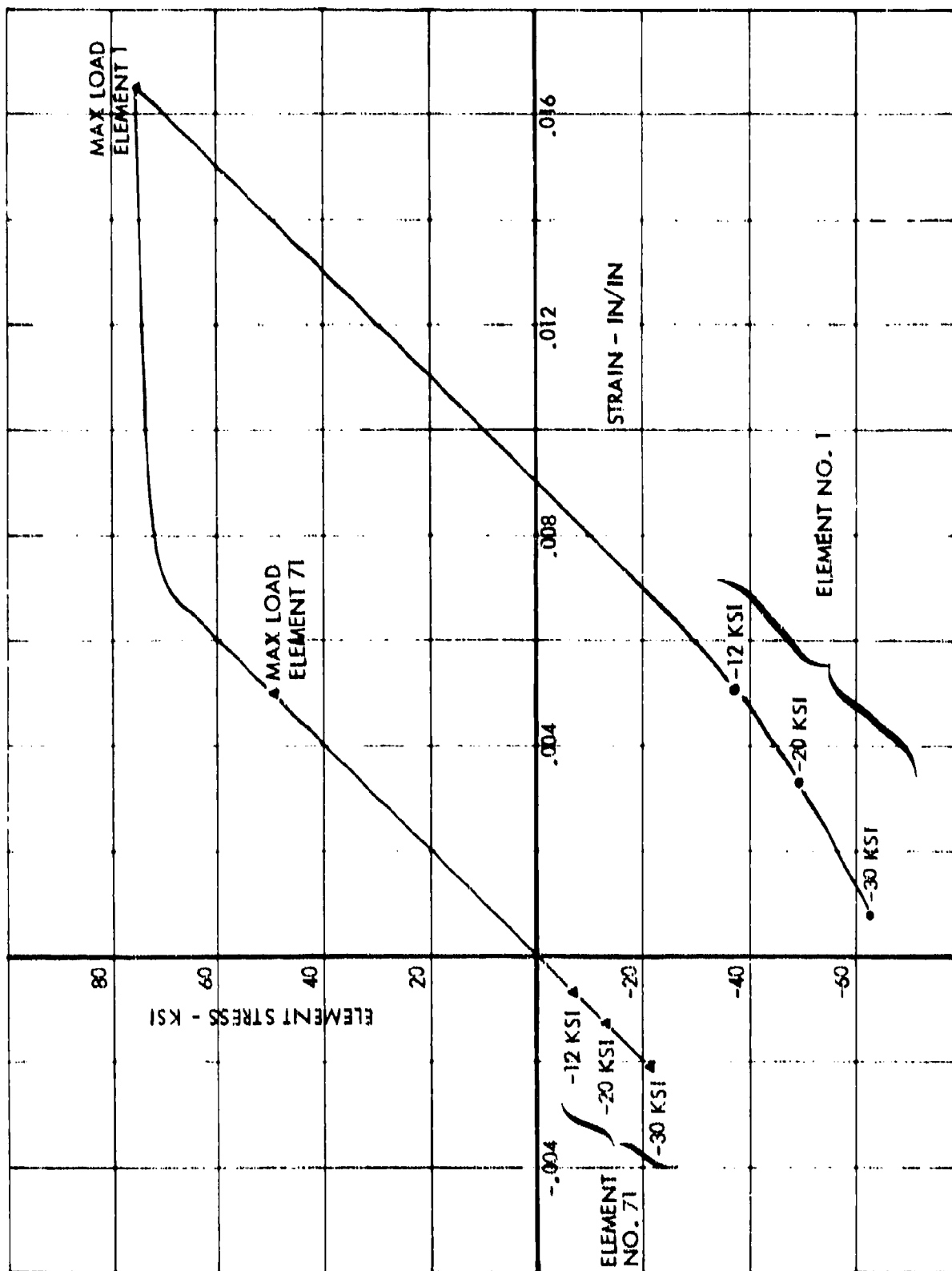


Figure 80. Analytical Stress-Strain Curve for Simplified Stress Concentration Specimen

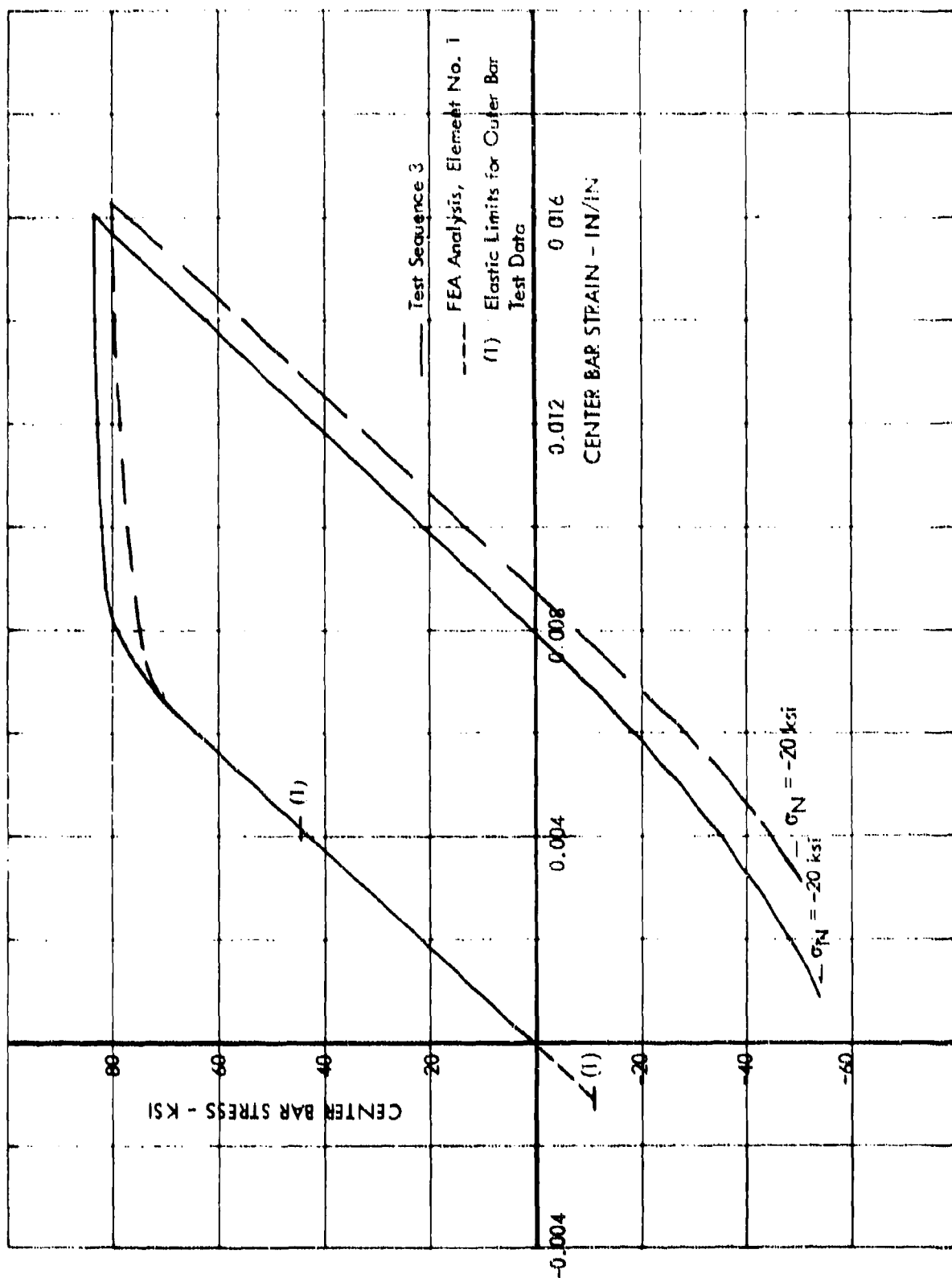


Figure 81. Analysis and Test Comparison for SSC Specimen Test Sequence No. 3, -20 ksi Compression

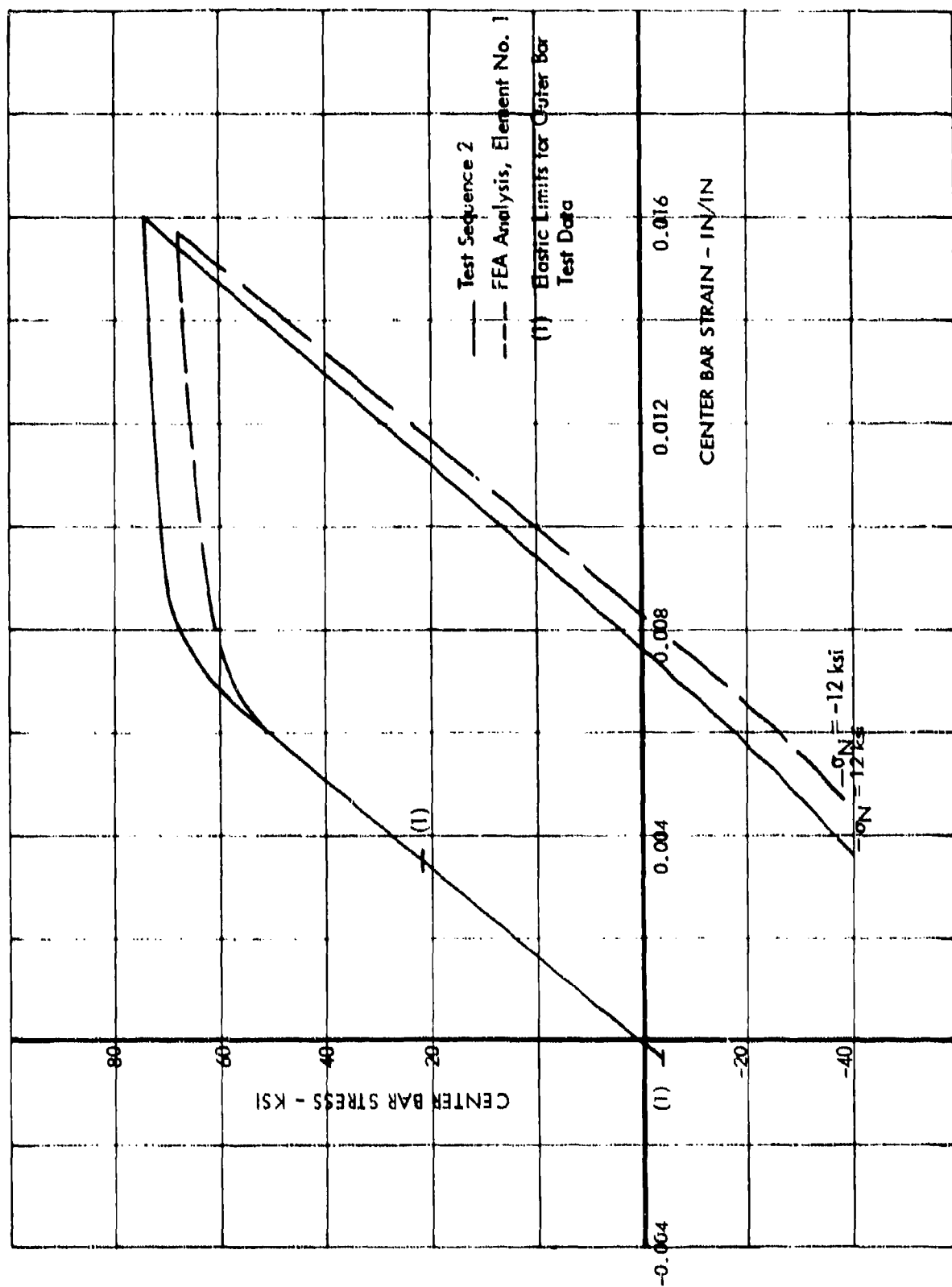


Figure 82. Analysis and Test Comparison For SSC Specimen Test Sequence No. 2, -12 ksi Compression

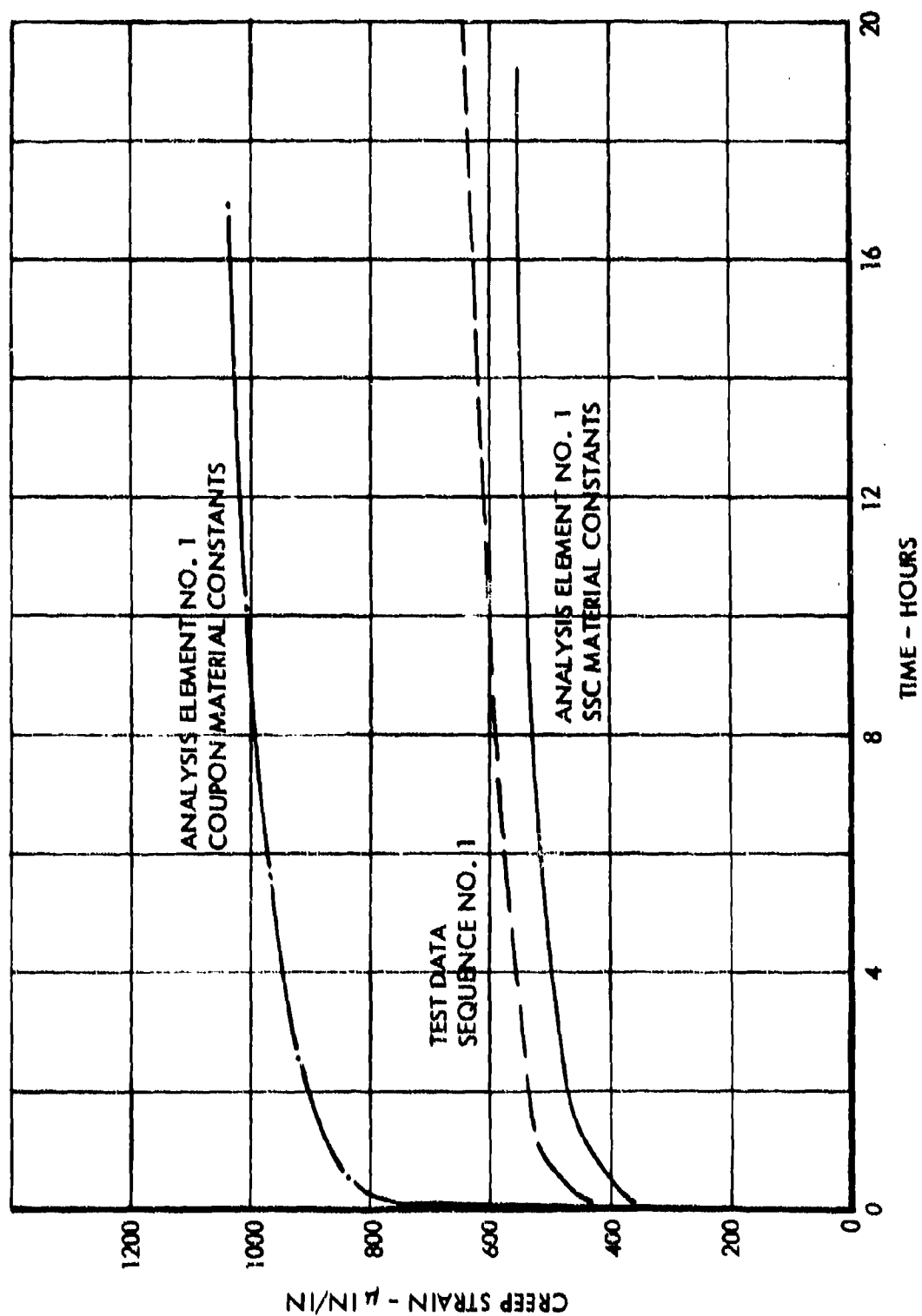


Figure 83. Analytical Creep Predictions Versus Center Bar Test Results - Simplified Stress Concentration Specimen, Applied Net Section Stress -20 ksi

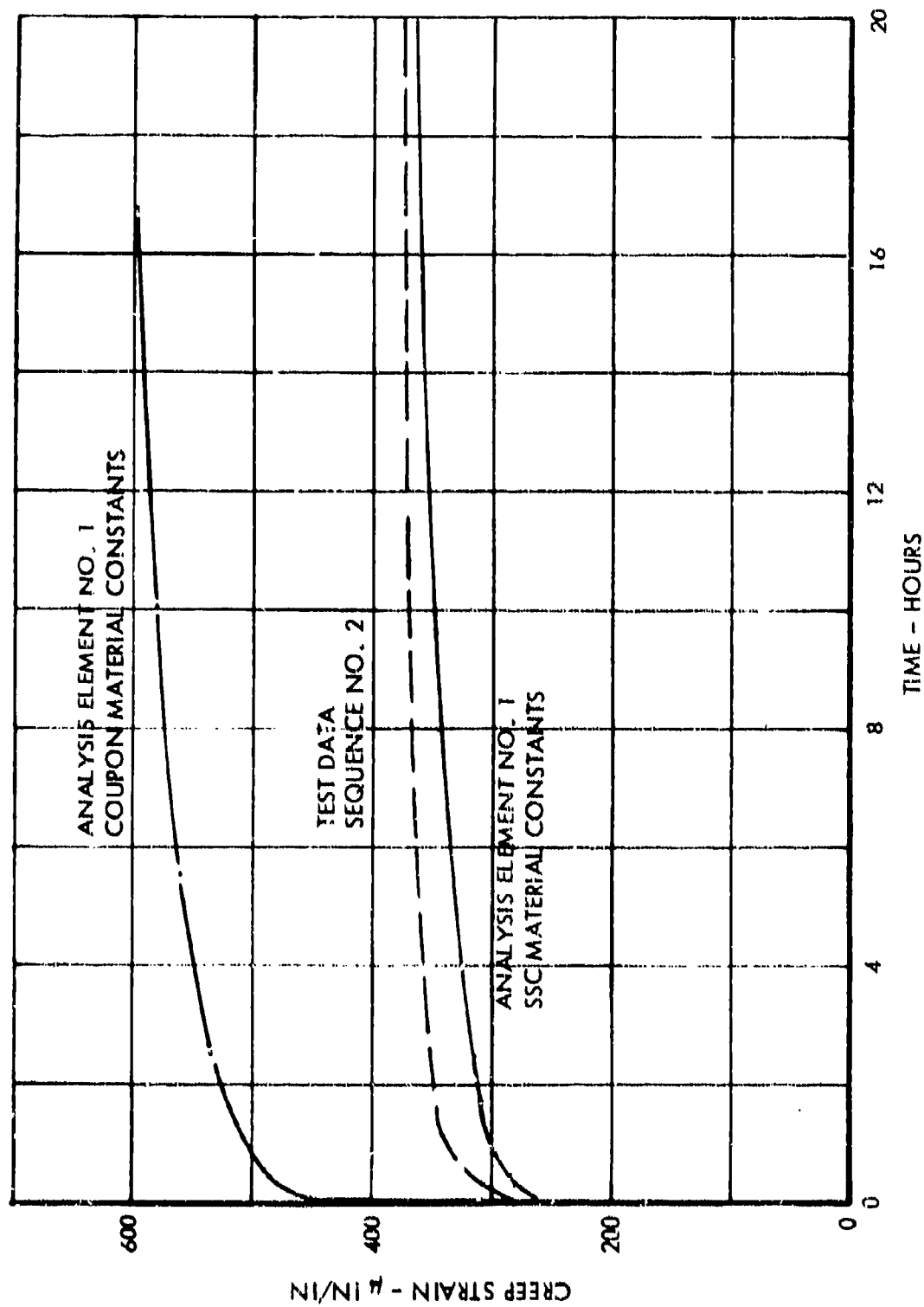


Figure 84. Analytical Creep Predictions Versus Center Bar Test Results - Simplified Stress Concentration Specimen - Applied Net Section Stress - 12.0 ksi

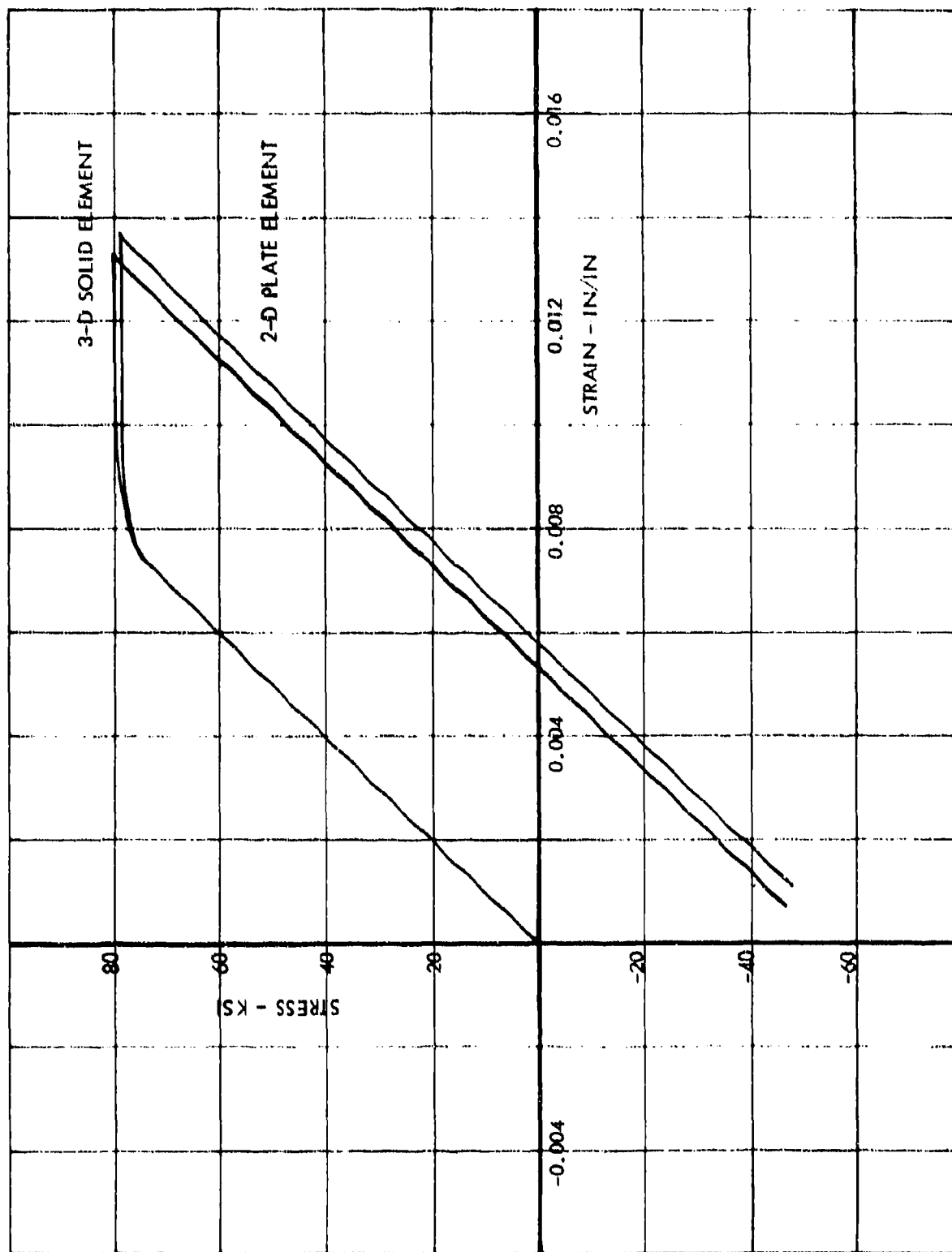
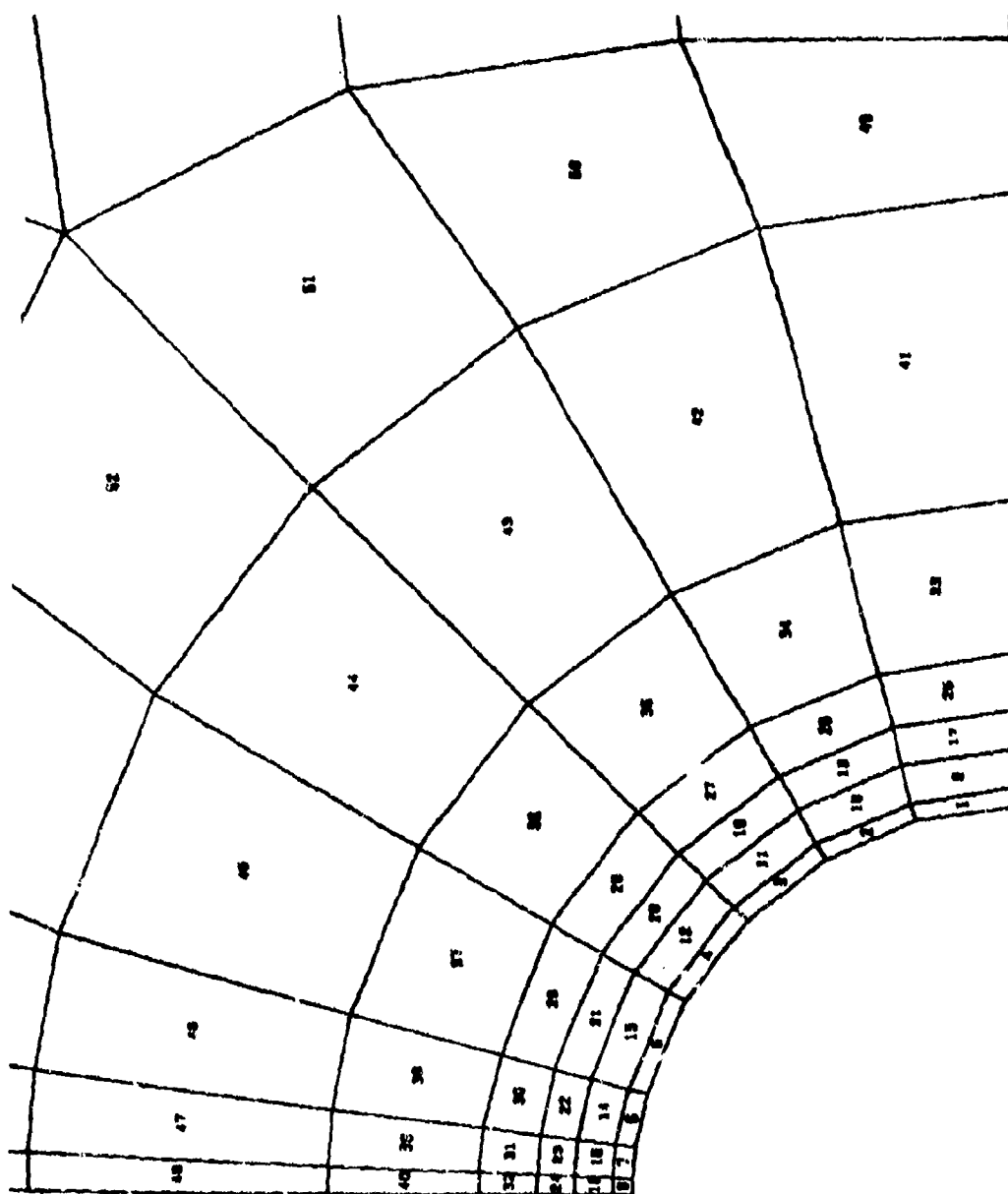


Figure 85. Average Stress-Strain Data for Solid and Plate Elements



PLASTIC STRESS LEVEL
(FAR-FIELD STRESS)

El. No. 6 = 20.2 ksi

7 = 21.5

6 = 22.5

16 = 24.5

15 = 25.5

14 = 27.5

24 = 29.5

23 = 29.5

5 = 29.5

22 = 31.5

13 = 33.5

31 = 33.5

32 = 33.5

30 = 34.5

PLASTIC ZONE
BOUNDARY

Figure 86. Plastic Zone Definition Around Stress Riser In Super-Scale Specimen

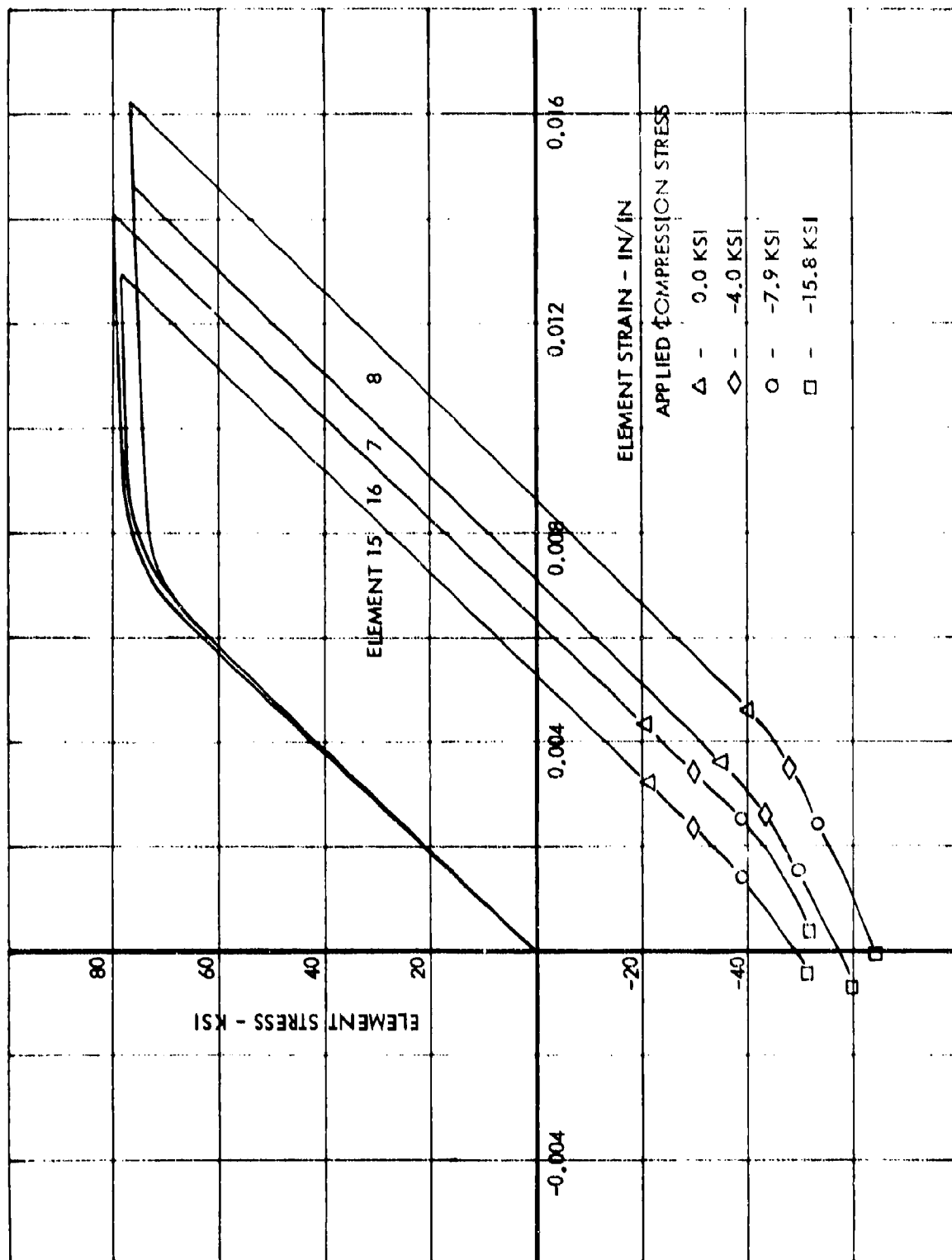


Figure 87. Stress-Strain Curve Specimen 2-D Finite Element Analysis Results

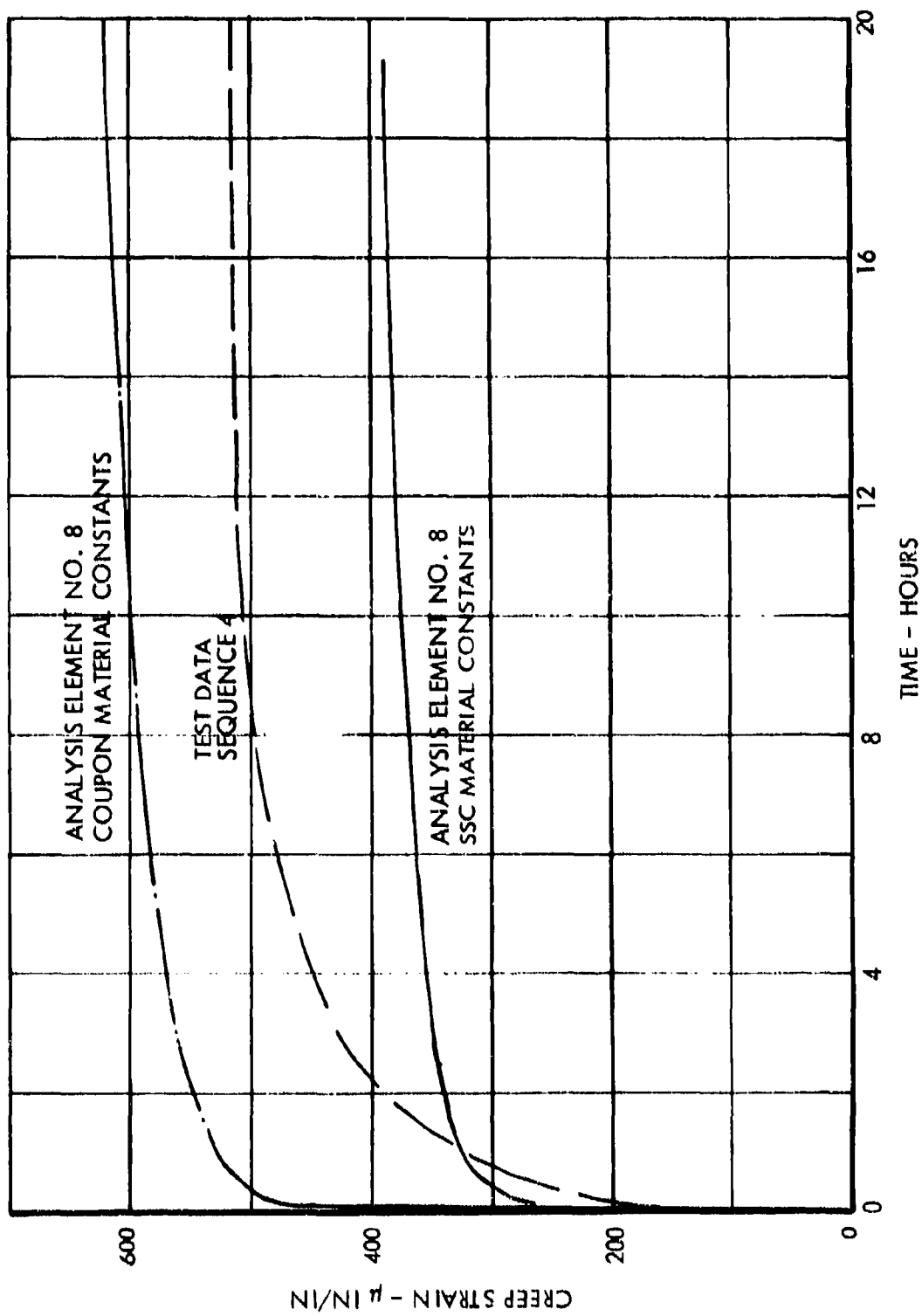


Figure 88. Analytical Creep Predictions Versus Transducer Measurements
Super-Scale Specimen, Zero Net Section Stress

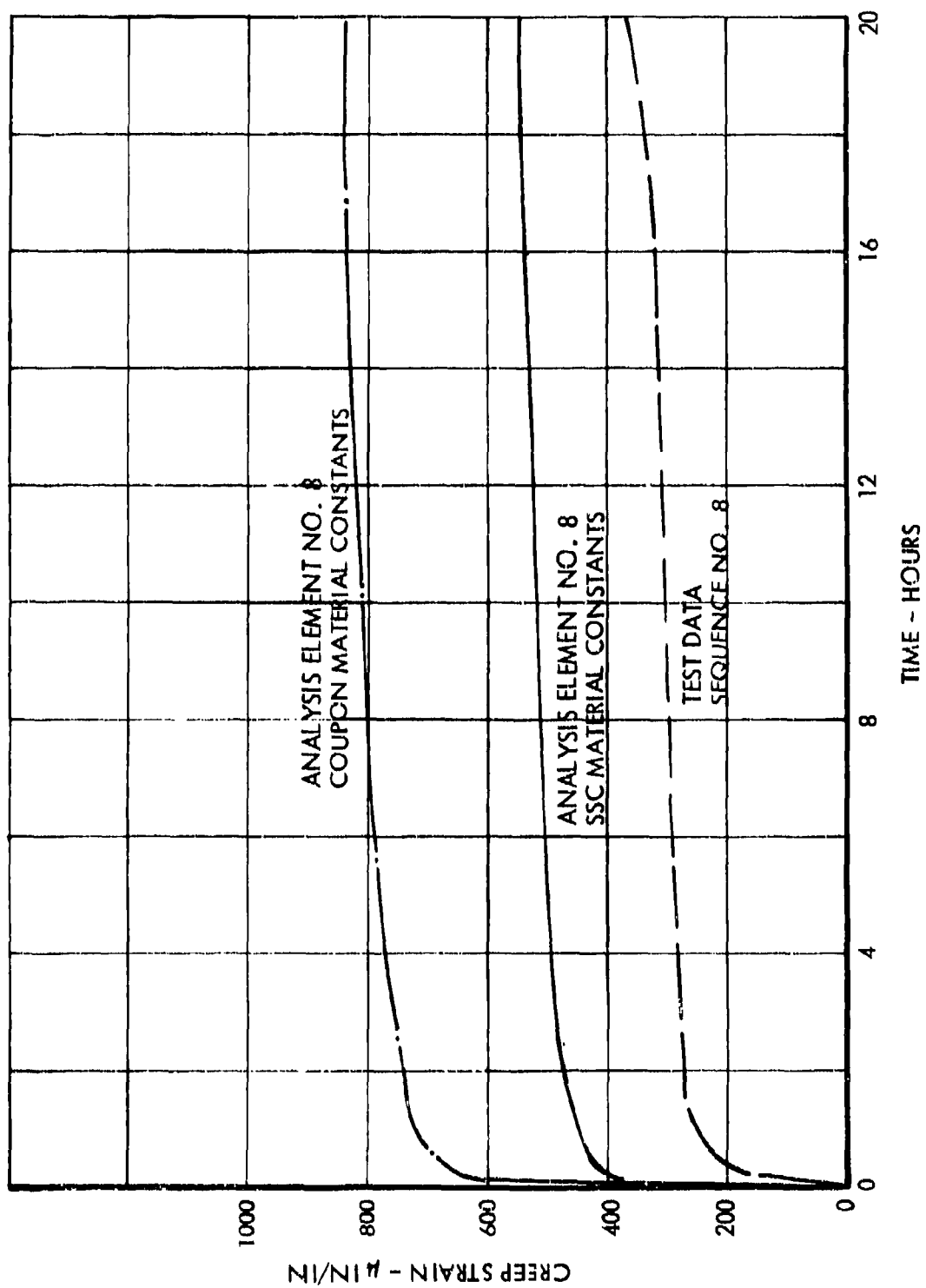


Figure 89. Analytical Creep Predictions Versus Transducer Measurements
Super-Scale Specimen, ~4.0 ksi Net Section Stress

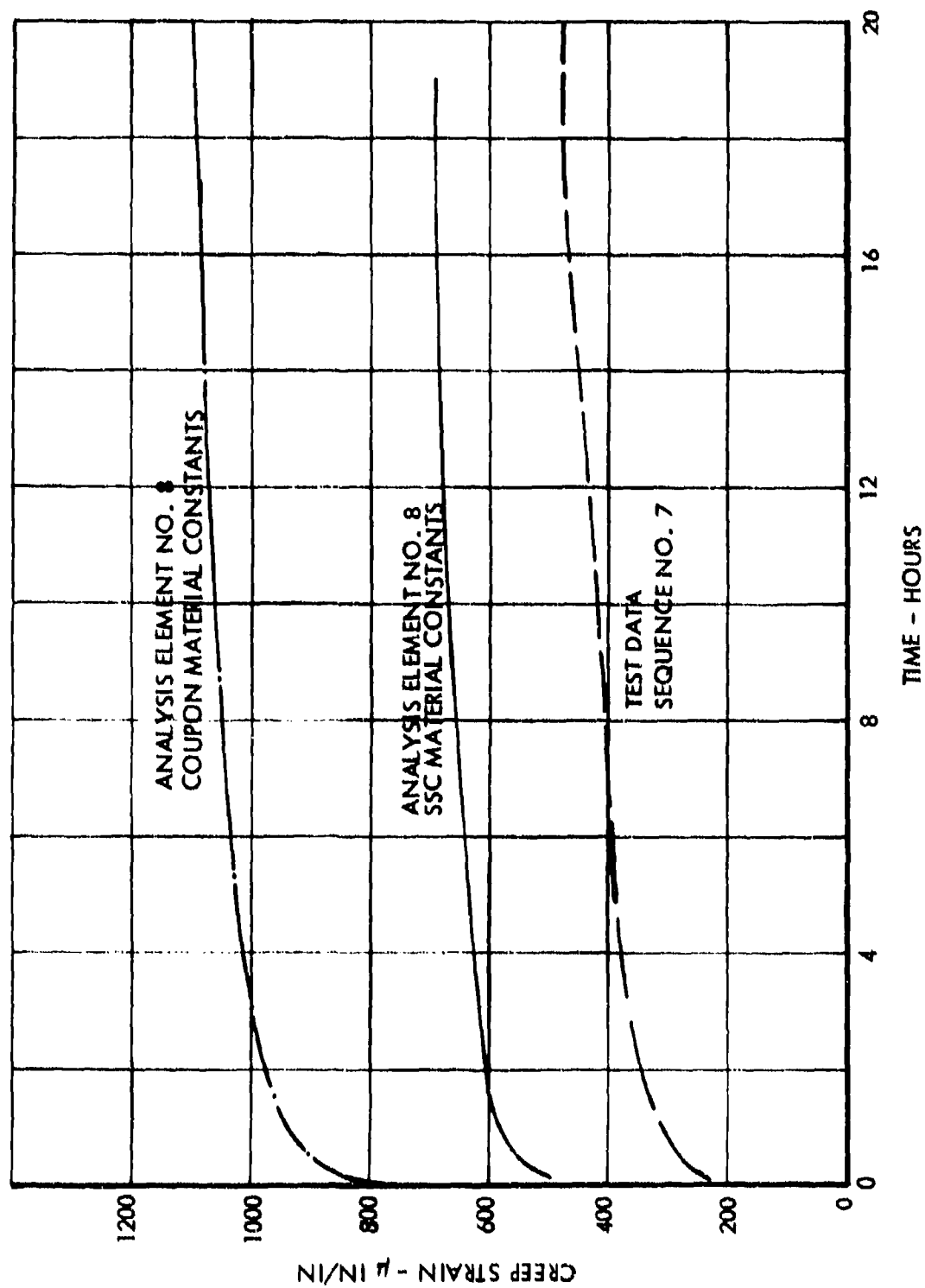


Figure 90. Analytical Creep Predictions Versus Transducer Measurements
Super-Scale Specimen, -7.9 ksi Net Section Stress

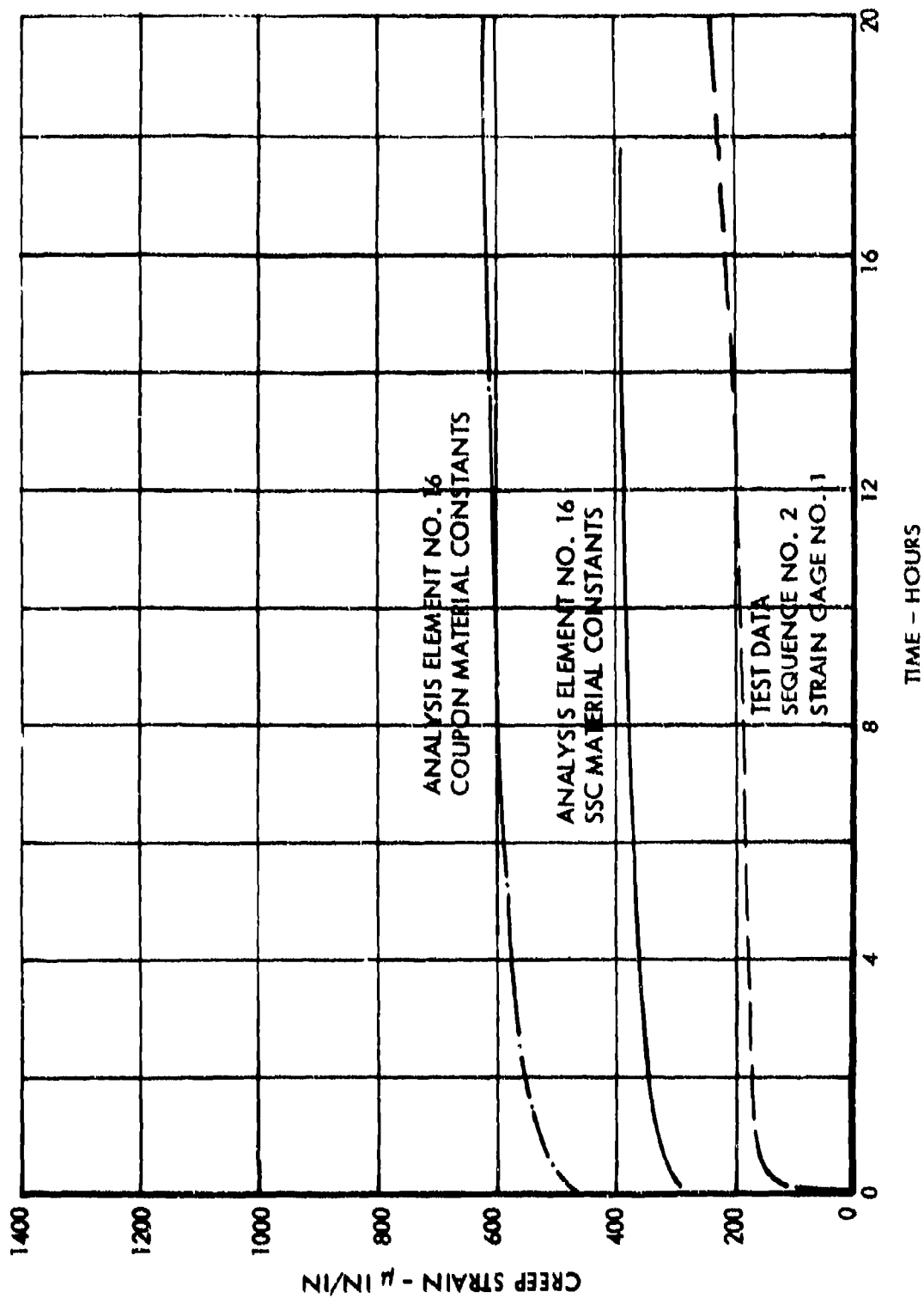


Figure 91. Analytical Creep Predictions Versus Strain Gage Measurements
Super-Scale Specimen, -7.9 ksi Net Section Stress

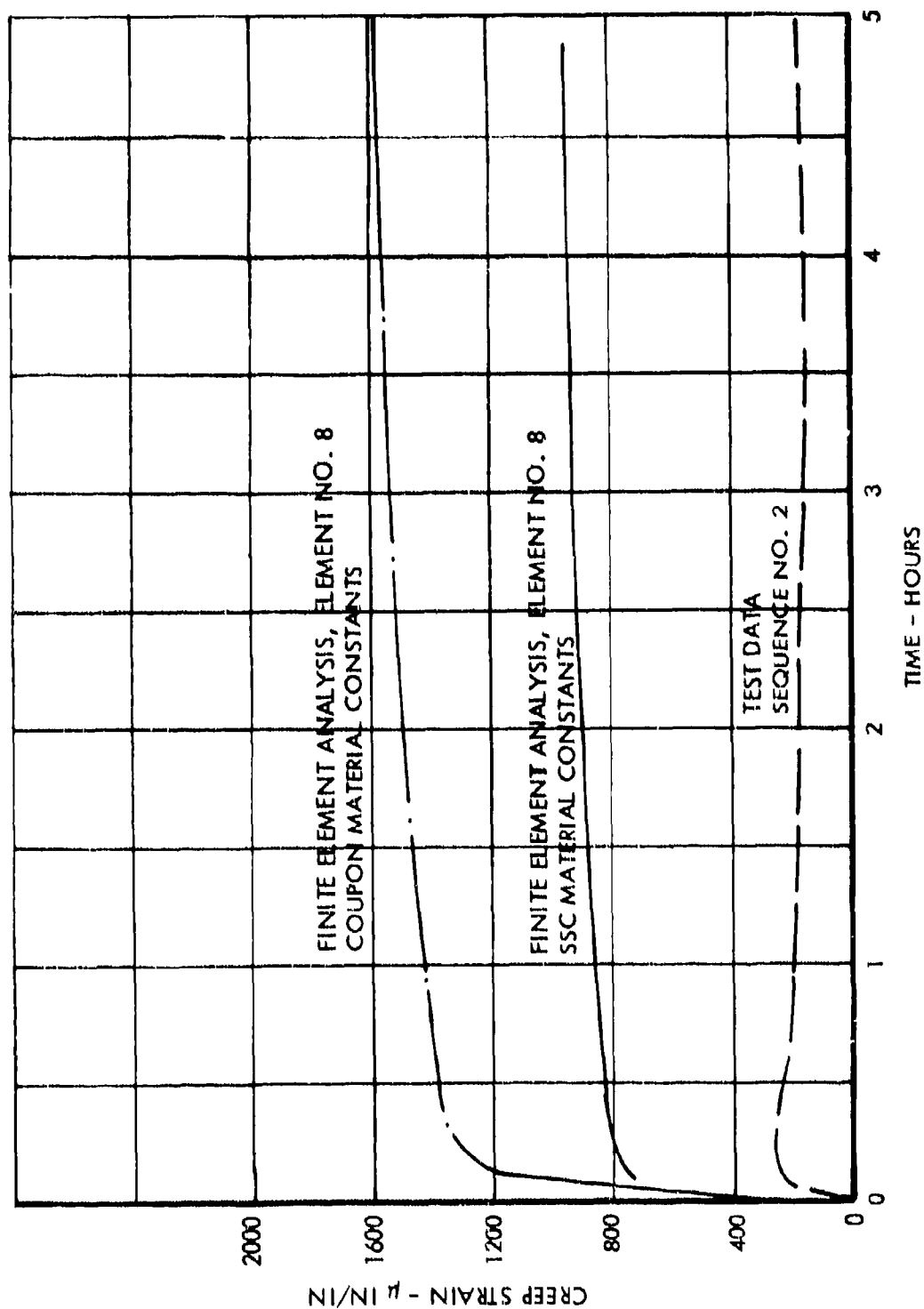


Figure 92. Analytical Creep Predictions Versus Transducer Measurements
Super-Scale Specimen, -15.8 ksi Net Section Stress

APPENDIX A
HYSTERESIS ANALYSIS COMPUTER PROGRAM DESCRIPTION

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A1.1 INTRODUCTION

Appendix A describes the computer program used to calculate fatigue damage. The program takes the true peak stresses for a notch and converts them into mean and alternating stresses through a rain flow counting technique. It then uses these stresses with unnotched S/N data and Miner's rule to calculate the damage. The derivation of the peak stresses for the notch is based on Neuber's equation. The effects of crack, stress relaxation, and material hardening or softening on these stresses are included in the program.

The following sections describe the main program and 21 subroutines comprising the program. The calling sequence for the subroutines is shown in Figure A1.

	MAIN	DRIVER	CREEPER	ADJUST	ORIGIN	MAKPLT	INCBC	ILABC	INCLC	LPTLC	LPTBC	RAIN
1. MAIN												
2. DRIVER	x											
3. CREEPER	x											
4. ADJUST	x											
5. ORIGIN			x			x	x	x				
6. MAKPLT	x	x										
7. INCBC		x	x									
8. INLABC			x	x								
9. INCLC		x										
10. LPTLC			x	x								
11. LPTBC					x							
12. RAIN	x											
13. SPECT	x											
14. CFVTRP	x											
15. CREEP			x									
16. LINBR								x				
17. BCRPLT			x			x						
18. FDSKT			x									
19. DAMAGE												x
20. DETERM									x			
21. LINSEC										x	x	
22. BIN7S							x					

Figure A1. Calling Sequence for the Subroutines

A1.2 LIST OF SYMBOLS

$\epsilon_{BC_B}, \sigma_{BC_B}$	arrays of strain, ϵ_{BC} , and stress, σ_{BC} , defining the branch curve in the branch curve system.
$\epsilon_{MCT}, \sigma_{MCT}$	arrays of strain, ϵ_{MCT} , and stress, σ_{MCT} , defining the tension part of the monotonic locus curve.
$\epsilon_{MCC}, \sigma_{MCC}$	arrays defining the compression part of the monotonic locus curve.
$\epsilon_{CCT}, \sigma_{CCT}$	arrays defining the tension part of the cyclic locus curve.
$\epsilon_{CCC}, \sigma_{CCC}$	arrays defining the compression part of the cyclic locus curve.
$\epsilon_{LCT}, \sigma_{LCT}$	arrays defining the tension part of the hardened locus curve.
$\epsilon_{LCC}, \sigma_{LCC}$	arrays defining the compression part of the hardened locus curve.
$D\epsilon, NCH$	arrays defining the number of cycles to fully harden, $D\epsilon$, versus half the plastic strain, $D\epsilon$.
SI, PC	arrays defining the % hardening, PC , versus $\sum 1/NCH$, SI .
ϵ, σ	general stress strain coordinates in the basic system.
B	subscript defining the branch curve coordinate system.
N, F	subscript N defines a coordinate system that has its origin at $(\epsilon_H(N), \sigma_H(N))$ with an orientation defined by F . If $F = +1$ the orientation is parallel to the basic system. $F = -1$ the orientation is rotated 180° to the basic system.
ϵ_H, σ_H	arrays defining the history of peak notch strains and stresses.
ϵ_P, σ_P	arrays defining all the peak notch strains and stresses.
$DLOAD$ or $DSKTS$	loading term used in Neuber's equation: $\frac{E \times \epsilon \times \sigma}{\left(1 - \left(\frac{ \epsilon }{A}\right)^2\right)^2} = DLOAD^2$
E	Young's modulus

λ, A
 \bar{E}, K, p, n

material constants.
creep constants.

A2.1 MAIN PROGRAM

Purpose

The main program reads the input data and controls the output and calls the various subroutines that execute the analysis.

Procedure

The flow for the main program shown in Figure A2 proceeds through the following stages:

- (1) The arrays defining the material stress/strain curves and the material constants are read. These are in general passed to the subroutines through common blocks.
- (2) The subroutine SPECT is called which reads and organizes the load spectrum into peak farfield stresses.
- (3) The main program computes the loading increments $DLOAD(J)$ from the peak farfield stresses.
- (4) Subroutine DRIVER calculates the peak notch strains and stresses (ϵ_H, σ_H) and (ϵ_P, σ_P) for a loading increment.
- (5) Subroutine CPVTRP hardens or softens the locus curves due to the current loading increment.
- (6) Subroutine ADJUST adjusts the history of peak notch strains and stresses (ϵ_H, σ_H) to reflect the hardening of the locus curve.
- (7) Subroutine CREEPER allows the notch stresses to creep if a hold period is called.

Stages (4) through (7) are repeated for all the loading increments.

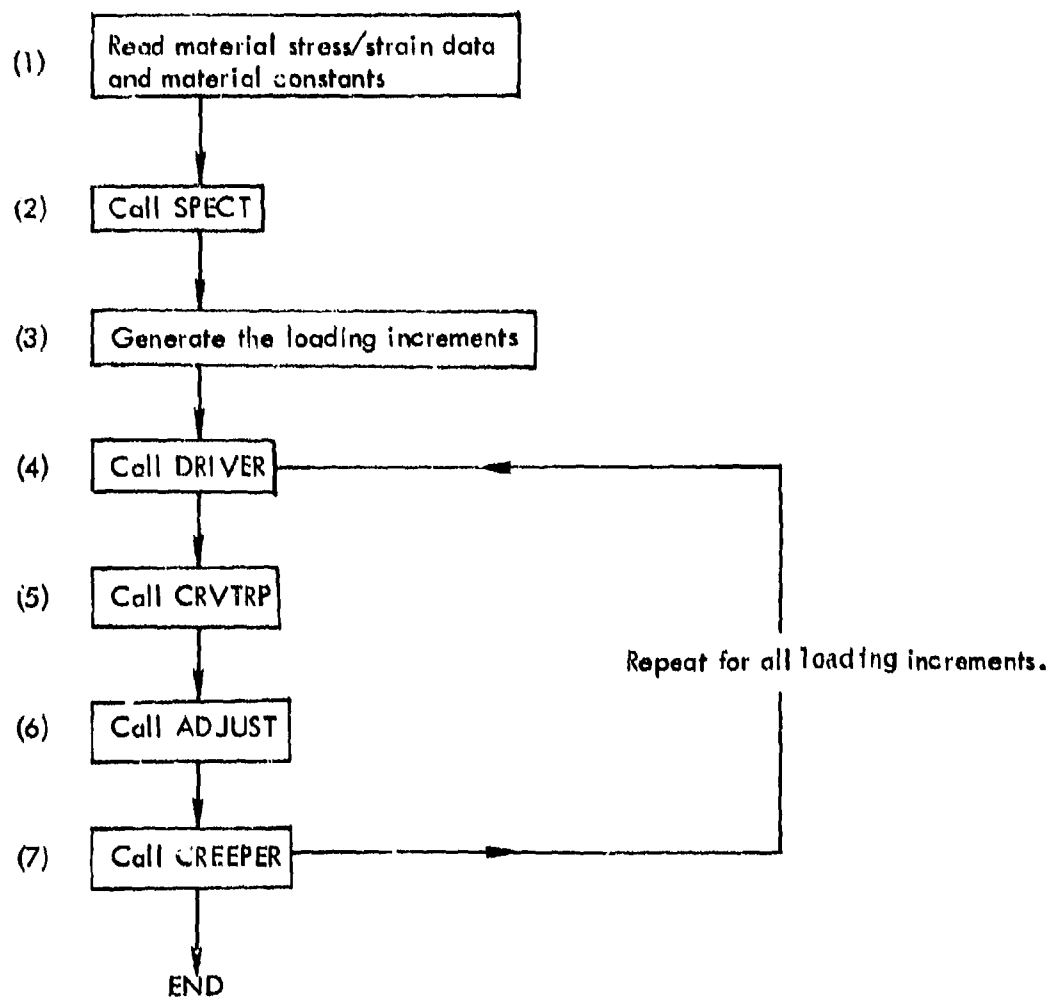


Figure A2. MAIN Program Flow

A2.2 SUBROUTINE DRIVER (DLOAD(J))

Purpose

This subroutine uses the J^{th} loading increment, DLOAD(J), to calculate the peak strains and stresses $\epsilon_P(J)$, $\sigma_P(J)$, and updates the history arrays of stresses and strains ϵ_H , σ_H , and the history array of the loading increments DSKTS.

Procedure

If NP is the current number of active history points, in entering subroutine DRIVER the analysis follows the following flow, see Figure A3.

- (1) If $NP \geq 3$ the last point calculated will be on a branch curve and the trace for the current loading increment will proceed from the point $(\epsilon_H(NP), \sigma_H(NP))$ along the branch curve that passes through the points $(\epsilon_H(NP-1), \sigma_H(NP-1))$ and $(\epsilon_H(NP), \sigma_H(NP))$.
- (2) If $|DLOAD(J)| < |DSKTS(NP-1)|$ the new point $(\epsilon_H(NP+1), \sigma_H(NP+1))$ will be found on this branch curve between the points $(\epsilon_H(NP-1), \sigma_H(NP-1))$ and $(\epsilon_H(NP), \sigma_H(NP))$, Figure A4.a.
- (3) If, however, $|DLOAD(J)| \geq |DSKTS(NP-1)|$ the current loading increment is numerically greater than the previous loading increment and the trace will pass beyond the point $(\epsilon_H(NP-1), \sigma_H(NP-1))$. In this instance the loading increment is adjusted so that it is relative to the point $(\epsilon_H(NP-2), \sigma_H(NP-2))$ and loading is considered from this point along a branch curve that passes through the points $(\epsilon_H(NP-2), \sigma_H(NP-2))$ and $(\epsilon_H(NP-3), \sigma_H(NP-3))$, Figure A4.b.
- (4) If $NP < 2$ loading is along the locus curve to point $(\epsilon_H(2), \sigma_H(2))$. A pseudo point $(\epsilon_H(1), \sigma_H(1))$ is found on the opposite end of the locus curve, Figure A5. This is necessary to position the branch curve for the next loading increment.

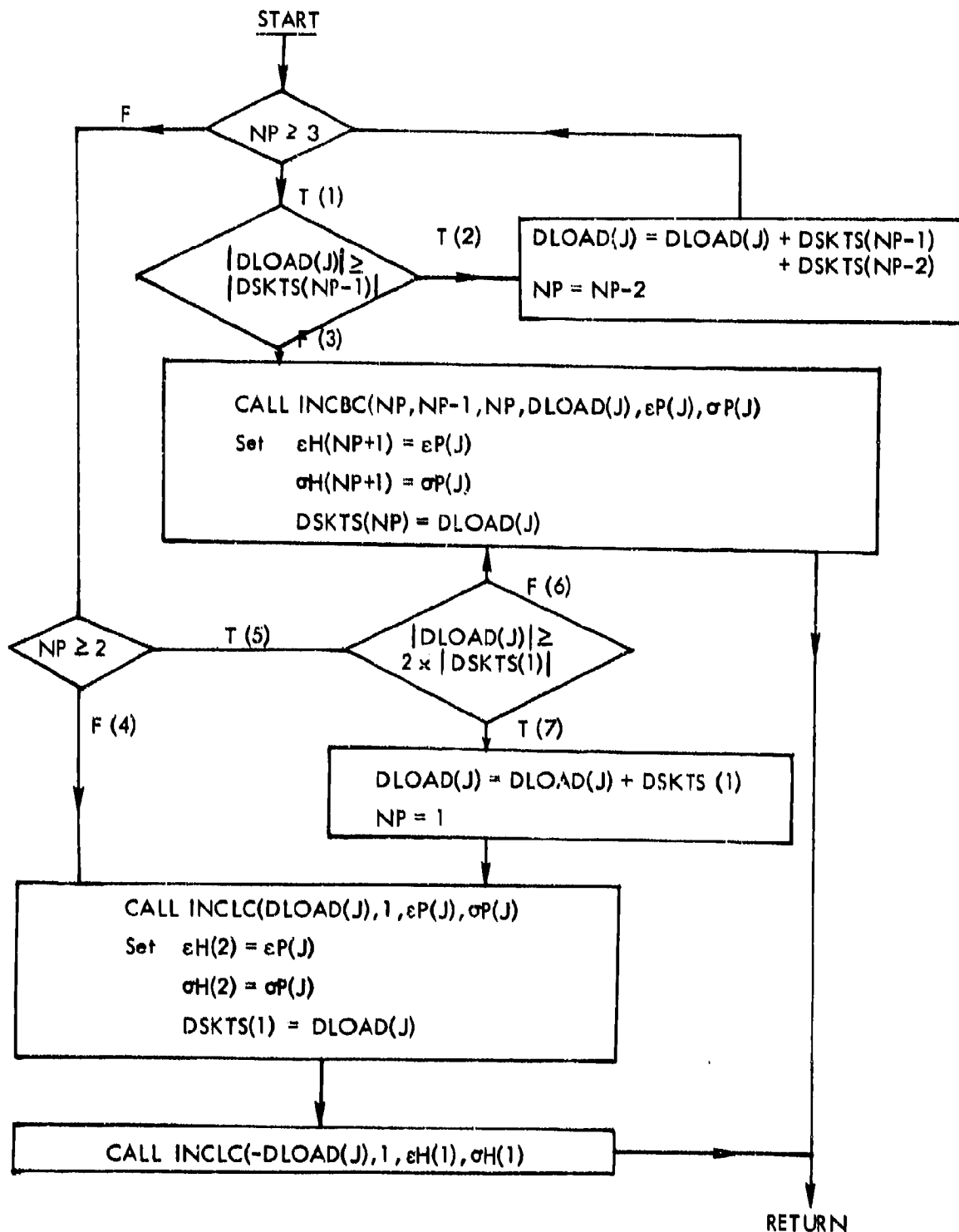


Figure A3. Analysis Flow for Subroutine DRIVER

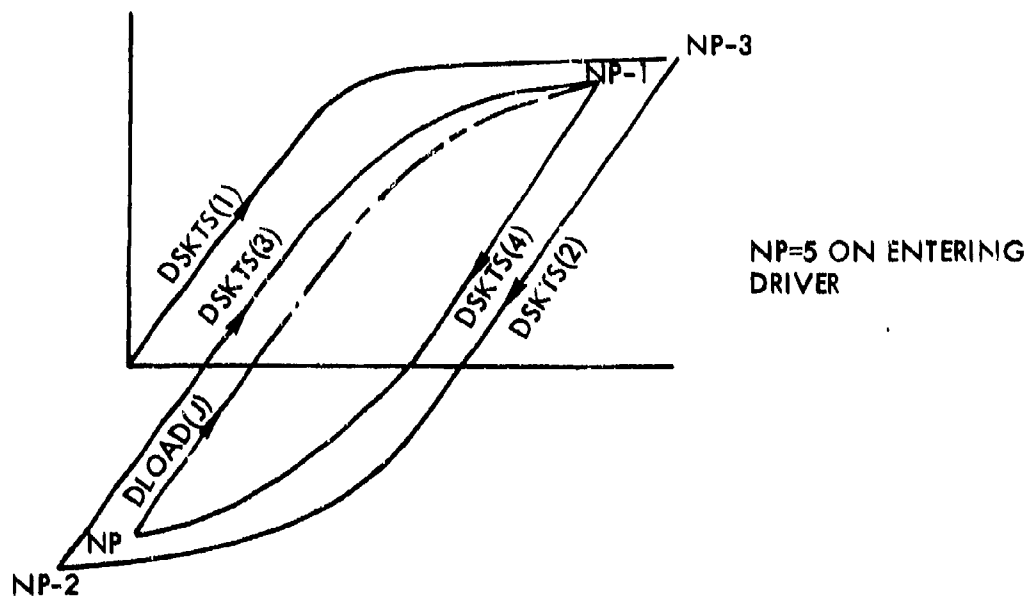


Figure A4.a. Current Loading Increment if $NP \geq 3$ and $|DLOAD(J)| < |DSKTS(NP-1)|$

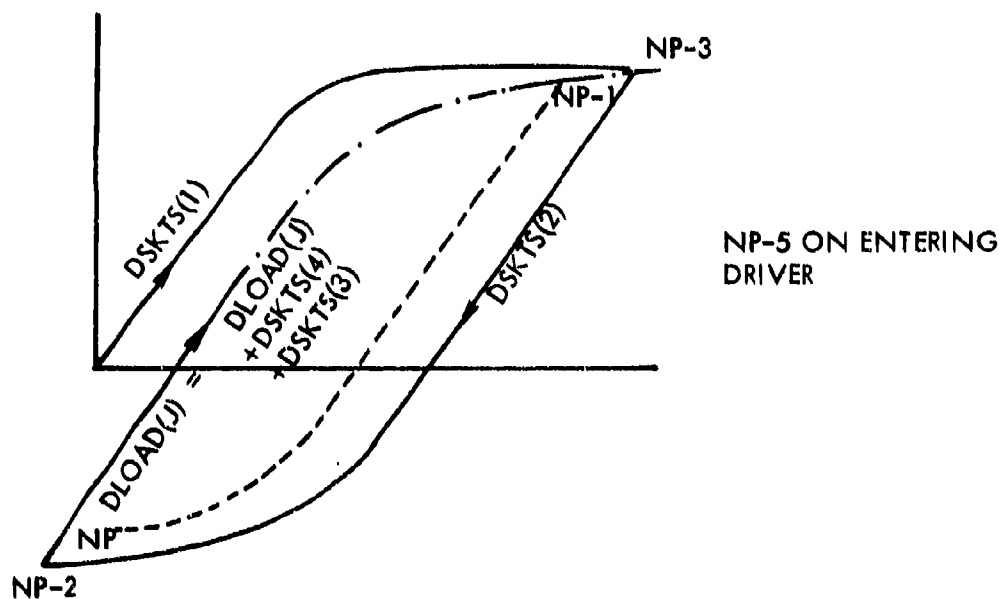


Figure A4.b. Current Loading Increment if $NP \geq 3$ and $|DLOAD(J)| \geq |DSKTS(NP-1)|$

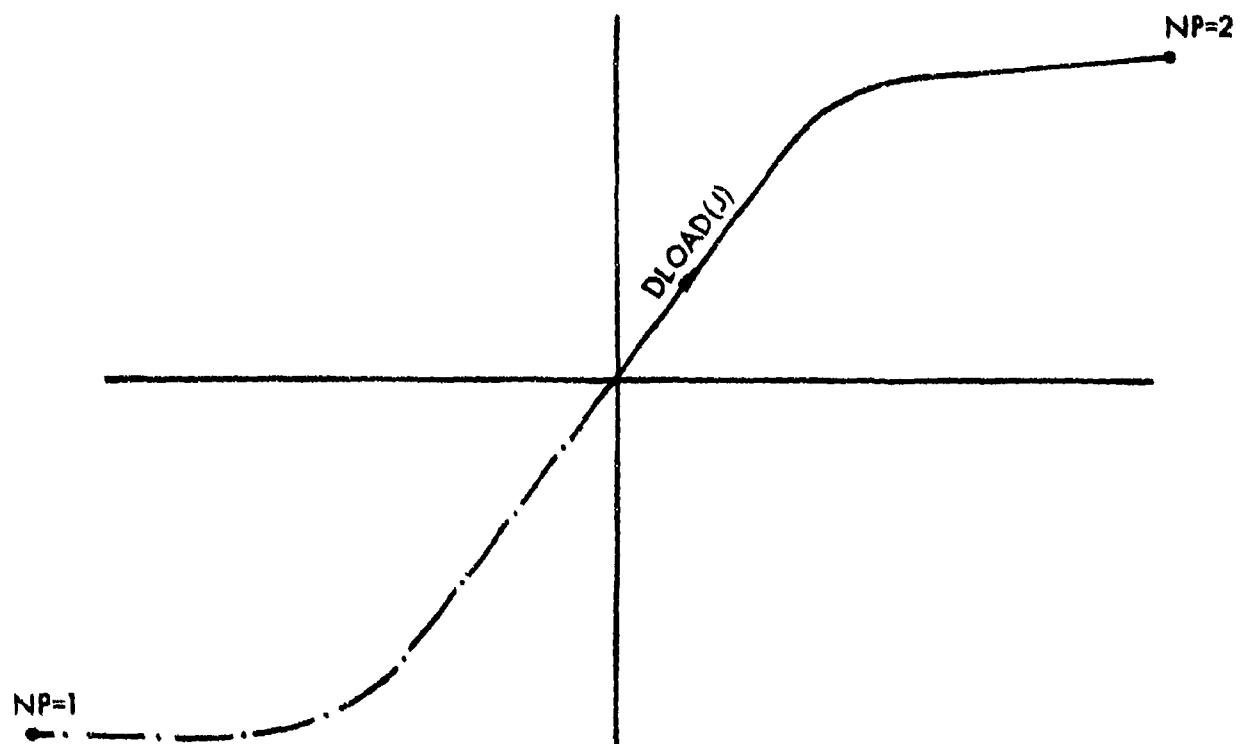


Figure A5. Loading Along the Locus Curve for $NP=2$

- (5) If $NP = 2$ the trace for the current loading increment proceeds from point $(\epsilon H(2), \sigma H(2))$ on the locus curve along the branch curve that passes through the points $(\epsilon H(2), \sigma H(2))$ and $(\epsilon H(1), \sigma H(1))$.
- (6) If $|DLOAD(J)| < 2 \times |DSKTS(1)|$ the new point $(\epsilon H(3), \sigma H(3))$ will be found on this branch curve between the points $(\epsilon H(1), \sigma H(1))$ and $(\epsilon H(2), \sigma H(2))$, Figure A6.a.
- (7) If, however, $|DLOAD(J)| \geq 2 \times |DSKTS(1)|$ the trace will pass through the point $(\epsilon H(1), \sigma H(1))$ and then follow the locus curve. In this instance the loading increment is adjusted so that it is relative to the origin of the plot and loading is considered to follow the locus curve, Figure A6.b.

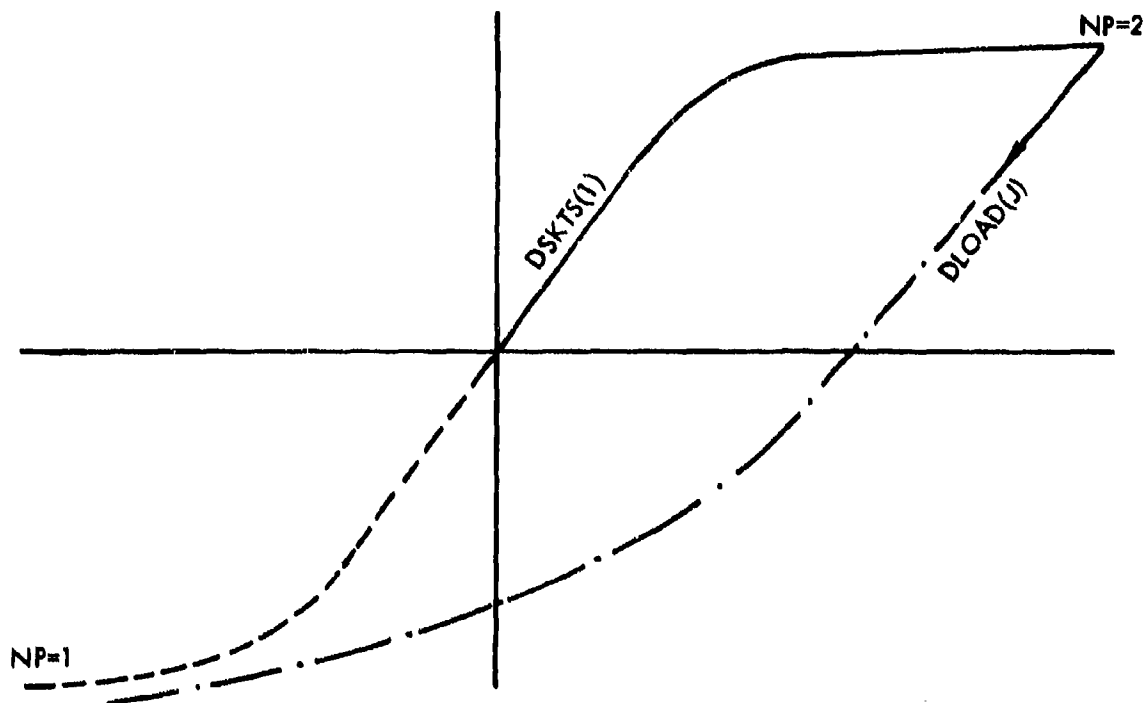


Figure A6.a. Loading for $NP=2$ and $|DLOAD(J)| < 2 \times |DSKTS(1)|$

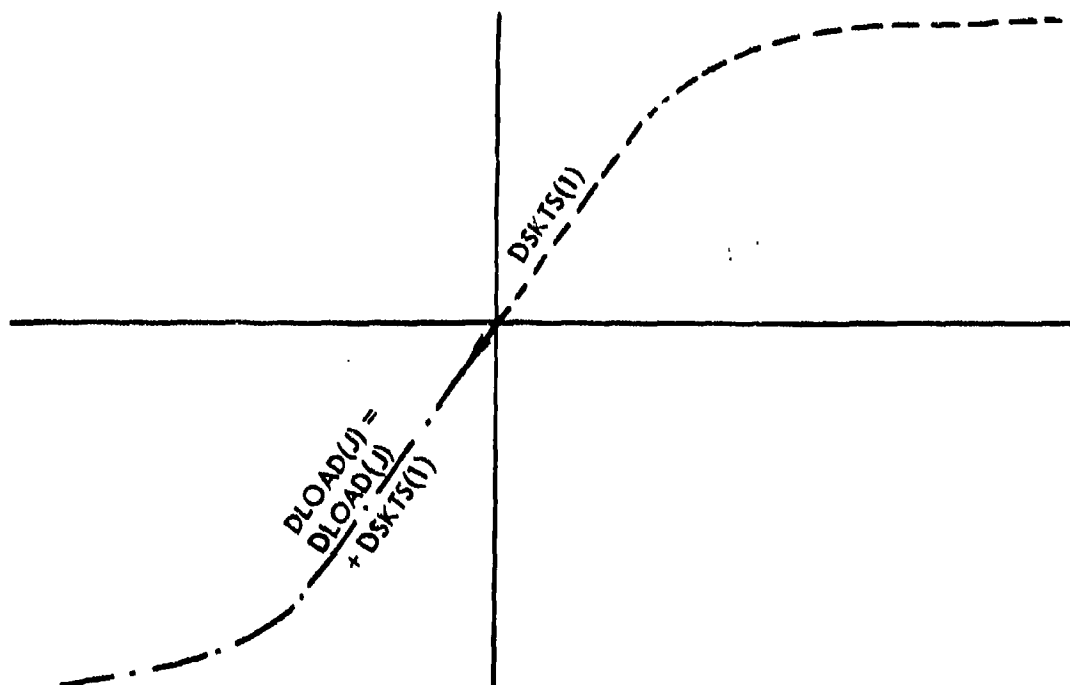


Figure A6.b. Loading for $NP=2$ and $|DLOAD(J)| \geq 2 \times |DSKTS(1)|$

A2.3 SUBROUTINE CREEPER(T)

Purpose

This subroutine calls the subroutine CREEP which calculates the creep strain and stress relaxation for the hold period T. The amount of creep for a hold period is affected by any previous creep, and this subroutine keeps up with the history of previous creeps and modifies the current effect appropriately. It also modifies the history arrays ϵH and σH of strains and stresses to reflect the creep.

Procedure

Consider a hold period, T, after the NP^{th} history point ($\epsilon H(NP), \sigma H(NP)$) has been calculated. Also consider that the effect of previous creeping is defined by the values CREP and TCREP then the effect on the current hold period is

$$PCT = \frac{|\sigma H(NP)^P \times TCREP^n - CPEP|}{|\sigma(NP)^P \times TCREP^n|} \quad (1)$$

If $PCT < 0$ no creep is allowed. The material is then allowed to creep has the point ($\epsilon H(NP), \sigma H(NP)$) to the new point ($\epsilon NP, \sigma NP$) by calling CREEP

$$CALL CREEP (\epsilon H, \sigma H, PCT, NP, T, \epsilon NP, \sigma NP) \quad (2)$$

This history of previous creeping is then modified

$$\begin{aligned} CREP &= CREP + |(\epsilon NP - \sigma NP/E) - (\epsilon H(NP) - \sigma H(NP)/E)| \\ TCREP &= T \end{aligned}$$

The history arrays ($\epsilon H, \sigma H$) are then modified to reflect this new point by calling subroutines ORIGIN, LPTLC, and ILABC.

A2.4 SUBROUTINE ADJUST (NI)

Purpose

This subroutine adjusts the NP history points (ϵ_H, σ_H) to be consistent with the hardened locus curve.

Procedure

The adjustment always starts with the point NI in the locus curve ($NI = 1$ or 2). This is done by projecting the initial point ($\epsilon_H(NI), \sigma_H(NI)$) into the new locus curve along a line whose slope is E, Figure A7. This is done by calling subroutine LPTLC

CALL LPTLC ($\epsilon_H(NI), \sigma_H(NI), E, \epsilon_{HA}, \sigma_{HA}$)

Then ($\epsilon_{HA}, \sigma_{HA}$) is taken as the new point ($\epsilon_H(NI), \sigma_H(NI)$).

The point $NI+2$ on the branch curve is adjusted in a similar manner Figure A8. This is done by calling the subroutine ILABC

CALL ILABC($NI, NI+1, \epsilon_H(NI+2), \sigma_H(NI+2), \epsilon_{HA}, \sigma_{HA}$)

Then ($\epsilon_{HA}, \sigma_{HA}$) is taken as the new point ($\epsilon_H(NI+2), \sigma_H(NI+2)$).

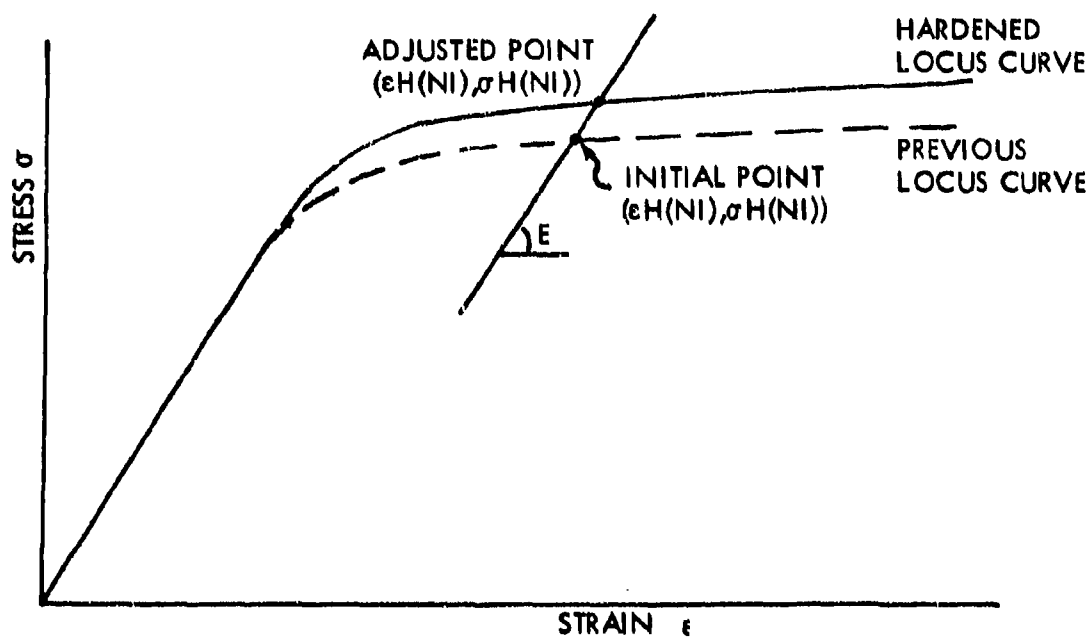


Figure A7. Adjustment of Points on the Locus Curve

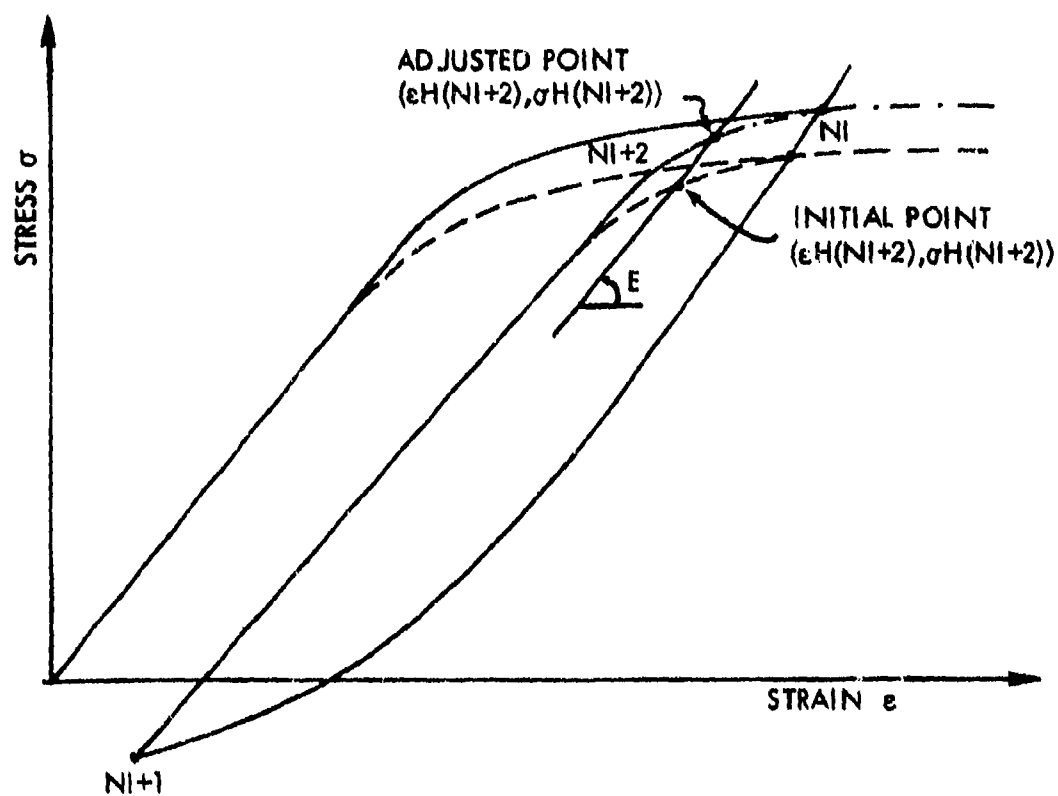


Figure A.8. Adjustment of Points on the Branch Curve

A2.5 SUBROUTINE ORIGIN (N1, N2, ϵBC_B , σBC_B , NPBC, ϵOBC , σOBC)

Purpose

This subroutine finds the origin (ϵOBC , σOBC) in the basic system of the branch curve defined by the NPBC points (ϵBC_B , σBC_B) such that it passes through the points ($\epsilon H(N1)$, $\sigma H(N1)$) and ($\epsilon H(N2)$, $\sigma H(N2)$).

Procedure

The algorithm considers the various locations of the branch curve in the basic system for which the points in the array (ϵBC_B , σBC_B) are coincident with the point ($\epsilon H(N1)$, $\sigma H(N1)$), Figure A9. Each point defining the branch curve is considered until a location is found for which the branch curve cuts just inside the point ($\epsilon H(N2)$, $\sigma H(N2)$). It is first necessary to develop the transformation from the basic system into the branch curve system.

The orientation of the branch curve relative to the basic system is defined by the points ($\epsilon H(N1)$, $\sigma H(N1)$) and ($\epsilon H(N2)$, $\sigma H(N2)$).

$$F = \frac{\epsilon H(N2) - \epsilon H(N1)}{|\epsilon H(N2) - \epsilon H(N1)|} \quad (1)$$

The origin of the branch curve in the basic system for the various locations is

$$\overline{\epsilon OBC} = \epsilon H(N1) - \epsilon BC_B(1) \times F \quad (2)$$

$$\overline{\sigma OBC} = \sigma H(N1) - \sigma BC_B(1) \times F$$

and the transformation from the basic system into the branch curve system is

$$\epsilon_B = (\epsilon - \overline{\epsilon OBC}) \times F \quad (3)$$

$$\sigma_B = (\sigma - \overline{\sigma OBC}) \times F$$

Substituting (2) into (3)

$$\epsilon_B = (\epsilon - H(N1)) \times F + \epsilon BC_B(1) \quad (4)$$

$$\sigma_B = (\sigma - \sigma H(N1)) \times F + \sigma BC_B(1)$$

For the point $(\epsilon BC_B(1), \sigma BC_B(1))$ coincident with the point $(\epsilon H(N1), \sigma H(N1))$ it is necessary to find the points $(\epsilon BC_B(J-1), \sigma BC_B(J-1))$ and $(\epsilon BC_B(J), \sigma BC_B(J))$ whose strains straddle the strain for the point $(\epsilon H(N2), \sigma H(N2))$ i.e.

$$\epsilon BC_B(J-1) < \epsilon H(N2) \leq \epsilon BC_B(J) \quad (5)$$

Transforming $\epsilon H(N2)$ through equation (4) gives

$$\epsilon BC_B(J-1) < (\epsilon H(N2) - \epsilon H(N1)) \times F + \epsilon BC_B(1) \leq \epsilon BC_B(J) \quad (6)$$

Now for the branch curve just cutting inside the point $(\epsilon H(N2), \sigma H(N2))$

$$\begin{aligned} \sigma H(N2) &> \sigma BC_B(J-1) + (\sigma BC_B(J) - \sigma BC_B(J-1)) \times \\ &\quad \frac{(\epsilon H(N2) - \epsilon BC_B(J-1))}{(\epsilon BC_B(J) - \epsilon BC_B(J-1))} \end{aligned} \quad (7)$$

Again transforming $\epsilon H(N2)$ and $\sigma H(N2)$ through equation (4) gives

$$\begin{aligned} &(\sigma H(N2) - \sigma H(N1)) \times F + \sigma BC_B(1) > \\ &\sigma BC_B(J-1) + (\sigma BC_B(J) - \sigma BC_B(J-1)) \times \\ &\quad \frac{((\epsilon H(N2) - \epsilon H(N1)) \times F + \epsilon BC_B(1) - \epsilon BC_B(J-1))}{(\epsilon BC_B(J) - \epsilon BC_B(J-1))} \end{aligned} \quad (8)$$

The algorithm uses equations (6) and (8) to find the point $(\epsilon BC_B(1), \sigma BC_B(1))$ such that when it is made coincident with the point $(\epsilon H(N1), \sigma H(N1))$ the branch curve just passes inside the point $(\epsilon H(N2), \sigma H(N2))$. The branch curve can now be made to pass through the points $(\epsilon H(N1), \sigma H(N1))$ and $(\epsilon H(N2), \sigma H(N2))$ by translating

it along the line defined by the points in the basic system ($\epsilon_{BC}(I-1)$, $\sigma_{BC}(I-1)$) and ($\epsilon_{BC}(I)$, $\sigma_{BC}(I)$) Figure A10. The amount of translation is determined by finding the intersection between the branch curve at its current location and a line that passes through the point ($\epsilon_H(N2)$, $\sigma_H(N2)$) having the same slope as the line along which the translation is to be made. The intersection ϵI_B , σI_B in the branch curve system is determined by making the call to the following subroutine

CALL LPTBC($\epsilon H_B(N2)$, $\sigma H_B(N2)$, θ , ϵI_B , σI_B)

Where $\epsilon H_B(N2)$, and $\sigma H_B(N2)$ are obtained by transforming the point ($\epsilon_H(N2)$, $\sigma_H(N2)$) through equation (4) and θ , the slope of the line is

$$\theta = \frac{\sigma_{BC_B}(I) - \sigma_{BC_B}(I-1)}{\epsilon_{BC_B}(I) - \epsilon_{BC_B}(I-1)} \quad (9)$$

Then the amount of translation ($\Delta\epsilon$, $\Delta\sigma$) is

$$\Delta\epsilon = \epsilon H_B(N2) - \epsilon I_B \quad (10)$$

$$\Delta\sigma = \sigma H_B(N2) - \sigma I_B$$

Transforming ($\epsilon H_B(N2)$, $\sigma H_B(N2)$) through equation (4)

$$\Delta\epsilon = (\epsilon_H(N2) - \epsilon_H(N1)) \times F + \epsilon_{BC_B}(I) - \epsilon I_B \quad (11)$$

$$\Delta\sigma = (\sigma_H(N2) - \sigma_H(N1)) \times F + \sigma_{BC_B}(I) - \sigma I_B$$

Then the final origin is

$$\epsilon_{OBC} = \overline{\epsilon_{OBC}} + \Delta\epsilon \times F \quad (12)$$

$$\sigma_{OBC} = \overline{\sigma_{OBC}} + \Delta\sigma \times F$$

and by substituting equations (2) and (11)

$$\epsilon_{OBC} = \epsilon_H(N2) - \epsilon I_B \times F \quad (13)$$

$$\sigma_{OBC} = \sigma_H(N2) - \sigma I_B \times F$$

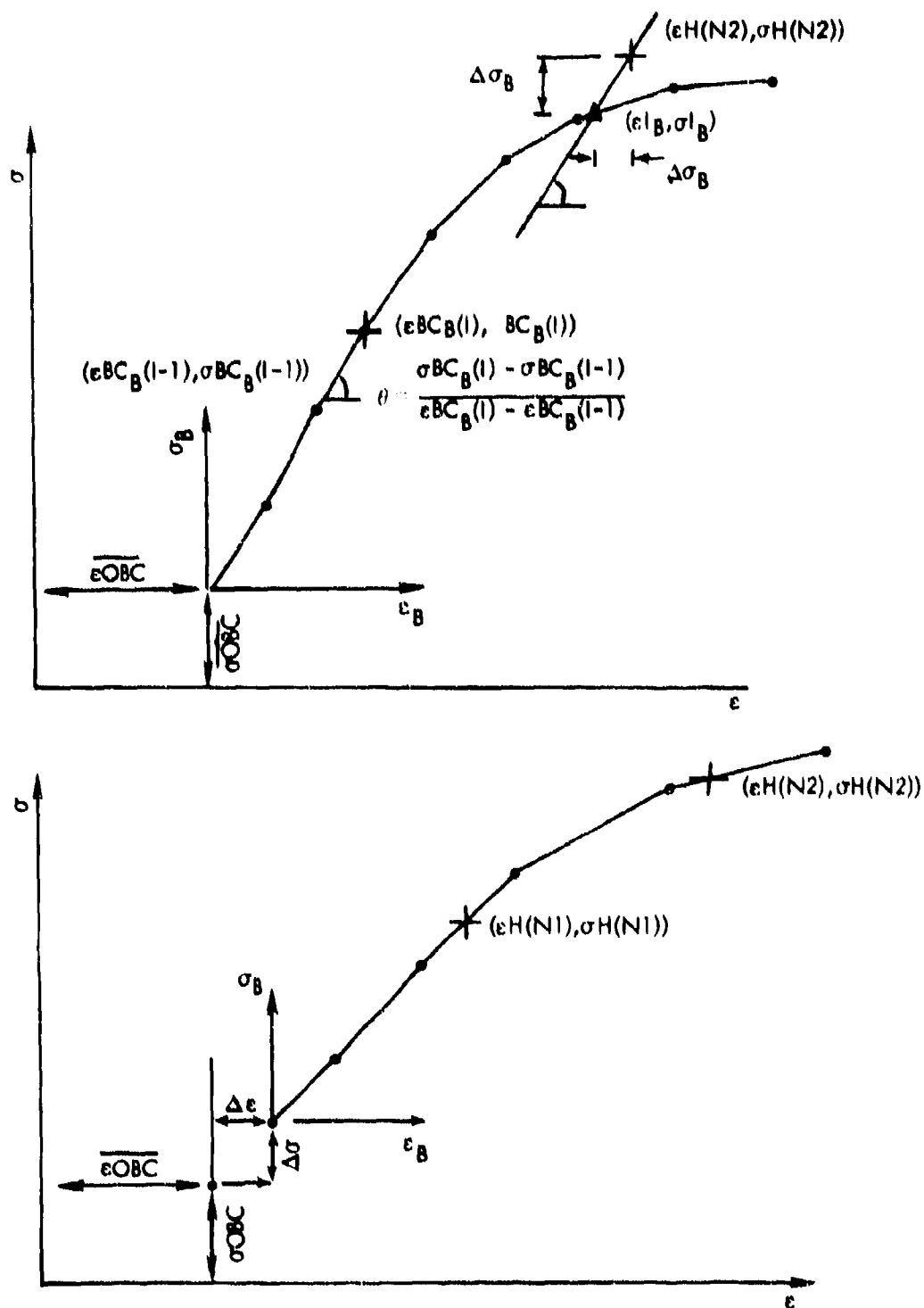


Figure A10. Translation of the Branch Curve

A2.6 SUBROUTINE MAKPLT(ϵ PLT(1), ϵ PLT(2), σ PLT(1), σ PLT(2),NFLAG)

Purpose

This subroutine is used to set up arrays ϵ PLOT and σ PLOT defining points that are to be joined together to form a curve between the two points (ϵ PLT(1), σ PLT(1)) and (ϵ PLT(2), σ PLT(2)). If $NFLAG > 1$ the curve is represented by the branch curve and if $NFLAG < 1$ it is represented by the locus curve.

Procedure

The start and end points in the array are always the points (ϵ PLT(1), σ PLT(1)) and (ϵ PLT(2), σ PLT(2)). If $NFLAG < 1$ the remaining points in the array are the points defining the locus curve whose strains lie between ϵ PLT(1) and ϵ PLT(2). If $NFLAG > 1$ it is necessary to first locate the branch curve in the basic system such that it passes through the points (ϵ PLT(1), σ PLT(1)) and (ϵ PLT(2), σ PLT(2)). This is done by calling the subroutine ORIGIN. Subroutine origin gives the location of the origin of the branch curve in the basic system. The points on the branch curve are now transformed into the basic system and the transformed points whose strains lie between ϵ PLT(1) and ϵ PLT(2) are used for the plot arrays.

A2.7 SUBROUTINE INCBC(N1,N2,N,DSKT, ϵ l, σ l)

Purpose

This subroutine finds the intersection (ϵ l, σ l) between Neuber's equation, defined relative to the point (ϵ H(N), σ H(N)) by the loading increment DSKT, with a branch curve which passes through the points (ϵ H(N1), σ H(N1)) and (ϵ H(N2), σ H(N2)).

Procedure

Neuber's equation defined relative to the point (ϵ H(N), σ H(N)) by the loading increment DSKT is

$$\frac{E \times \epsilon_N \times \sigma_N}{\left(1 - \left(\frac{|\epsilon_N|}{A}\right)^d\right)^2} = \text{DSKT}^2 \quad (1)$$

Consider the transformation from the basic system to a system which has an origin at (ϵ H(N), σ H(N)) Figure A11.

$$\epsilon_N = (\epsilon - \epsilon H(N)) \times F \quad (2)$$

$$\sigma_N = (\sigma - \sigma H(N)) \times F$$

where

$$F = \text{DSKT}/|\text{DSKT}| \quad (3)$$

Substituting (2) into (1)

$$\frac{E \times (\epsilon - \epsilon H(N)) \times (\sigma - \sigma H(N))}{\left(1 - \left(\frac{\epsilon - \epsilon H(N)}{A}\right)^d\right)^2} = \text{DSKT}^2 \quad (4)$$

Next it is necessary to find the origin (ϵ OBC, σ OBC) in the basic system of the branch curve which passes through the points (ϵ H(N1), σ H(N1)) and (ϵ H(N2), σ H(N2)).

This is done by making the following call to subroutine ORIGIN

$$\text{CALL ORIGIN (N1,N2,\epsilon BC,\sigma BC, NPBC,\epsilon OBC,\sigma OBC)} \quad (5)$$

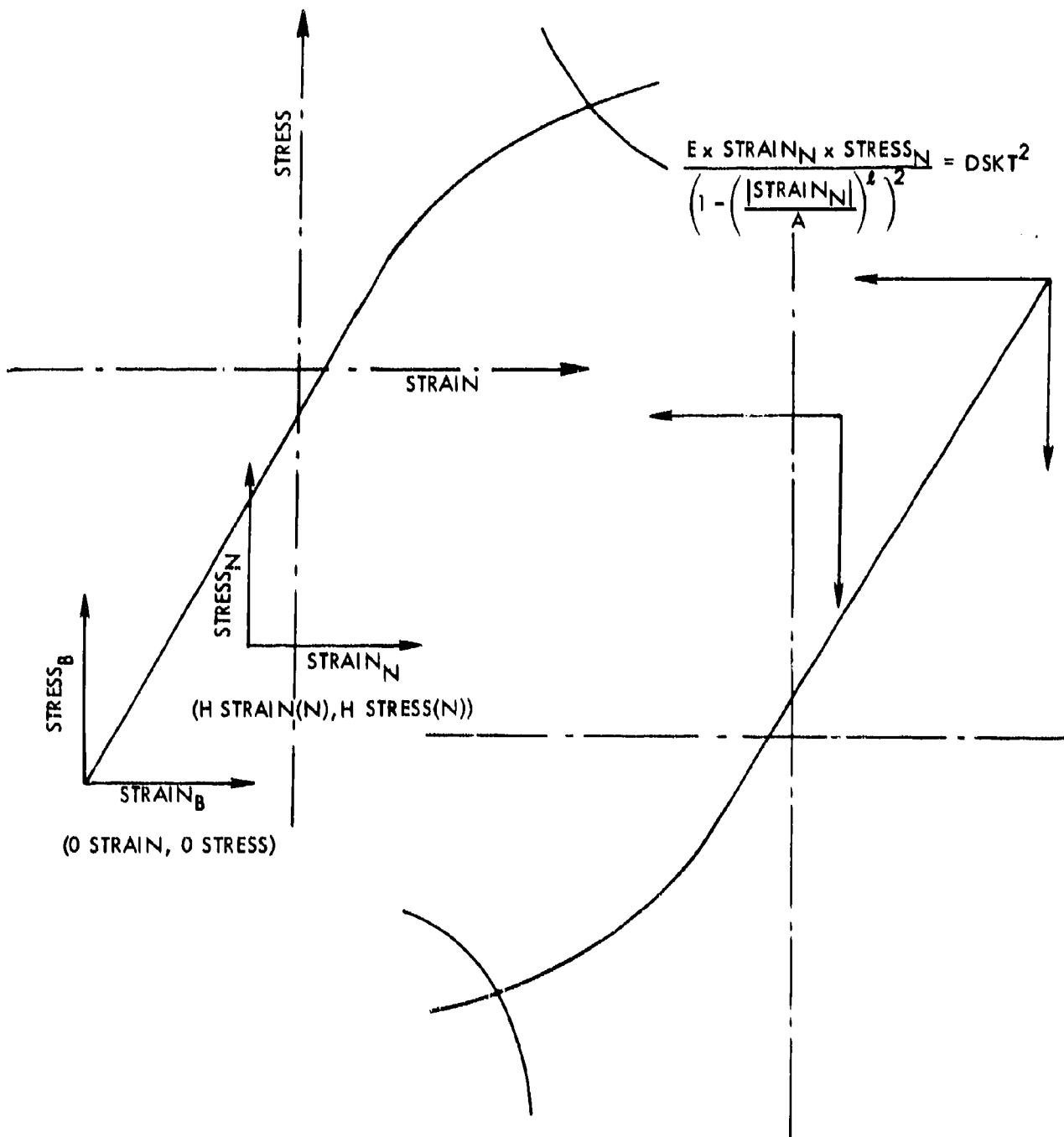


Figure A11. Transformation to a System with Its Origin at $(\epsilon_H(N), \sigma_H(N))$

Now consider the transformation from the branch curve system ϵ_B, σ_B to the basic system

$$\begin{aligned}\epsilon &= \epsilon_B \times F + \epsilon OBC \\ \sigma &= \sigma_B \times F + \sigma OBC\end{aligned}\tag{6}$$

Substituting (6) into (4)

$$\frac{E \times (\epsilon_B \times F + \epsilon OBC - \epsilon H(N)) \times (\sigma_B \times F + \sigma OBC - \sigma H(N))}{\left(1 - \left(\frac{|\epsilon_B \times F + \epsilon OBC - \epsilon H(N)|}{A}\right)^4\right)^2} = DSKT^2\tag{7}$$

This equation is used in the following algorithm to find the intersection

- Consider the points defining the branch curve relative to its origin $(\epsilon BC, \sigma BC)$. These points are sequentially substituted for (ϵ_B, σ_B) in equation (7) until a pair $(\epsilon BC_B(l-1), \sigma BC_B(l-1))$, $(\epsilon BC_B(l), \sigma BC_B(l))$ are found which give solutions that straddle the value $DSKT^2$.
- The problem is now reduced to finding the intersection of a line joining the points $(\epsilon BC_B(l-1), \sigma BC_B(l-1))$ and $(\epsilon BC_B(l), \sigma BC_B(l))$ with Neuber's equation defined relative to the point $(\epsilon H(N), \sigma H(N))$. This is done by making the following call to subroutine BINTS

CALL BINTS $(\epsilon BC_B(l-1), \epsilon BC_B(l), \sigma BC_B(l-1), \sigma BC_B(l), N, DSKT^2,$
 $\epsilon OBC, \sigma OBC, F, \epsilon l_B, \sigma l_B)$

Finally ϵl_B and σl_B are substituted into equations (6) to find the point of intersection $(\epsilon l, \sigma l)$ relative to the basic system.

A2.8 SUBROUTINE ILABC(N1,N2,εP,σP,εI,σI)

COMMON/SLOPES/θ

Purpose

This subroutine finds the intersection εI,σI of the branch curve, defined by the array εBC,σBC, which passes through the points (εH(N1),σH(N1)) and (εH(N2),σH(N2)) with a line that passes through the point (εP,σP) with a slope θ.

Procedure

It is first necessary to find the origin (εOBC,σOBC) of the branch curve such that it passes through the points (εH(N1),σH(N1)) and (εH(N2),σH(N2)). This is done by making the following call to the subroutine ORIGIN

CALL ORIGIN (N1,N2,εBC_B,σBC_B,NPBC,εOBC,σOBC)

Next, consider the transformation from the basic system into the branch system

$$\varepsilon_B = (\varepsilon - \varepsilon_{OBC}) \times F \quad (1)$$

$$\sigma_B = (\sigma - \sigma_{OBC}) \times F$$

where

$$F = \frac{H(N2) - H(N1)}{|H(N2) - H(N1)|} \quad (2)$$

Find the projection $\overline{\varepsilon P}_B$ of the point (εP,σP) in the branch curve strain axis by translating along a line with slope θ, Figure A12.

$$\overline{\varepsilon P}_B = \varepsilon P_B - \sigma P_B / \theta \quad (3)$$

Substituting (1)

$$\overline{\varepsilon P}_B = ((\varepsilon - \sigma/\theta) - (\varepsilon_{OBC} - \sigma_{OBC}/\theta))F \quad (4)$$

Similarly translating the points $(\epsilon BC_B, \sigma BC_B)$ defining the branch curve into the strain axis

$$\overline{\epsilon BC_B} = \epsilon BC_B - \sigma BC_B / \theta \quad (5)$$

These values are scanned until a pair $\overline{\epsilon BC_B}(J-1), \overline{\epsilon BC_B}(J)$ are found which straddle the value $\overline{\epsilon P_B}$ i.e.

$$\overline{\epsilon BC_B}(J-1) < \overline{\epsilon P_B} < \overline{\epsilon BC_B}(J) \quad (6)$$

The problem is now simply one of finding the intersection $(\epsilon I, \sigma I)$ between two lines, one which passes through the points $(\epsilon BC_B(J-1), \sigma BC_B(J-1))$ and $(\epsilon BC_B(J), \sigma BC_B(J))$, the other which passes through the point $(\epsilon P, \sigma P)$ with slope E. This is done by calling subroutine LINBR as follows

```
CALL LINBR( $\epsilon BC_B(J-1)$ ,  $\epsilon BC_B(J)$ ,  $\sigma BC_B(J-1)$ ,  $\sigma BC_B(J)$ ,  $\epsilon P$ ,  $\sigma P$ ,  
            $\epsilon OBC$ ,  $\sigma OBC$ , F,  $\epsilon I$ ,  $\sigma I$ )
```

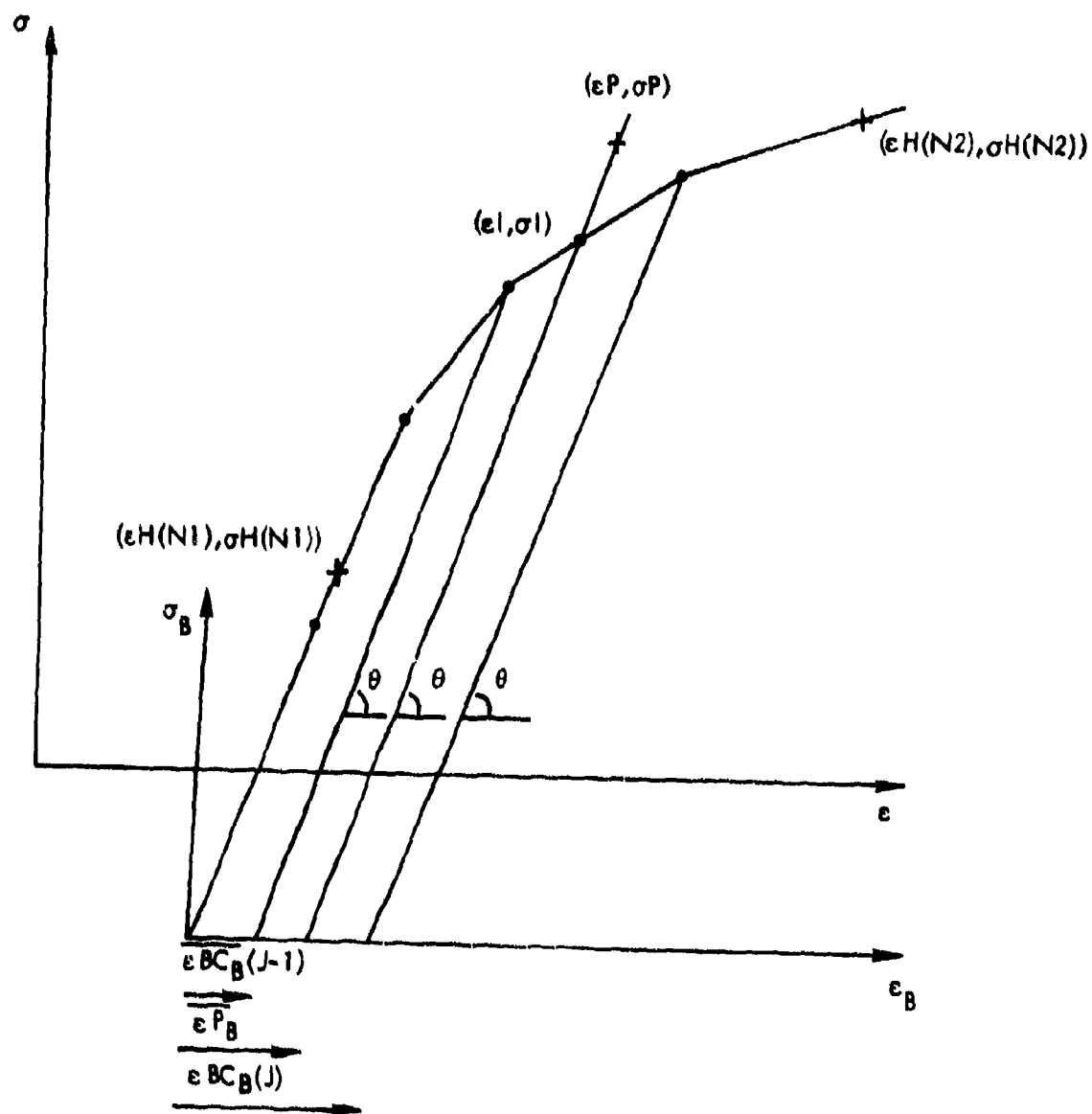



Figure A12. Projection into the Branch Curve Strain Axis

A2.9 SUBROUTINE INCLC(DSKT,N, ϵ l, σ l)

Purpose

This subroutine finds the intersection (ϵ l, σ l) between Neuber's equation, defined relative to the point (ϵ H(N), σ H(N)) by the loading increment DSKT, with the locus curve.

Procedure

Neuber's equation defined relative to the point (ϵ H(N), σ H(N)) by the loading increment DSKT is

$$\frac{E \times \epsilon_N \times \sigma_N}{\left(1 - \left(\frac{|\epsilon_N|}{A}\right)^4\right)^2} = \text{DSKT}^2 \quad (1)$$

Consider the transformation from the basic system to a system which has an origin at (ϵ H(N), σ H(N)) Figure A10.

$$\epsilon_N = (\epsilon - \epsilon_H(N)) \times F \quad (2)$$

$$\sigma_N = (\sigma - \sigma_H(N)) \times F$$

where

$$F = \text{DSKT}/|\text{DSKT}| \quad (3)$$

Substituting (2) into (1)

$$\frac{E \times (\epsilon - \epsilon_H(N)) \times (\sigma - \sigma_H(N))}{\left(1 - \left(\frac{|\epsilon - \epsilon_H(N)|}{A}\right)^4\right)^2} = \text{DSKT}^2 \quad (4)$$

Now if $F > 0$ the intersection will be with the tension half of the locus curve and if $F < 0$ it will be with the compression half of the locus curve. Define the array ϵ LC, σ LC as the specific half of the locus curve being considered. This array together with equation (4) is used in the following algorithm to find the intersection.

- Sequentially substitute the points $(\epsilon_{LC}, \sigma_{LC})$ into equation (4) until a pair, $(\epsilon_{LC}(I-1), \sigma_{LC}(I-1))$ and $(\epsilon_{LC}(I), \sigma_{LC}(I))$, are found which give solutions that straddle the value $DSKT^2$.
- The problem is now reduced to finding the intersection of a line joining the points $(\epsilon_{LC}(I-1), \sigma_{LC}(I-1))$ and $(\epsilon_{LC}(I), \sigma_{LC}(I))$ with Neuber's equation defined relative to the point $(\epsilon_H(N), \sigma_H(N))$. This is done by making the following call to subroutine DETERM

CALL DETERM ($\epsilon_{LC}(I-1), \epsilon_{LC}(I), \sigma_{LC}(I-1), \sigma_{LC}(I),$
 $N, DSKT^2, \epsilon_I, \sigma_I)$

A2.10 SUBROUTINE LPTLC($\epsilon P, \sigma P, \theta, \epsilon l, \sigma l$)

Purpose

This subroutine finds the intersection ($\epsilon l, \sigma l$) between the locus curve and a line that passes through the point ($\epsilon P, \sigma P$), with a slope θ .

Procedure

Consider the projection $\overline{\epsilon P}$ of the point ($\epsilon P, \sigma P$) on the strain axis by translating along the line with slope θ , Figure A13.

$$\overline{\epsilon P} = \epsilon P - \sigma P / \theta \quad (1)$$

If $\overline{\epsilon P} > 0$ the intersection will be with the tension half of the locus curve and if $\overline{\epsilon P} < 0$ the intersection will be with the compression half. Define the array $\epsilon LC, \sigma LC$ as the specific half of the locus curve being considered. Now consider the projections $\overline{\epsilon LC}$ of the points of this array on the strain axis by translating along the line with slope θ .

$$\overline{\epsilon LC} = \epsilon LC - \sigma LC / \theta \quad (2)$$

These are scanned until a pair $\overline{\epsilon LC}(J-1), \overline{\epsilon LC}(J)$ are found which straddle the value $\overline{\epsilon P}$ i.e.

$$\overline{\epsilon LC}(J-1) < \overline{\epsilon P} < \overline{\epsilon LC}(J) \quad (3)$$

The problem is now simply one of finding the intersection of two lines, one which passes through the points ($\epsilon LC(J-1), \sigma LC(J-1)$) and ($\epsilon LC(J), \sigma LC(J)$), the other which intersects the strain axis at $\overline{\epsilon P}$ with a slope θ . This is done by making the following call to subroutine LINSEC.

```
CALL LINSEC( $\epsilon LC(J-1), \epsilon LC(J), \sigma LC(J-1), \sigma LC(J), \theta, \epsilon l, \sigma l$ )
COMMON LINE  $\overline{\epsilon P}$ 
```

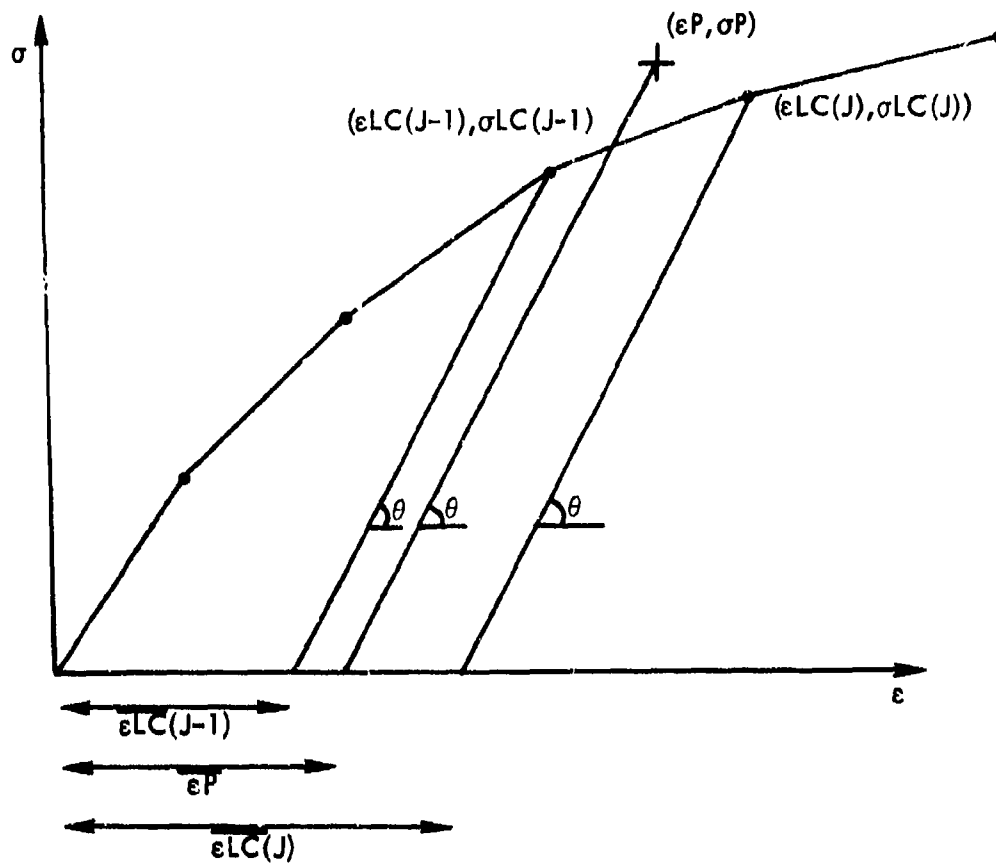


Figure A13. Projection into the Strain Axis

A2.11 SUBROUTINE LPTBC($\epsilon P_B, \sigma P_B, \theta, \epsilon l_B, \sigma l_B$)

Purpose

This subroutine finds the intersection ($\epsilon l_B, \sigma l_B$) between the branch curve defined by the array ($\epsilon BC_B, \sigma BC_B$) and a line which passes through the point ($\epsilon P_B, \sigma P_B$) with a slope of θ .

Procedure

Consider the projection $\overline{\epsilon BC}_B$ of the array ($\epsilon BC_B, \sigma BC_B$) on the strain axis by translating along a line with slope θ , Figure A14.

$$\overline{\epsilon BC}_B = \epsilon BC_B - \sigma BC_B / \theta \quad (1)$$

Similarly consider the projection $\overline{\epsilon P}_B$ of the point ($\epsilon P_B, \sigma P_B$)

$$\overline{\epsilon P}_B = \epsilon P_B - \sigma P_B / \theta \quad (2)$$

The algorithm simply scans the array $\overline{\epsilon BC}_B$ to find a pair of values $\epsilon BC_B(J-1)$ and $\epsilon BC_B(J)$ whose projections straddle $\overline{\epsilon P}_B$ i.e.

$$\overline{\epsilon BC}_B(J-1) < \overline{\epsilon P}_B < \overline{\epsilon BC}_B(J) \quad (3)$$

The problem is now simply one of finding the intersection of two lines, one which passes through the points ($\epsilon BC_B(J-1), \sigma BC_B(J-1)$) and ($\epsilon BC_B(J), \sigma BC_B(J)$), the other which intersects the strain axis at $\overline{\epsilon P}_B$ with a slope θ . This is done by making the following call to subroutine LINSEC

CALL LINSEC ($\epsilon BC_B(J-1), \epsilon BC_B(J), \sigma BC_B(J-1), \sigma BC_B(J), \theta, \epsilon l_B, \sigma l_B$)

COMMON LINE | $\overline{\epsilon P}_B$ |

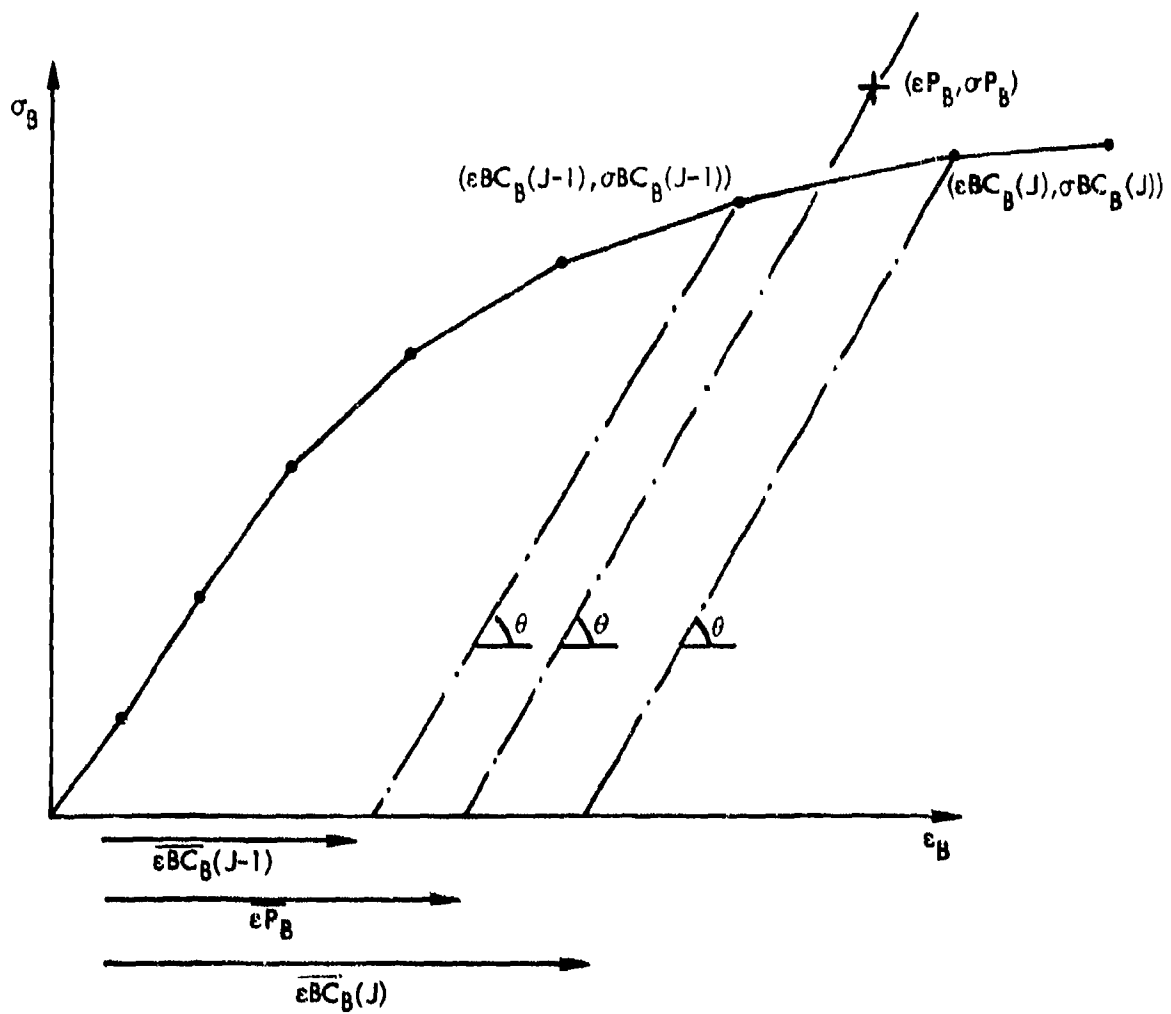


Figure A14. Projection into the Strain Axis

A2.12 SUBROUTINE RAIN (S,JMIN,JMAX)

This subroutine employs the rain flow counting method to generate the analysis spectrum from the peak spectrum S. JMIN and JMAX contain indexes for minimum peak strain or stress and maximum peak strain or stress respectively.

A2.13 SUBROUTINE SPECT (S,NS,NLOADS,S1,S2,N)

This subroutine reads block spectrum values S1, S2 and N and converts it into peak spectrum for hysteresis loop analysis.

A2.14 SUBROUTINE CRVTRP($\Delta\epsilon/2$)

Purpose

This subroutine takes half the plastic strain increment, $\Delta\epsilon/2$, for the current loading increment and computes the hardened locus curve.

Procedure

If $\Delta\epsilon/2$ is <0 the analysis hardens the tension locus curve and if it is >0 it hardens the compression locus curve. Consider $\Delta\epsilon/2 > 0$, the analysis uses this value to interpret the number of cycles to hardening, NTC, from the arrays, $\Delta\epsilon$, NCH, Figure A15. The sum, SNC, of the inverse of the number of cycles to hardening for all the tension cycles is then computed. This value, SNC, is then used to interpret the % hardening, PCC, of the compression locus curve from the arrays SI, PC Figure A16. The compression part of the hardened locus curve is then computed by interpreting between the monotonic compression locus curve and the cyclic locus curve using PCC i.e.

$$\sigma_{LCC} = \sigma_{MCC} + (\sigma_{CCC} - \sigma_{MCC}) + PCC$$

ϵ_{MCC} , ϵ_{CCC} , and ϵ_{LCC} are assumed to be equivalent

The identical procedure is used to harden the tension side of the locus curve when $\Delta\epsilon/2 < 0$.

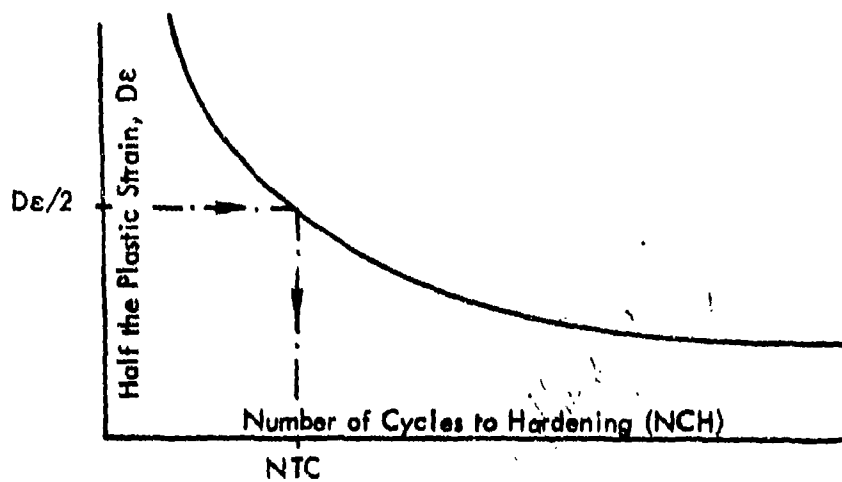


Figure 15. Interpretation of NTC

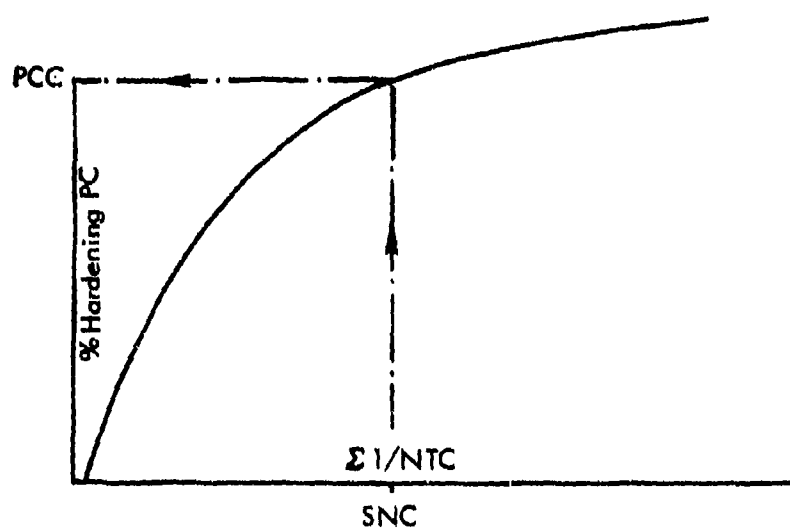


Figure 16. Interpretation of PCC

A2.15 SUBROUTINE CREEP ($\epsilon_H, \sigma_H, PCT, NP, T, \epsilon_{NP}, \sigma_{NP}$)

Purpose

This subroutine calculates the point ($\epsilon_{NP}, \sigma_{NP}$) considering the material to have crept from the point ($\epsilon_H(NP), \sigma_H(NP)$) over a time T . The term PCT reflects the effect of previous hold periods.

Procedure

The amount of stress relaxation σ_R is obtained by finding the root to the following function

$$f(\sigma_R) = \sigma_R + K\bar{E} (\sigma_H(NP) - \sigma_R)^{p+1}, n \quad (1)$$

$$f'(\sigma_R) = 1 + K\bar{E} (p+1) (\sigma_H(NP) - \sigma_R)^p, n \quad (2)$$

Hence using the Newton's method

$$\sigma_{R_{i+1}} = \sigma_{R_i} - \frac{f(\sigma_{R_i})}{f'(\sigma_{R_i})} \quad (3)$$

Equation (3) is iterated until $\sigma_{R_{i+1}}$ and σ_{R_i} are sufficiently close to each other. Knowing σ_R the creep strain ϵ_C is found from the equation

$$\epsilon_C = \sigma_R / (\bar{E} (\sigma_H(NP) - \sigma_R)) \quad (4)$$

Hence

$$\begin{aligned} \epsilon_{NP} &= \epsilon_H(NP) - \epsilon_C \\ \sigma_{NP} &= \sigma_H(NP) - \sigma_R \end{aligned} \quad (5)$$

A2.16 SUBROUTINE LINBR($\epsilon_D(1), \epsilon_D(2), \sigma_D(1), \sigma_D(2), \epsilon_P, \sigma_P, F, \epsilon_l, \sigma_l$)

Purpose

This subroutine finds the intersection (ϵ_l, σ_l) between a line defined by the points ($\epsilon_D(1), \sigma_D(1)$) and ($\epsilon_D(2), \sigma_D(2)$) in the branch curve system which has its origin at (ϵ_OBC, σ_OBC) in the basic system with another line which passes through the point ϵ_P, σ_P having a slope E .

Procedure

The transformation from the basic system into the branch curve system is

$$\begin{aligned}\epsilon_B &= (\epsilon - \epsilon_OBC) \times F \\ \sigma_B &= (\sigma - \sigma_OBC) \times F\end{aligned}\quad (1)$$

The projection of the point (ϵ_P, σ_P) in the strain axis of the branch curve system along a line whose slope is E is, Figure A17.

$$\overline{\epsilon_P}_B = \epsilon_P_B - \sigma_P_B / E \quad (2)$$

Transforming through (1)

$$\overline{\epsilon_P}_B = ((\epsilon - \sigma/E) - (\epsilon_OBC - \sigma_OBC/E))F \quad (3)$$

The algorithm starts by finding the mid point (ϵ_l, σ_l) of the line defined by the two points ($\epsilon_D(1), \sigma_D(1)$) and ($\epsilon_D(2), \sigma_D(2)$)

$$\begin{aligned}\epsilon_l_B &= (\epsilon_D(1) + \epsilon_D(2))/2 \\ \sigma_l_B &= (\sigma_D(1) + \sigma_D(2))/2\end{aligned}\quad (4)$$

This point is then projected into the strain axis along a line which has a slope E

$$\epsilon_l_B = \epsilon_l_B = \sigma_l_B / E \quad (5)$$

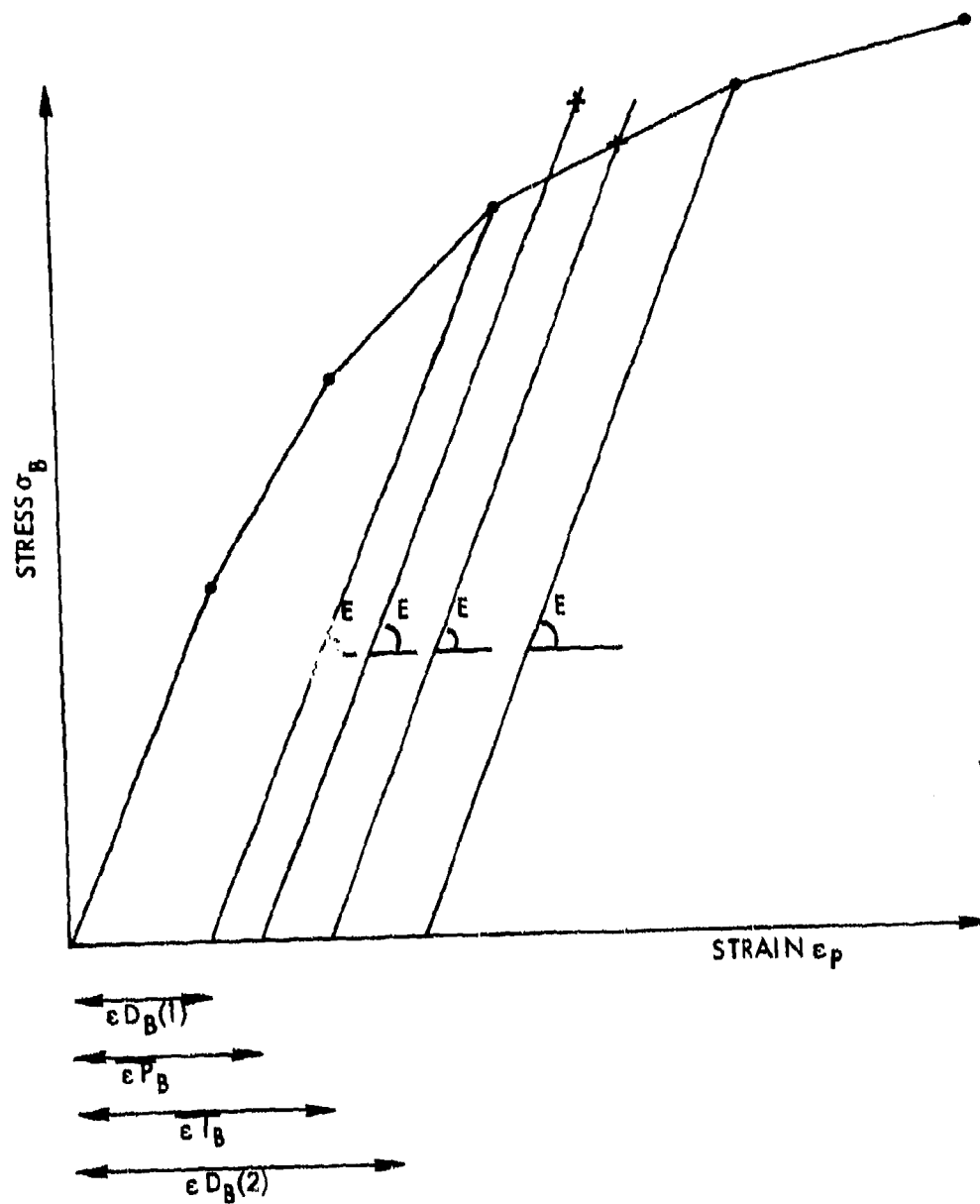


Figure A17. Projection into the Branch Curve Strain Axis

If $\overline{\epsilon P}_B > \overline{\epsilon I}_B$ the point $(\epsilon D_B(2), \sigma D_B(2))$ is set equal to the point $(\epsilon I_B, \sigma I_B)$ but if $\overline{\epsilon P}_B < \overline{\epsilon I}_B$ the point $(\epsilon D_B(1), \sigma D_B(1))$ is set equal to the point $(\epsilon I_B, \sigma I_B)$.

This is repeated until $\overline{\epsilon I}_B$ is sufficiently close to $\overline{\epsilon P}_B$.

Finally the point $(\overline{\epsilon I}_B, \sigma I_B)$ is transformed into the basic system through equation (4)

$$\begin{aligned}\epsilon I &= \overline{\epsilon I}_B \times F + \epsilon OBC \\ \sigma I &= \sigma I_B \times F + \sigma OBC\end{aligned}\tag{6}$$

A2.17 SUBROUTINE BCRPLT(PLT, σ PLT,NP)

Purpose

This subroutine simply takes the NP points defined by the arrays PLT, σ PLT and draws straight lines between them on the CALCOMP plotter.

A2.18 SUBROUTINE FDSKT(N)

This subroutine computes DSKTS(N) for the creep analysis. CREPER calls FDSKT after the amount of creep has been calculated and the history arrays (EX,SY) have been adjusted for the computed creep.

A2.19 SUBROUTINE DAMAGE

This subroutine calculates fatigue damage using the conventional Palmgren-Miner's rule of cumulative fatigue damage. This routine is called by the RAIN subroutine.

A2.20 SUBROUTINE DETERM ($\epsilon D(1), \epsilon D(2), \sigma D(1), \sigma D(2), N, DSKT^2, \epsilon I, \sigma I$)

Purpose

This subroutine finds the intersection ($\epsilon I, \sigma I$) between a line defined by the two points ($\epsilon D(1), \sigma D(1)$) and ($\epsilon D(2), \sigma D(2)$) and Neuber's equation defined relative to the point ($\epsilon H(N), \sigma H(N)$) by the term $DSKT^2$.

Procedure

From subroutine INCLC, Neuber's equation relative to the point ($\epsilon H(N), \sigma H(N)$) is

$$\frac{E \times (\epsilon - \epsilon H(N)) \times (\sigma - \sigma H(N))}{\left(1 - \left(\frac{\epsilon - \epsilon H(N)}{A}\right)^2\right)^2} = DSKTS^2 \quad (1)$$

The algorithm starts by finding the midpoint ($\epsilon I, \sigma I$) of the line defined by the two points ($\epsilon D(1), \sigma D(1)$) and ($\epsilon D(2), \sigma D(2)$)

$$\begin{aligned} \epsilon I &= (\epsilon D(1) + \epsilon D(2))/2 \\ \sigma I &= (\sigma D(1) + \sigma D(2))/2 \end{aligned} \quad (2)$$

This point is then substituted into Equation (1). Then, if the solution to this equation is greater than $DSKTS^2$, the point ($\epsilon D(2), \sigma D(2)$) is set equal to the point ($\epsilon I, \sigma I$); and, if the solution is less than $DSKTS^2$, the point ($\epsilon D(1), \sigma D(1)$) is set equal to the point ($\epsilon I, \sigma I$). This procedure is repeated until the solution is sufficiently close to $DSKTS^2$.

A2.21 SUBROUTINE LINSEC($\epsilon D(1)$, $\epsilon D(2)$, $\sigma D(1)$, $\sigma D(2)$, θ , ϵI , σI)

COMMON LINE $|\overline{\epsilon P}|$

Purpose

This subroutine finds the intersection (ϵI , σI) between a line defined by the two points ($\epsilon D(1)$, $\sigma D(1)$) and ($\epsilon D(2)$, $\sigma D(2)$) and another line which intersects the strain axis at ϵP with a slope θ .

Procedure

The algorithm starts by finding the mid point (ϵI , σI) of the line defined by the two points ($\epsilon D(1)$, $\sigma D(1)$) and ($\epsilon D(2)$, $\sigma D(2)$)

$$\begin{aligned}\epsilon I &= (\epsilon D(1) + \epsilon D(2))/2 \\ \sigma I &= (\sigma D(1) + \sigma D(2))/2\end{aligned}\tag{1}$$

This point is then projected into the strain axis along a line which has a slope of θ Figure A18.

$$\overline{\epsilon I} = \epsilon I - \sigma I / \theta\tag{2}$$

Then if $|\overline{\epsilon I}| > |\epsilon P|$ the point ($\epsilon D(2)$, $\sigma D(2)$) is set equal to the point (ϵI , σI) and if $|\overline{\epsilon I}| < |\epsilon P|$ the point ($\epsilon D(1)$, $\sigma D(1)$) is set equal to the point (ϵI , σI).

This procedure is repeated until $\overline{\epsilon I}$ is sufficiently close to ϵP .

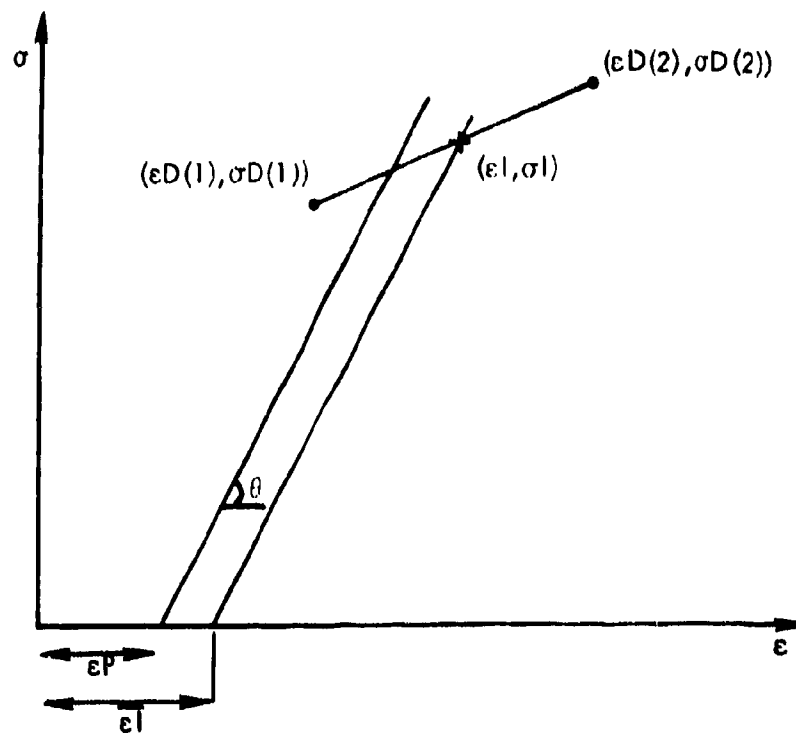


Figure A18. Projection into the Strain Axis

A2.22 SUBROUTINE BINT5($\epsilon D_B(1), \epsilon D_B(2), \sigma D_B(1), \sigma D_B(2), N, DSKT^2, \epsilon OBC, \sigma OBC, F, \epsilon I_B, \sigma I_B$)

Purpose

This subroutine puts the intersection ($\epsilon I_B, \sigma I_B$) in the branch curve system between Neuber's equation defined by the loading increment DSKT relative to the point ($\epsilon H(N), \sigma H(N)$) with a line defined by the points ($\epsilon D_B(1), \sigma D_B(1)$) and ($\epsilon D_B(2), \sigma D_B(2)$).

Procedure

From equation (3) in the writeup for subroutine, INCBC Neuber's equation defined by DSKT relative to the point ($\epsilon H(N), \sigma H(N)$) in terms of the branch curve coordinates that have an origin at ($\epsilon OBC, \sigma OBC$) is

$$\frac{E \times (\epsilon_B \times F + \epsilon OBC - \epsilon H(N)) \times (\sigma_B \times F + \sigma OBC - \sigma H(N))}{\left(1 - \frac{(\epsilon_B \times F + \epsilon OBC - \epsilon H(N))^2}{A}\right)^2} = DSKT^2 \quad (1)$$

The points ($\epsilon D_B(1), \sigma D_B(1)$) and ($\epsilon D_B(2), \sigma D_B(2)$) are substituted in turn into the left side of equation (1) to obtain the solutions $DSKT1^2$ and $DSKT2^2$. Then

$$DSKT1^2 < DSKT^2 < DSKT2^2$$

Next, the point ($\epsilon I_B, \sigma I_B$) midway between ($\epsilon D_B(1), \sigma D_B(1)$) and ($\epsilon D_B(2), \sigma D_B(2)$) is computed

$$\begin{aligned} \epsilon I_B &= \epsilon D_B(1) + (\epsilon D_B(2) - \epsilon D_B(1))/2 \\ \sigma I_B &= \sigma D_B(1) + (\sigma D_B(2) - \sigma D_B(1))/2 \end{aligned} \quad (2)$$

This is similarly substituted into the left side of (1) to obtain the solution $DSKT_1^2$. Then if $DSKT_1^2 > DSKT^2$ the point (e_{1B}, σ_{1B}) is substituted for the point $(e_{2B}(2), \sigma_{2B}(2))$ but if $DSKT_1^2 < DSKT^2$ it is substituted for the point $(e_{2B}(1), \sigma_{2B}(1))$. This process is repeated until $DSKT_1^2$ is sufficiently close to $DSKT^2$.

APPENDIX B
HYSTERESIS ANALYSIS USER'S GUIDE

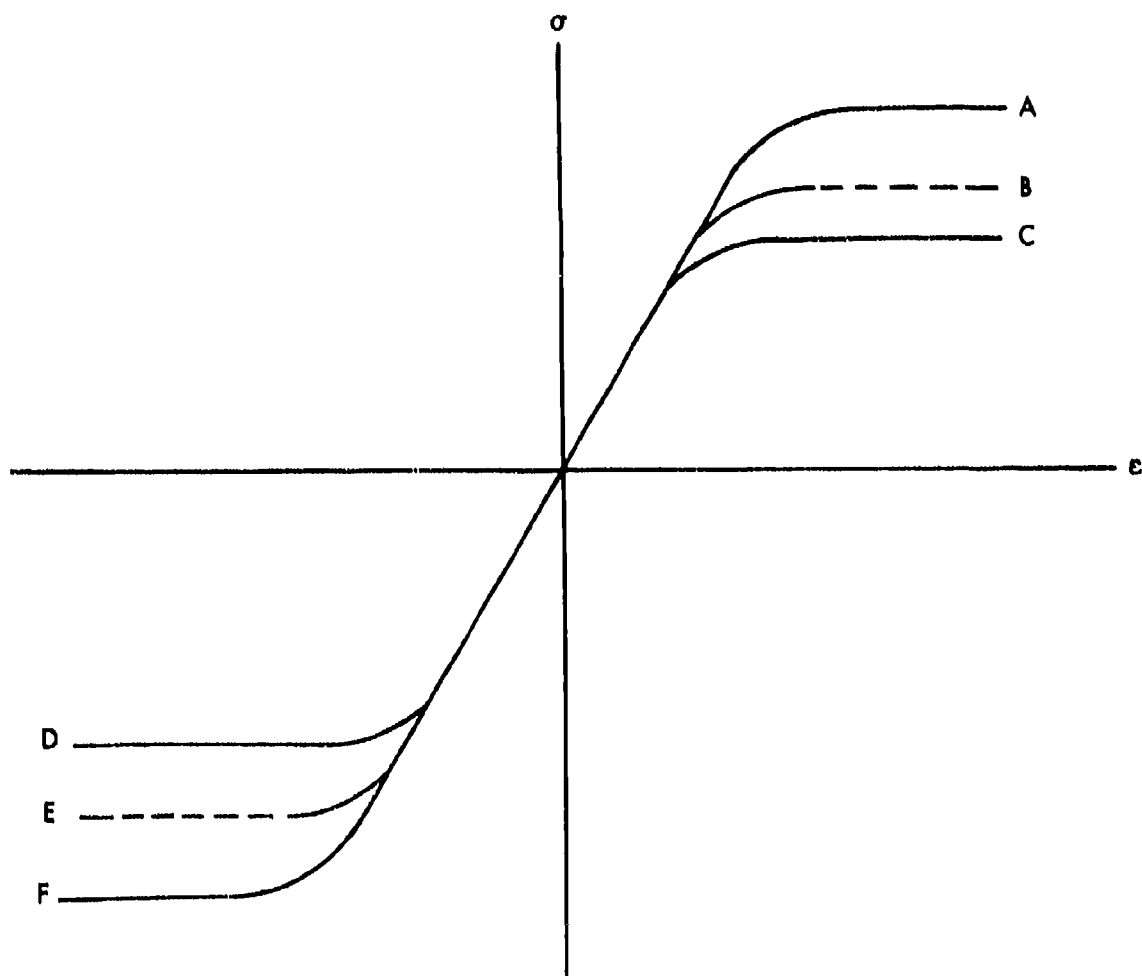
1. INPUT DATA REQUIREMENTS

Input to the Hysteresis Loop Analysis program consists primarily of stresses with the corresponding number of applied cycles. The stress can be input as unnotched peak stresses, mean and alternate stresses, or as maximum and minimum stresses. The input stresses are converted to program units and stored in DLOAD.

Other input also comes from material properties and includes branch curves, locus curves, fully reversed cycles to harden curve and normalized hardening curve. Examples are illustrated in Figures B1 and B2. There are also two sets of material constants, one set for creep calculations and one set for the hysteresis loop analysis.

In addition to the above there are various plot option flags, print flag spectrum input option flags and a flag to compute or not compute hardening. These are all explained in the Input Instructions.

Sample input for an example problem is included in Appendix C.



- A. CYCLIC TENSION LOCUS CURVE (TLOCCE, TLOCCS)
- B. PARTIALLY HARDENED LOCUS CURVE (COMPUTED)
- C. MONOTONIC TENSION LOCUS CURVE (TLOCME, TLOCMS)
- D. MONOTONIC COMPRESSION LOCUS CURVE (CLOCME, CLOCMS)
- E. PARTIALLY HARDENED COMPRESSION LOCUS CURVE (COMPUTED)
- F. CYCLIC COMPRESSION LOCUS CURVE (CLOCCE, CLOCCS)

Figure B1. Input Example of Locus Curves

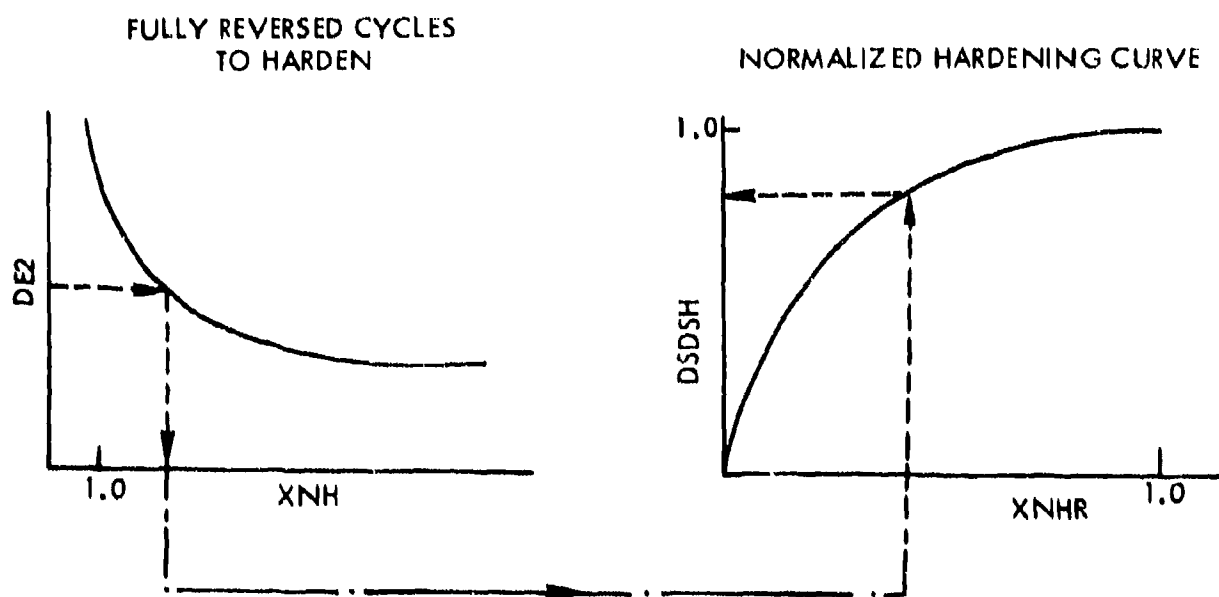


Figure B2. Example of Hardening Curves

2. INPUT INSTRUCTIONS

1. Monotonic tension and compression locus curves

READ(5,1) NLCMT,NLCMC

1 FORMAT(4I5)

NLCMT = the number of points defining the tension monotonic
locus curve ≤ 25

NLCMC = the number of points defining the compression monotonic
locus curve ≤ 25

READ(5,6) (TLOCME(I), I=1, NLCMT)

6 FORMAT(8F10.0)

TLOCME = the strain values for the tension monotonic locus curve

READ(5,6) (TLOCMS(I), I=1, NLCMT)

TLOCMS = the stress values for the tension monotonic locus curve

READ(5,6) (CLOCME(I), I=1, NLCMC)

CLOCME = the strain values for the compression monotonic locus curve

READ(5,6) (CLOCMS(I), I=1, NLCMC)

CLOCMS = the stress values for the compression monotonic locus curve

2. Cyclic tension and compression curves

READ(5,1) NLCCT,NLCCC

NLCCT = the number of points defining the cyclic tension
locus curve ≤ 25

NLCCC = the number of points defining the cyclic compression
locus curve ≤ 25

READ(5,6) (TLOCCE(I), I=1, NLCCT)

TLOCCE = the strain values for the cyclic tension locus curve

READ(5,6) (TLOCCS(I), I=1, NLCCT)

TLOCCS = the stress values for the cyclic tension locus curve

READ(5,6) (CLOCCE(I), I=1, NLCCC)

CLOCCE = the strain values for the cyclic compression locus curve

READ(5,6) (CLOCCS(I), I=1, NLCCC)

CLOCCS = the stress values for the cyclic compression locus curve

3. Upper and lower branch curves

READ(5,1) NUBC, NLBC

NUBC = the number of values defining the upper branch curve ≤ 30

NLBC = the number of values defining the lower branch curve ≤ 30

READ(5,6) (EUBC(I), I=1, NUBC)

EUBC = the strain values for the upper branch curve

READ(5,6) (SUBC(I), I=1, NUBC)

SUBC = the stress values for the upper branch curve

READ(5,6) (ELBC(I), I=1, NLBC)

ELBC = the strain values for the lower branch curve

READ(5,6) (SLBC(I), I=1, NLBC)

SLBC = the stress values for the lower branch curve

4. Fully reversed cycles to harden curve

READ(5,1) NRP

NRP = the number of values input to define the fully reversed
cycles to harden curve, ≤ 20

READ(5,6) (XNH(I), I=1, NRP)

XNH = the number of fully reversed cycles to hardening

READ(5,6) (DE2(I), I=1, NRP)

DE2 = $\Delta\epsilon/2$ (half the Δ strain)

5. Normalized hardening curve

READ(5,1) NHC

NHC = the number of values input to define the normalized
hardening curve, ≤ 20

READ(5,6) (XNHR(I), I=1, NHC)

XNHR = Normalized cycles to harden

READ(5,6) (DSDSH(I), I=1, NHC)

DSDSH = Normalized $\Delta\sigma$

6. Plot, map, print and hardening flags

READ(5,1) MPLOT, IMAP, IPRINT, IHARD

MPLOT = Plot flag = 0 no plots; Plot Flag = 1 plot
 IMAP = Select plot variables
 = 0 plot applied loads vs. notch strain
 = 1 plots notch stress vs. notch strain
 IPRINT = Print flag = 0, no diagnostic print
 = 1 diagnostic print
 IHARD = Hardening flag
 = 0 no hardening is calculated
 = 1 hardening is calculated

7. Plot variables (Input only if MPLOT=1)

READ(5,6) XXS, YXS, XL, XSTRT, XSCAL

XXS = X coordinate for X-axis in inches from origin
 (will be negative)
 YXS = Y coordinate for X-axis in inches from origin
 (will be negative)
 XL = Length of X-axis in inches
 XSTRT = Starting label value on X-axis; units = in/in
 XSCAL = Axis label values per inch (also used to scale X values);
 units = in/in

READ(5,6) YYS, YYL, YSTRT, YSCAL

YYS = X coordinate for Y-axis in inches from origin
 (will be negative)
 YYL = Y coordinate for Y-axis in inches from origin
 (will be negative)
 YL = Length of Y-axis in inches
 YSTRT = Starting label value on the Y-axis
 IMAP = 1, units = ksi; IMAP = 0, units = kips
 YSCAL = Y-axis label values per inch (also used to scale Y values)
 IMAP = 1, units = ksi; IMAP = 0, units = kips

8. Read program options

READ(5,1) IOP1, IOP2, IOP3, IOP4

- IOP1
 - = Spectrum input options
 - = 1 Block spectrum, input mean and alternate stresses
(Skip input items 11)
 - = 2 Block spectrum, input maximum and minimum stresses
(Skip input items 11)
 - = 3 Peak spectrum is input
(Skip input items 10)
 - = 9999 no more cases - signals end of computer run
- IOP2
 - = Output option
 - = 1 writes out calculated peak spectrum
 - = 2 does not write out calculated peak spectrum
- IOP3
 - = Output option
 - = 1 write notch strains and notch stresses
 - = 2 does not write notch strains and notch stresses
- IOP4
 - = Output debug option
 - = 1 write stress interpolation information
 - = 2 do not write stress interpolation information

9. Material constants

READ(5,8) CL,A,E,K1,SLOCU,SLOCL

8 FORMAT(6F10.0)

- CL
 - = Material constant for modified Neuber's equation
- A
 - = Area material constant for modified Neuber's equation
- E
 - = Slope of the elastic portion of the locus curve (elastic modulus)
- K1
 - = Stress concentration factor
- SLOCU
 - = last stress value on elastic portion of tension locus curve
(cyclic or monotonic. Elastic portion is the same for both
curves)

SLOCL = Last stress value on the elastic portion of compression locus curve (cyclic or monotonic. Elastic portion is the same for both curves)

10. Block spectrum input (IOP1=3 do not input)

READ(5,100) NPASS,NLAYER,NC

100 FORMAT(3I5)

NPASS = the number of passes to be analyzed

NLAYER = the number of layers in each pass, ≤ 8000

NC = the number of cycles in a block of cycles selected for hysteresis analysis

READ(5,10) S1(I),S2(I),N(I)

10 FORMAT(2F10.0,I10)

S1 = Mean stress if IOP1=1; Max. stress if IOP1=2, stress in psi

S2 = Alternate stress if IOP1=1; Min. stress if IOP1=2, stress in psi

N = Applied cycles
(Repeat NLAYER times) I=1, NLAYER

11. Peak stress input IOP1=(1 or 2 not input)

READ(5,1) NLOADS

NLOADS = the number of peak stresses
(unnotched peak stress)

READ(5,101) (S(I),NS(I),T(I), I=1,NLOADS)

101 FORMAT(3F10.0)

S = Unnotched peak stress, in psi
NS = Applied cycles
T = Time in the hold condition for this peak stress, in hours

3. OUTPUT DESCRIPTION

The program output consists of three basic sections plus various options for plotting and debugging operations. The first section of the output is a printing of the material constants used in the Neuber analysis and are listed as a check of the input. Next, the strains and stresses (STRAIN and SI) are printed in eight columns across the page. Strain and stress for load cases 1 through 4 are on the first line, strain and stress for load cases 5 through 8 are on the second line, etc. through N loadings.

The third basic output consists of eleven columns of data pertaining to the damage calculations. These columns are identified by the following:

DAMG	=	the accumulative damage.
XXI	=	the damage at each load level.
TN	=	the number of accumulative applied cycles.
SIJ	=	the maximum notched stress at the corresponding load level in ksi.
SII	=	the minimum notched stress at the corresponding load level in ksi.
J	=	the index of the selected maximum load from the order of input. Example: If J=4 then this maximum stress was selected from the calculations for the fourth load that was input. This is determined from the RAIN analysis.
I	=	the index of the selected minimum load; the same as J for the maximum load.
NH	=	the number of accumulative half cycles as determined by the RAIN analysis.
NF	=	the number of accumulative whole cycles as determined by the RAIN analysis.
NS(J)	=	the applied cycles for the maximum stress from the input selected by the index J.

NS(I) = the applied cycles for the minimum stress from the input selected by the index I.

DAMAGE = the total damage for this input spectrum.

There are also four output options that can be called by input flag notes. These are:

- o MPlot
- o IPRINT
- o IOP2
- o IOP4

MPlot simply generates a tape for stress or load versus strain plots. The IPRINT option (IPRINT \neq 0) is for program diagnostic purposes only. When this option is specified, the subroutine name is printed along with pertinent program variables (name and value are printed). When one hundred iterations (maximum allowed) are needed to solve for the intersections of curves, the program prints the number of iterations along with the variable names and values involved in the particular algorithm.

Two other options, IOP2 and IOP4, may also be selected for print-out. IOP2 will print the calculated peak stress spectrum (from input data) and will print peak and valley stresses and number of cycles. The IOP4 option is a debug option and writes the stress interpolation information from the interpolation of the S-N data used in the damage calculations.

Sample output for an example problem is included in Appendix C.

4. LIST OF COMPUTER PROGRAM SYMBOLS AND DEFINITIONS

PROGRAM SYMBOLS (MAIN)

MPLOT	Plot option - plot or no plot condition
IMAP	Selects plot variables - applied loads or notch stress vs. notch strain
XXS	X-coordinate for X-axis in inches from origin (will be negative)
YXS	Y-coordinate for X-axis in inches from origin (will be negative)
XL	Length of X-axis in inches
XSTR	Starting label value on X-axis
XSCAL	X-axis label values per inch (also value used to scale X values)
XYS	X-coordinate for Y-axis in inches from origin (will be negative)
YYS	Y-coordinate for Y-axis in inches from origin (will be negative)
YL	Length of Y-axis in inches
YSTR	Starting label value on Y-axis
YSCAL	Y-axis label values per inch (also value used to scale Y values)
IOP1	Spectrum input options
IOP2	Spectrum output options
IOP3	Spectrum output options
IOP4	De-bug option - gives additional print
CL	Material constant for modified Neuber's equation
A	Material constant for modified Neuber's equation
E	Elastic modulus
K1	Stress concentration factor
NPASS	Number of passes to be analyzed
NLAYER	Number of layers in each pass
NC	Number of cycles in a block of cycles selected for hysteresis analysis
S1	Mean stress if IOP1=1; Max stress if IOP1=2
S2	Alternating stress if IOP1=1; Min stress if IOP1=2
N	Number of applied cycles
NLOADS	Number of peak stresses to be input
S	Unnotched peak stress
NS	Applied cycles

LIST OF COMPUTER PROGRAM SYMBOLS AND DEFINITIONS
PROGRAM SYMBOLS (MAIN) (Continued)

NUBC	Number of points in upper branch curve
EUBC	X-axis values (stress) of upper branch curve
SUBC	Y-axis values (strain) of upper branch curve
NLBC	Number of points to define lower branch curve
ELBC	X-axis values (stress) of lower branch curve
SLBC	Y-axis values (strain) of lower branch curve
NTLC	Number of points to define tension portion of locus curve
EHTLC	X-axis values (stress) of locus curve (tension half)
SHTLC	Y-axis values (strain) of locus curve (tension half)
NCLC	Number of points to define compression portion of locus curve
EHCLC	X-axis values (stress) of locus curve (compression half)
SHCLC	Y-axis values (strain) of locus curve (compression half)
XS	Array containing computed X values (stress) used in damage calculations
YS	Array containing computed Y values (strain) used in damage calculations
NP	Pointer to current values in EX, SY arrays used in computations
SEQ	Test sequence I.D. used to identify plotted output
SAVE	Previously computed Y value (plot value)
IFLG	Flags plot routine to put new axis on for plot (Multi case computer run)
INIT	Initializes the KS index for storing stress strain history arrays XS, YS
EX	Array containing (X) computed end points of plotted curve
SY	Array containing (Y) computed end points of plotted curve
DSKTS	Δ SKT values (Neuber's term as defined in loop analysis)
DLOAD	Array containing the Δ loads (input or computed from input)
SLOCL	Last Y value on straight portion of lower locus curve
SLOCU	Last Y value on straight portion of upper locus curve
NLCCT	Number of points defining tension cyclic locus curve

LIST OF COMPUTER PROGRAM SYMBOLS AND DEFINITIONS PROGRAM SYMBOLS (MAIN) (Continued)

NLCCC	Number of points defining compression cyclic locus curve
TLOCCE	Tension strain values for cyclic locus curve
TLOCCS	Tension stress values for cyclic locus curve
CLOCCE	Compression strain values for cyclic locus curve
CLOCCS	Compression stress values for cyclic locus curve
NLCMT	Number of points defining compression monotonic locus curve
NLCMC	Number of points defining compression monotonic locus curve
TLOCME	Tension strain values for monotonic locus curve
TLOCMS	Tension stress values for monotonic locus curve
CLOCME	Compression strain values for monotonic locus curve
CLOCMS	Compression stress values for monotonic locus curve
I	Do loop index
DAMG	Final computed damage
T	Time a given load is in hold mode
SNHT	Sumation cycles for tension hardening
SNHC	Sumation cycles for compression hardening
HARDT	Tension hardening flag
HARDC	Compression hardening flag
TCREP	Time from previous hold condition
CREP	Computed creep value
X1	Intermediate storage for X-value of end point
Y1	Intermediate storage for Y-value of end point
NHFLG	Hardening flag

PROGRAM SYMBOLS (ADJUST)

NI	Pointer used in selecting first or second values in end point arrays EX and SY
EA	Selected EX value using NI
SA	Selected SY value using NI

LIST OF COMPUTER PROGRAM SYMBOLS AND DEFINITIONS PROGRAM SYMBOLS (ADJUST) (Continued)

NP1 One less than NI
NP2 Two less than NI

PROGRAM SYMBOLS (CRVTRP)

DS2 Half the delta strain
NCYC The number of reversals to harden
NCYCSM The sum of the number of reversals to harden
XNH X value on number of fully reversed cycles to harden curve
DE2 Absolute value of DS2
XNHR X-value on normalized hardening curve
DSDSH Y-value on normalized hardening curve
NRP Number of points defining number of reversals to harden curve
NHC Number of points defining normalized hardening curve
XLOG Log of the interpolated X value from fully reversed cycles to
 harden curve
XN Anti log of XLOG
SNHC Sumation cycles for compression hardening
I Index for loop to bracket DS2 on number of cycles to harden curve
NI Pointer for bracketing value on cycles to harden curve
SNH Intermediate value of SNHC and SNHT
HARDC Compression hardening flag
PCT Percent hardened
SNHT Sum of cycles to harden (tension)
HARDT Tension hardening flag

LIST OF COMPUTER PROGRAM SYMBOLS AND DEFINITIONS PROGRAM SYMBOLS (MAPP)

PLOTX	X values to be plotted
PLOTY	Y values to be plotted
IP	No. of points (X,Y values)
NFLG	Greater than 1 doubles the A constant in Neuber's term.
DX	Delta X difference between X values
DY	Delta Y difference between Y values
EXY	Intermediate value in converting from notch stress to applied load
F	Sets sign + or minus of Delta SKT
SAVE	Save previous point on plot

PROGRAM SYMBOLS (DRIVER)

DSKT	Input or computed delta loads
EPLT, SPLT	Intermediate points to plot
IEQ	Flag set when consecutive loads are equal
KS	Index on history array used in damage calculation
INIT	Initialization flag for first loading
NP	Index for end point arrays (EX, SY)
DSKTS	Loads array - one for each EX, SY
SLOCU	Last Y value on elastic portion of tension locus curve
SLOCL	Last Y value on elastic portion of compression locus curve
IPLT	Plot flag = 1 tension; = 2 compression
NP1	NP plus 1

PROGRAM SYMBOLS (ILABC)

N1	Pointer to a value in end points arrays EX and SY
N2	Pointer to a value in end points arrays EX and SY
EA, SA	Point defining line
E	Slope of elastic portion of locus curve

PROGRAM SYMBOLS (ILABC) (Continued)

F	Plus or minus value of 1.0 signals tension or compression
PART1	Distance from line to origin
PART2	Distance from translated line to origin
XO,YO	Translation values for branch curve
I	Index for loop on branch curve points

PROGRAM SYMBOLS (LINBR)

XB,YB	Points on the branch curve that bracket the intersection
EA,SA	Point on the line that defines the line along with slope E
XO,YO	Translation parameters for the branch curve
F	Plus or minus 1.0 that signals tension or compression
X,Y	Point on branch curve where intersection occurs
XAB,YAB	Point on branch curve above intersection
XBE,YBE	Point on branch curve below intersection
KOUNT	Count of iterations needed to find (X,Y)
PART1	Distance of line from branch curve origin
PART2	Distance of line from translated branch curve origin

PROGRAM SYMBOLS (BCRPLT)

X	X values to be plotted
Y	Y values to be plotted
IPRINT	=0 No intermediate print, >1 gives intermediate print
XX	Scaled X values
YY	Scaled Y values
NT	Pointer to proper Y title
NS	No. of words in Y title
XXS	X coordinate for X-axis in inches from origin (will be negative)
YXS	Y coordinate for X-axis in inches from origin (will be negative)
XL	Length of X-axis in inches
XSTRT	Starting label value on X-axis

PROGRAM SYMBOLS (BCRPLT) (Continued)

XSCAL	X-axis label values per inch (also value used to scale X values).
XYS	X coordinate for Y-axis in inches from origin (will be negative)
YYS	Y coordinate for Y-axis in inches from origin (will be negative)
YL	Length of Y-axis in inches
YSTRT	Starting label value on Y-axis
YSCAL	Y-axis label values per inch (also value used to scale Y values)

PROGRAM SYMBOLS (MAKPLT)

NFLG	=1 plot locus curve, >1 plot branch curve
IP	Count of points to plot
X	X-value of intersection on the appropriate curve (locus or branch)
Y	Y-value of intersection on the appropriate curve (locus or branch)
PLOTX	X-values plotted (contains end points and points between).
NS	Index used in searching input curves for intervening points, s-start
NE	Index used in searching input curves for intervening points, E-end
PLOTY	Y values plotted (from input locus and branch curve)
F	Used in effecting a sign change
NTLC	
EHTLC	
SHTLC	
NCLC	
EHCLC	
SHCLC	

PROGRAM SYMBOLS (LINSEC)

X	X-value of point half way between (XAB,YAB) and (XBE,YBE)
Y	Y-value of point half way between (XAB,YAB) and (XBE,YBE)
KOUNT	Counts the number of iterations needed to find the intersection
VALUE	Used in determining accuracy of the solution
TEST	The tolerance used in accepting a solution (epsilon)

PROGRAM SYMBOLS (LINSEC) (Continued)

XAB,YAB Point on locus curve above intersection
 XBE,YBE Point on locus curve below intersection

PROGRAM SYMBOLS (INCLC)

DSQSKT Neuber term squared
 IP Counter of points along locus curve
 EVAL1 Value of Neuber's term last time in this routine (previous value)
 EVAL2 Value of Neuber's term
 EVALN Numerator of Neuber's term
 EVALD Denominator of Neuber's term
 EH X-values on locus curve (around point of intersection of Neubers)
 SH Y-values on locus curve (curve and locus curve)
 JP Index of point below intersection on Neuber's curve

PROGRAM SYMBOLS (DETERM)

X2 X-value half way between (XAB,YAB) and (XBE,YBE)
 Y2 Y-value half way between (XAB,YAB) and (XBE,YBE)
 KOUNT Count of number of times through loop to get convergence
 FN Numerator of Neuber's term
 FD Denominator of Neuber's term
 FSUBI Value of Neuber's term
 ABSFI Absolute value of difference of Neuber's term (FSUBI) and previous
 Squared value of Neuber's term passed in calling sequence

PROGRAM SYMBOLS (INCBC)

A2 2 times the material constant A
 F Sets the sign in numerator of Neuber's term by taking difference
 Of end points
 N1 Pointers in EX,SY array for first point for BRCH.C. translation
 N2 Pointers in EX,SY array for second point for BRCH.C. translation

PROGRAM SYMBOLS (INCBC) (Continued)

TOP	Numerator of Neuber's term
BOT	Denominator of Neuber's term
FUNCT	Value of Neuber's term
FUNCT1	Previous value of Neuber's term

PROGRAM SYMBOLS (BINTS)

X	X-value half way between (XAB and XBE)
Y	Y-value half way between (YAB and YBE)
KOUNT	Number of iterations to find intersection
A2	Two times material constant A
FN	Numerator in Neuber's term
FD	Denominator in Neuber's term
FV	Value of Neuber's term

PROGRAM SYMBOLS (ORIGIN)

YO	Y-value for origin of branch curve
F	Sets sign to flip branch curve + for upper- for lower
EUBC	X values defining branch curve
SUBC	Y values defining branch curve
ET	Slope of branch curve at current points
EA	DX translation value of branch curve
SA	DY translation value of branch curve
XO	X-value for origin of branch curve

PROGRAM SYMBOLS (SPECT)

NPASS	Number of passes to be analyzed
NLAYER	Number of layers in each pass
NC	Number of cycles in a block of cycles selected for hysteresis analysis
S1	Mean stress if OP =1, max. stress if OP =2
S2	Alt. stress if OP =1, min. stress if OP =2

PROGRAM SYMBOLS (SPECT) (Continued)

N	Applied cycles
SMAX	Maximum stress
SMIN	Minimum stress

PROGRAM SYMBOLS (DAMAGE)

DAMG	Total damage
XXI	Current incremental damage
STR	Notch root strain
TN	Total number of cycles in external spectrum
SII	Notch stress (minimum)
SIJ	Notch stress (maximum)
NH	Number of half cycles
NF	Number of full cycles
SM	Mean stress in S-N data
VS	Variable stress in S-N data
ALN	Number of allowable cycles in S-N data
AN	Interpolated number of cycles for failure for given mean and alternating stresses

PROGRAM SYMBOLS (RAIN)

JMAX	Array for maximum peak strain or stress
JMIN	Array for minimum peak strain or stress
NTOT	Total number of cycles for rain flow counting

PROGRAM SYMBOLS (LPTLC)

EA,SA	Point defining a line with slope SLP
SLP	Slope of line intersecting locus curve
X,Y	Point of intersection of a line with a locus curve
DIFF	Value that determines tension or compression locus curve
JP	Pointer in locus curve to a point below intersection

PROGRAM SYMBOL (LPTBC)

TEST	Tolerance value passed to LINSEC
DIFF	Same as TEST but used as local variable
	Both are distance to origin from line
EA, SA	Point defining line that intersects branch curve
SLP	The slope of line that intersects branch curve
I	Index on branch curve points, used in search for points that bracket intersection
J	Index on loop to find points that bracket intersection
JP	Pointer to points that bracket intersection

PROGRAM SYMBOLS (FDSKT)

FNUM	The numerator of the Δ SKT computation
FDNOM	The denominator of Δ SKT computation
DSKTS	The Δ SKT value

PROGRAM SYMBOLS (CREPER)

T	The time for this loading condition in hold, mode
PCT	The percent of creep that has occurred
TCREP	The time in the hold condition
CREP	Computed creep. Creep is only allowed where $SY(NP) < SY(NP-1)$
PCTN	The stress relaxation functional minus CREP
PCTD	The stress relaxation function
P	Material constant
PN	Material constant
EI, SI	Intersection of locus curve and a line
NP	The pointer to current values in history arrays EX, SY (end points)

PROGRAM SYMBOLS (CREEP)

K	Material constant used for creep calculation
EBAR	Material constant used for creep calculation

PROGRAM SYMBOLS (CREEP) (Continued)

P	Material constant used for creep calculation
PN	Material constant used for creep calculation
KOUNT	Counter for iterations needed to solve for the stress relaxation Sigma R
SIGR	The stress relaxation; current value of Sigma R (initial value = 0).
FSIGR	Function of Sigma R
FPSIGR	Derivative of FSIGR
SIGRI	Previous value of SIGR in iterative solution
CE	Creep strain

APPENDIX C

EXAMPLE PROBLEM WITH INPUT AND OUTPUT
DATA LISTING

INPUT AND OUTPUT FOR A SELECTED EXAMPLE PROBLEM

To illustrate the capabilities of the hysteresis analysis program an example problem is presented here along with the input data and program output. The example problem presented here is one of the flight-by-flight spectra used in the Phase II experimental program. Spectrum 2 as discussed in Section 4.4 was used. This spectrum includes three flights, one flight with a large tension overload, plus sustained load hold periods. Typically, the spectrum can be described by,

$$50[(C + \text{Hold}) + 150(2A + B)] + N(2A + B)$$

where;

A, B, and C are flights as defined in Figure 67.

Hold is a 1-hour sustained load hold period at -9.4 ksi after each Flight C.

N is the number of times 2A + B is repeated to produce damage at $\sum \frac{n}{N} = 1.0$

This example illustrates the analysis capabilities of the program for complex loading sequences as well as the creep and stress relaxation module for evaluating the time dependent effects or damage.

000037	000	17500-	1-0
000038	000	11200-	1-0
000039	000	17500-	1-0
000040	000	17500-	1-0
000041	000	17500-	1-0
000042	000	19100-	1-0
000043	000	9600-	1-0
000044	000	22200-	1-0
000045	000	10800-	1-0
000046	000	20600-	1-0
000047	000	12400-	1-0
000048	000	20600-	1-0
000049	000	12400-	1-0
000050	000	20600-	1-0
000051	000	12400-	1-0
000052	000	20600-	1-0
000053	000	12400-	1-0
000054	000	25200-	1-0
000055	000	5600-	1-0
000056	000	25200-	1-0
000057	000	5600-	1-0
000058	000	25200-	1-0
000059	000	5600-	1-0
000060	000	25200-	1-0
000061	000	5600-	1-0
000062	000	25200-	1-0
000063	000	5600-	1-0
000064	000	25200-	1-0
000065	000	5600-	1-0
000066	000	25200-	1-0
000067	000	5600-	1-0
000068	000	25200-	1-0
000069	000	5600-	1-0
000070	000	25200-	1-0
000071	000	5600-	1-0
000072	000	25200-	1-0
000073	000	5600-	1-0
000074	000	25200-	1-0
000075	000	5600-	1-0
000076	000	25200-	1-0
000077	000	5600-	1-0
000078	000	25200-	1-0
000079	000	5600-	1-0
000080	000	25200-	1-0
000081	000	5600-	1-0
000082	000	25200-	1-0
000083	000	5600-	1-0
000084	000	25200-	1-0
000085	000	5600-	1-0
000086	000	25200-	1-0
000087	000	5600-	1-0
000088	000	25200-	1-0
000089	000	5600-	1-0
000090	000	25200-	1-0
000091	000	5600-	1-0
000092	000	25200-	1-0
000093	000	5600-	1-0
000094	000	25200-	1-0
000095	000	5600-	1-0
000096	000	25200-	1-0
000097	000	5600-	1-0
000098	000	25200-	1-0
000099	000	5600-	1-0
000100	000	25200-	1-0
000101	000	5600-	1-0
000102	000	25200-	1-0
000103	000	5600-	1-0
000104	000	25200-	1-0
000105	000	5600-	1-0
000106	000	25200-	1-0
000107	000	5600-	1-0
000108	000	25200-	1-0
000109	000	5600-	1-0
000110	000	25200-	1-0
000111	000	5600-	1-0
000112	000	25200-	1-0
000113	000	5600-	1-0
000114	000	25200-	1-0

24.0

Figure C1. Input For Example Problem (Continued)

000115	000	18800-	1-0
000116	000	10100-	1-0
000117	000	18800-	1-0
000118	000	10100-	1-0
000119	000	18800-	1-0
000120	000	10100-	1-0
000121	000	18800-	1-0
000122	000	10100-	1-0
000123	000	25600-	1-0
000124	000	8400-	1-0
000125	000	21600-	1-0
000126	000	9950-	1-0
000127	000	19900-	1-0
000128	000	11700-	1-0
000129	000	19900-	1-0
000130	000	11700-	1-0
000131	000	19900-	1-0
000132	000	11700-	1-0
000133	000	19900-	1-0
000134	000	11700-	1-0
000135	000	22000-	1-0
000136	000	11900-	1-0
000137	000	22000-	1-0
000138	000	11900-	1-0
000139	000	22000-	1-0
000140	000	11900-	1-0
000141	000	22000-	1-0
000142	000	11900-	1-0
000143	000	21300-	1-0
000144	000	17400-	1-0
000145	000	11500-	1-0
000146	000	11500-	1-0
000147	000	11500-	1-0
000148	000	11500-	1-0
000149	000	11500-	1-0
000150	000	11500-	1-0
000151	000	11500-	1-0
000152	000	1200-	1-0
000153	000	15100-	1-0
000154	000	9800-	1-0
000155	000	22200-	1-0
000156	000	10800-	1-0
000157	000	20600-	1-0
000158	000	12400-	1-0
000159	000	20600-	1-0
000160	000	12400-	1-0
000161	000	20600-	1-0
000162	000	12400-	1-0
000163	000	20600-	1-0
000164	000	12400-	1-0
000165	000	25200-	1-0
000166	000	5600-	1-0
000167	000	25200-	1-0
000168	000	5600-	1-0
000169	000	25200-	1-0
000170	000	5600-	1-0
000171	000	25200-	1-0
000172	000	5600-	1-0

Figure C1. Input For Example Problem (Continued)

[illegible]

000231	000	22000-	1-000000
000232	000	11900-	1-000000
000233	000	23900-	1-000000
000234	000	17500-	1-000000
000235	000	11200-	1-000000
000236	000	11200-	1-000000
000237	000	11200-	1-000000
000238	000	11200-	1-000000
000239	000	11200-	1-000000
000240	000	11200-	1-000000
000241	000	11200-	1-000000
000242	000	11200-	1-000000
000243	000	19100-	1-000000
000244	000	9600-	1-000000
000245	000	22200-	1-000000
000246	000	10800-	1-000000
000247	000	20600-	1-000000
000248	000	12400-	1-000000
000249	000	20600-	1-000000
000250	000	12400-	1-000000
000251	000	20600-	1-000000
000252	000	12400-	1-000000
000253	000	20600-	1-000000
000254	000	12400-	1-000000
000255	000	25200-	1-000000
000256	000	5600-	1-000000
000257	000	25200-	1-000000
000258	000	5600-	1-000000
000259	000	25200-	1-000000
000260	000	5600-	1-000000
000261	000	25200-	1-000000
000262	000	5600-	1-000000
000263	000	27800-	1-000000
000264	000	-9400-	1-000000
000265	000	18800-	1-000000
000266	000	10100-	1-000000
000267	000	18800-	1-000000
000268	000	10100-	1-000000
000269	000	18800-	1-000000
000270	000	10100-	1-000000
000271	000	18800-	1-000000
000272	000	10100-	1-000000
000273	000	20600-	1-000000
000274	000	6400-	1-000000
000275	000	21600-	1-000000
000276	000	9950-	1-000000
000277	000	19300-	1-000000
000278	000	11100-	1-000000
000279	000	19300-	1-000000
000280	000	11100-	1-000000
000281	000	19300-	1-000000
000282	000	11100-	1-000000
000283	000	19300-	1-000000
000284	000	11100-	1-000000
000285	000	22600-	1-000000
000286	000	11900-	1-000000
000287	000	22900-	1-000000
000288	000	11900-	1-000000

Figure C1. Input For Example Problem (Continued)

000289	000	22000-	1-0
000290	000	11900-	1-0
000291	000	22000-	1-0
000292	000	11900-	1-0
000293	000	23900-	1-0
000294	000	18000-	1-0
000295	000	10100-	1-0
000296	000	18000-	1-0
000297	000	10100-	1-0
000298	000	10100-	1-0
000299	000	18000-	1-0
000300	000	10100-	1-0
000301	000	18000-	1-0
000302	000	10100-	1-0
000303	000	20600-	1-0
000304	000	8400-	1-0
000305	000	21600-	1-0
000306	000	9950-	1-0
000307	000	19900-	1-0
000308	000	11700-	1-0
000309	000	19900-	1-0
000310	000	11700-	1-0
000311	000	19900-	1-0
000312	000	11700-	1-0
000313	000	19900-	1-0
000314	000	11700-	1-0
000315	000	22000-	1-0
000316	000	11900-	1-0
000317	000	22000-	1-0
000318	000	11900-	1-0
000319	000	22000-	1-0
000320	000	11900-	1-0
000321	000	22000-	1-0
000322	000	11900-	1-0
000323	000	23900-	1-0
000324	000	9400-	1-0
000325	000	17500-	1-0
000326	000	11200-	1-0
000327	000	17500-	1-0
000328	000	11200-	1-0
000329	000	17500-	1-0
000330	000	11200-	1-0
000331	000	17500-	1-0
000332	000	11200-	1-0
000333	000	19100-	1-0
000334	000	9600-	1-0
000335	000	22200-	1-0
000336	000	10800-	1-0
000337	000	20600-	1-0
000338	000	12400-	1-0
000339	000	20600-	1-0
000340	000	12400-	1-0
000341	000	20600-	1-0
000342	000	12400-	1-0
000343	000	20600-	1-0
000344	000	12400-	1-0
000345	000	25200-	1-0
000346	000	5600-	1-0

Figure C1. Input For Example Problem (Continued)

000347	000	25200.-	1.0
000348	000	26000.-	1.0
000349	000	26200.-	1.0
000350	000	26400.-	1.0
000351	000	26600.-	1.0
000352	000	26800.-	1.0
000353	000	27000.-	1.0
000354	000	27800.-	1.0
000355	000	0.0	1.0
000356	000	99999	1.0

END ELT
TASK UNITS/ 0 PGM SIZE/ 6144

*END IGNORED - IN CONTROL MODE

*XQT -ARS

Figure C1. Input For Example Problem (Concluded)

MATERIAL CONSTANTS

CL,A,E,K1,SLOCU,SLOCCL .150700+01 .745000-01 .110000+01 .243000+01 .750000+05 -.640000+05
 KOUNT= 101 X= .27000-01 Y= .10500+06 VALUE= -.69182-J2 TEST= -.52194-J2

Figure C2. Output for Example Problem

[illegible]

243

1955-01-22	19422+15	22X-3-32	+6875+22	3855-3-32	19422+85	2005-3-02	+6875+02
1955-01-26	2422+05	2013-02	+5153+03	+330-0-02	+4222+85	2051-3-02	+5153+02
1955-01-30	2422+05	2013-02	+5153+03	+340-0-02	+4222+85	2251-3-32	+5153+03
1955-01-30	2797+05	27167-02	-49172+05	3520-0-02	16222+05	16613-02	-49172+05
1955-01-30	16222+05	16613-02	+3549+04	3520-0-02	16222+05	16613-02	+3549+04
1955-01-30	16222+15	16613-32	-3549+04	4733-3-32	17171-32	17171-32	-3549+04
1955-01-30	23424+05	16153-32	-40156+04	3253-0-02	19422+05	2005-3-02	-76984+86
1955-01-30	19422+05	20063-02	+6875+02	3560-0-02	17422+05	2005-3-02	+6875+02
1955-01-30	2422+05	20063-02	+6875+02	4340-0-02	17422+05	2005-3-02	+6875+02
1955-01-30	2422+05	20513-32	+5153+03	4340-0-02	2422+05	2051-3-02	+5153+03
1955-01-30	2422+05	20513-32	+5153+03	4765-3-32	2422+05	2051-3-02	+5153+03
1955-01-30	13953+05	19163-02	-89063+03	3340-0-02	13953+05	19163-02	-89063+03
1955-01-30	13953+05	19163-02	-89063+03	3340-0-02	13953+05	19163-02	-89063+03
1955-01-30	13953+05	19163-02	-89063+03	3340-0-02	13953+05	19163-02	-89063+03
1955-01-30	23984+05	21563-02	-16949+04	40150-0-02	20994+05	21563-02	-16949+04
1955-01-30	23984+05	21563-02	-16949+04	40150-0-02	20994+05	21563-02	-16949+04
1955-01-30	31609+05	31125-03	-14464+05	41153-3-32	31609+05	31125-03	-14464+05
1955-01-30	31609+05	31125-03	-14464+05	52350-0-02	31609+05	31125-03	-14464+05
1955-01-30	37737+35	37737-33	-24454+05	50330-0-02	31609+05	31125-03	-14464+05
1955-01-30	37737+35	37737-33	-24454+05	50330-0-02	31609+05	31125-03	-14464+05

CUMULATIVE DAMAGE

DATE	XXI	TN	SI J	SEI	J	NIH	MF	MSL(J)	MSL(J)
104-01	.000	1.500	75.304	-22.733	1	1	0	1.000	1.000
104-03	.000	1.000	42.241	-22.733	2	2	0	1.000	1.000
104-05	.000	1.000	42.041	-22.733	3	3	1	1.000	1.000
104-07	.000	1.000	42.041	-22.733	4	4	2	1.000	1.000
104-09	.000	1.000	42.041	-22.733	5	5	3	1.000	1.000
104-11	.000	1.000	42.041	-22.733	6	6	4	1.000	1.000
104-13	.000	1.000	42.041	-22.733	7	7	5	1.000	1.000
104-15	.000	1.000	42.041	-22.733	8	8	6	1.000	1.000
104-17	.000	1.000	42.041	-22.733	9	9	7	1.000	1.000
104-19	.000	1.000	42.041	-22.733	10	10	8	1.000	1.000
104-21	.000	1.000	42.041	-22.733	11	11	9	1.000	1.000
104-23	.000	1.000	42.041	-22.733	12	12	10	1.000	1.000
104-25	.000	1.000	42.041	-22.733	13	13	11	1.000	1.000
104-27	.000	1.000	42.041	-22.733	14	14	12	1.000	1.000
104-29	.000	1.000	42.041	-22.733	15	15	13	1.000	1.000
104-31	.000	1.000	42.041	-22.733	16	16	14	1.000	1.000
104-33	.000	1.000	42.041	-22.733	17	17	15	1.000	1.000
104-35	.000	1.000	42.041	-22.733	18	18	16	1.000	1.000
104-37	.000	1.000	42.041	-22.733	19	19	17	1.000	1.000
104-39	.000	1.000	42.041	-22.733	20	20	18	1.000	1.000
104-41	.000	1.000	42.041	-22.733	21	21	19	1.000	1.000
104-43	.000	1.000	42.041	-22.733	22	22	20	1.000	1.000
104-45	.000	1.000	42.041	-22.733	23	23	21	1.000	1.000
104-47	.000	1.000	42.041	-22.733	24	24	22	1.000	1.000
104-49	.000	1.000	42.041	-22.733	25	25	23	1.000	1.000
104-51	.000	1.000	42.041	-22.733	26	26	24	1.000	1.000
104-53	.000	1.000	42.041	-22.733	27	27	25	1.000	1.000
104-55	.000	1.000	42.041	-22.733	28	28	26	1.000	1.000
104-57	.000	1.000	42.041	-22.733	29	29	27	1.000	1.000
104-59	.000	1.000	42.041	-22.733	30	30	28	1.000	1.000
104-61	.000	1.000	42.041	-22.733	31	31	29	1.000	1.000
104-63	.000	1.000	42.041	-22.733	32	32	30	1.000	1.000
104-65	.000	1.000	42.041	-22.733	33	33	31	1.000	1.000
104-67	.000	1.000	42.041	-22.733	34	34	32	1.000	1.000
104-69	.000	1.000	42.041	-22.733	35	35	33	1.000	1.000
104-71	.000	1.000	42.041	-22.733	36	36	34	1.000	1.000
104-73	.000	1.000	42.041	-22.733	37	37	35	1.000	1.000
104-75	.000	1.000	42.041	-22.733	38	38	36	1.000	1.000
104-77	.000	1.000	42.041	-22.733	39	39	37	1.000	1.000
104-79	.000	1.000	42.041	-22.733	40	40	38	1.000	1.000
104-81	.000	1.000	42.041	-22.733	41	41	39	1.000	1.000
104-83	.000	1.000	42.041	-22.733	42	42	40	1.000	1.000
104-85	.000	1.000	42.041	-22.733	43	43	41	1.000	1.000
104-87	.000	1.000	42.041	-22.733	44	44	42	1.000	1.000
104-89	.000	1.000	42.041	-22.733	45	45	43	1.000	1.000
104-91	.000	1.000	42.041	-22.733	46	46	44	1.000	1.000
104-93	.000	1.000	42.041	-22.733	47	47	45	1.000	1.000
104-95	.000	1.000	42.041	-22.733	48	48	46	1.000	1.000
104-97	.000	1.000	42.041	-22.733	49	49	47	1.000	1.000
104-99	.000	1.000	42.041	-22.733	50	50	48	1.000	1.000
104-101	.000	1.000	42.041	-22.733	51	51	49	1.000	1.000
104-103	.000	1.000	42.041	-22.733	52	52	50	1.000	1.000
104-105	.000	1.000	42.041	-22.733	53	53	51	1.000	1.000
104-107	.000	1.000	42.041	-22.733	54	54	52	1.000	1.000

Figure C2. Output for Example Problem (Continued)

CUMULATIVE DAMAGE

107-02	.000	55.000	20.984	1.603	109	112	4	53	1.000	1.000
107-02	.000	55.000	20.984	1.603	111	112	4	54	1.000	1.000
107-02	.000	55.000	20.984	1.603	113	114	4	55	1.000	1.000
107-02	.000	55.000	20.984	1.603	115	116	4	56	1.000	1.000
107-02	.000	55.000	20.984	1.603	117	118	4	57	1.000	1.000
107-02	.000	55.000	20.984	1.603	119	120	4	58	1.000	1.000
107-02	.000	55.000	20.984	1.603	121	122	4	59	1.000	1.000
107-02	.000	55.000	20.984	1.603	123	124	4	60	1.000	1.000
107-02	.000	55.000	20.984	1.603	125	126	4	61	1.000	1.000
107-02	.000	55.000	20.984	1.603	127	128	4	62	1.000	1.000
107-02	.000	55.000	20.984	1.603	129	130	4	63	1.000	1.000
107-02	.000	55.000	20.984	1.603	131	132	4	64	1.000	1.000
107-02	.000	55.000	20.984	1.603	133	134	4	65	1.000	1.000
107-02	.000	55.000	20.984	1.603	135	136	4	66	1.000	1.000
107-02	.000	55.000	20.984	1.603	137	138	4	67	1.000	1.000
107-02	.000	55.000	20.984	1.603	139	140	4	68	1.000	1.000
107-02	.000	55.000	20.984	1.603	141	142	4	69	1.000	1.000
107-02	.000	55.000	20.984	1.603	143	144	4	70	1.000	1.000
107-02	.000	55.000	20.984	1.603	145	146	4	71	1.000	1.000
107-02	.000	55.000	20.984	1.603	147	148	4	72	1.000	1.000
107-02	.000	55.000	20.984	1.603	149	150	4	73	1.000	1.000
107-02	.000	55.000	20.984	1.603	151	152	4	74	1.000	1.000
107-02	.000	55.000	20.984	1.603	153	154	4	75	1.000	1.000
107-02	.000	55.000	20.984	1.603	155	156	4	76	1.000	1.000
107-02	.000	55.000	20.984	1.603	157	158	4	77	1.000	1.000
107-02	.000	55.000	20.984	1.603	159	160	4	78	1.000	1.000
107-02	.000	55.000	20.984	1.603	161	162	4	79	1.000	1.000
107-02	.000	55.000	20.984	1.603	163	164	4	80	1.000	1.000
107-02	.000	55.000	20.984	1.603	165	166	4	81	1.000	1.000
107-02	.000	55.000	20.984	1.603	167	168	4	82	1.000	1.000
107-02	.000	55.000	20.984	1.603	169	170	4	83	1.000	1.000
107-02	.000	55.000	20.984	1.603	171	172	4	84	1.000	1.000
107-02	.000	55.000	20.984	1.603	173	174	4	85	1.000	1.000
107-02	.000	55.000	20.984	1.603	175	176	4	86	1.000	1.000
107-02	.000	55.000	20.984	1.603	177	178	4	87	1.000	1.000
107-02	.000	55.000	20.984	1.603	179	180	4	88	1.000	1.000
107-02	.000	55.000	20.984	1.603	181	182	4	89	1.000	1.000
107-02	.000	55.000	20.984	1.603	183	184	4	90	1.000	1.000
107-02	.000	55.000	20.984	1.603	185	186	4	91	1.000	1.000
107-02	.000	55.000	20.984	1.603	187	188	4	92	1.000	1.000
107-02	.000	55.000	20.984	1.603	189	190	4	93	1.000	1.000
107-02	.000	55.000	20.984	1.603	191	192	4	94	1.000	1.000
107-02	.000	55.000	20.984	1.603	193	194	4	95	1.000	1.000
107-02	.000	55.000	20.984	1.603	195	196	4	96	1.000	1.000
107-02	.000	55.000	20.984	1.603	197	198	4	97	1.000	1.000
107-02	.000	55.000	20.984	1.603	199	200	4	98	1.000	1.000
107-02	.000	55.000	20.984	1.603	201	202	4	99	1.000	1.000
107-02	.000	55.000	20.984	1.603	203	204	4	100	1.000	1.000
107-02	.000	55.000	20.984	1.603	205	206	4	101	1.000	1.000
107-02	.000	55.000	20.984	1.603	207	208	4	102	1.000	1.000
107-02	.000	55.000	20.984	1.603	209	210	4	103	1.000	1.000
107-02	.000	55.000	20.984	1.603	211	212	4	104	1.000	1.000
107-02	.000	55.000	20.984	1.603	213	214	4	105	1.000	1.000
107-02	.000	55.000	20.984	1.603	215	216	4	106	1.000	1.000
107-02	.000	55.000	20.984	1.603	217	218	4	107	1.000	1.000
107-02	.000	55.000	20.984	1.603	219	220	4	108	1.000	1.000
107-02	.000	55.000	20.984	1.603	221	222	4	109	1.000	1.000
107-02	.000	55.000	20.984	1.603	223	224	4	110	1.000	1.000

Figure C2. Output for Example Problem (Continued)

11-02	.000	111.000	21.141	-7.603	221	222	8	107	1.000
11-02	.000	112.000	19.434	-4.315	223	224	8	108	1.000
11-02	.000	113.000	19.422	.047	225	226	8	109	1.000
11-02	.000	114.000	19.422	.047	227	228	8	110	1.000
11-02	.000	115.000	19.422	.047	229	230	8	111	1.000
11-02	.000	116.000	19.422	.047	231	232	8	112	1.000
11-02	.000	117.000	24.422	.515	233	234	8	113	1.000
11-02	.000	118.000	24.422	.515	235	236	8	114	1.000
11-02	.000	119.000	24.422	.515	237	238	8	115	1.000
11-02	.000	120.000	24.422	.515	239	240	8	116	1.000
11-02	.000	121.000	29.797	-9.172	241	242	8	117	1.000
11-02	.000	122.000	16.922	-3.547	243	244	8	118	1.000
11-02	.000	123.000	16.922	-3.547	245	246	8	119	1.000
11-02	.000	124.000	16.922	-3.547	247	248	8	120	1.000
11-02	.000	125.000	16.922	-3.547	249	250	8	121	1.000
11-02	.000	126.000	23.484	-7.603	251	252	8	122	1.000
11-02	.000	127.000	19.422	-4.016	253	254	8	123	1.000
11-02	.000	128.000	19.422	.047	255	256	8	124	1.000
11-02	.000	129.000	19.422	.047	257	258	8	125	1.000
11-02	.000	130.000	19.422	.047	259	260	8	126	1.000
11-02	.000	131.000	19.422	.047	261	262	8	127	1.000
11-02	.000	132.000	26.422	.515	263	264	8	128	1.000
11-02	.000	133.000	26.422	.515	265	266	8	129	1.000
11-02	.000	134.000	26.422	.515	267	268	8	130	1.000
11-02	.000	135.000	26.422	.515	269	270	8	131	1.000
11-02	.000	136.000	28.797	-9.172	271	272	8	132	1.000
11-02	.000	137.000	13.953	-.631	273	274	8	133	1.000
11-02	.000	138.000	13.953	-.631	275	276	8	134	1.000
11-02	.000	139.000	13.953	-.631	277	278	8	135	1.000
11-02	.000	140.000	17.957	-.631	279	280	8	136	1.000
11-02	.000	141.000	24.734	-4.777	281	282	8	137	1.000
11-02	.000	142.000	24.734	-2.111	283	284	8	138	1.000
11-02	.000	143.000	20.984	1.609	285	286	8	139	1.000
11-02	.000	144.000	20.984	1.609	287	288	8	140	1.000
11-02	.000	145.000	20.984	1.609	289	290	8	141	1.000
11-02	.000	146.000	20.984	1.609	291	292	8	142	1.000
11-02	.000	147.000	31.609	-14.484	293	294	8	143	1.000
11-02	.000	148.000	31.609	-14.484	295	296	8	144	1.000
11-02	.000	149.000	31.609	-14.484	297	298	8	145	1.000
11-02	.000	150.000	31.609	-14.484	299	300	8	146	1.000
11-02	.000	151.000	31.609	-14.484	301	302	9	146	1.000
11-02	.000	152.000	31.609	-14.484	301	302	9	146	1.000
11-02	.000	153.000	31.609	-14.484	301	302	9	146	1.

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